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Effects of chemical reaction on moving isothermal vertical plate with variable mass diffusion

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Abstract

An exact solution to the problem of flow past an impulsively started infinite vertical isothermal plate with variable mass diffusion is presented here, taking into account of the homogeneous chemical reaction of first-order. The dimensionless governing equations are solved by using the Laplace-transform technique. The velocity and skin-friction are studied for different parameters like chemical reaction parameter, Schmidt number and buoyancy ratio parameter. It is observed that the velocity increases with decreasing chemical reaction parameter and increases with increasing buoyancy ratio parameter.

Keywords: *first order chemical reaction, moving isothermal vertical plate, mass diffusion, heat transfer, variable buoyancy ratio*

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Nomenclature

A	constants
C'	species concentration in the fluid $mol.m^{-3}$
C	dimensionless concentration
C_p	specific heat at constant pressure $J.kg^{-1}.K^{-1}$
D	mass diffusion coefficient $m^2.s^{-1}$
Gm	mass Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity $m.s^{-2}$
k	thermal conductivity $J.m^{-1}.K^{-1}$
K_l	chemical reaction parameter J
K	dimensionless chemical reaction parameter
N	buoyancy ratio parameter
Pr	Prandtl number
Sc	Schmidt number
T'	temperature of the fluid near the plate K
t'	time s
t	dimensionless time
u'	velocity of the fluid in the x'-direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
u	dimensionless velocity
y'	coordinate axis normal to the plate m
y	dimensionless coordinate axis normal to the plate

Greek symbols

α	thermal diffusivity $m^2.s^{-1}$
β	volumetric coefficient of thermal expansion K^{-1}
β^*	volumetric coefficient of expansion with concentration K^{-1}
μ	coefficient of viscosity $Pa.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid $kg.m^{-3}$
τ	dimensionless skin-friction
θ	dimensionless temperature
η	similarity parameter
$erfc$	complementary error function

Subscripts

w	conditions at the wall
∞	conditions in the free stream

1 Introduction

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to concentration (Cussler [1]). In many chemical engineering processes, there is the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production. Bourne and Dixon[2] analyzed the cooling of fibres in the formation process. Thermal boundary layer growth on continuously moving horizontal belts studied by Griffin and Throne[3].

Soundalgekar[4] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate with mass transfer. Muthucumaraswamy and Ganesan[5] have studied numerical solution of flow past an impulsively started semi-infinite isothermal vertical plate with uniform mass diffusion. Das et al.[6] have considered the effects of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique.

It is proposed to study the flow past an impulsively started infinite isothermal vertical plate in the presence of variable mass transfer by the Laplace-transform technique. In this paper, the buoyancy ratio parameter is introduced. When the temperature is high enough, buoyancy effects also generate a significant flow which aids or opposes this induced flow. There are many situations in the manufacturing industry, especially in metal forming and heat treatment, in which one encounters energy transfer to the surroundings, from a moving material.

2 Mathematical Analysis

Here the flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate with variable mass diffusion is considered. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. Initially, the plate and the fluid are of the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with uniform velocity u_0, the plate temperature is raised to T'_w and the species concentration level near the plate is made to rise linearly with time. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2}, \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\,\frac{\partial^2 T'}{\partial y'^2}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\,\frac{\partial^2 C'}{\partial y'^2} - K_l\,C', \tag{3}$$

with the following initial and boundary conditions:

$$
\begin{aligned}
& u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad for\ all \quad y', t' \leq 0; \\
t' > 0: \quad & u' = u_0, \quad T' = T'_w, \\
& C' = C'_\infty + (C'_w - C'_\infty)\ A\ t', \qquad at \quad y' = 0; \\
& u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad as \quad y' \to \infty,
\end{aligned}
\tag{4}
$$

where $A = \dfrac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$
u = \frac{u'}{u_0 Gr},\ t = \frac{t' u_0^2}{\nu},\ y = \frac{y' u_0}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},
$$

$$
Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gm = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \tag{5}
$$

$$
Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ N = \frac{Gm}{Gr},\ K = \frac{\nu K_l}{u_0^2}
$$

in equations (1) to (4), leads to

$$
\frac{\partial u}{\partial t} = \theta + N\ C + \frac{\partial^2 u}{\partial y^2}, \tag{6}
$$

$$
\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\ \frac{\partial^2 \theta}{\partial y^2}, \tag{7}
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{Sc}\ \frac{\partial^2 C}{\partial y^2} - K\ C. \tag{8}
$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & u=0, & \theta=0, & C=0, & for\ all & y,t\leq 0 \\ t>0: & u=\dfrac{1}{Gr}, & \theta=1, & C=t, & at & y=0 \\ & u=0, & \theta\to 0, & C\to 0 & as & y\to\infty. \end{array} \tag{9}$$

3 Results and Discussion

The equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc(\eta\sqrt{Pr}), \tag{10}$$

$$\begin{aligned} C = \tfrac{t}{2}\ &\Big[exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \\ &+ exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big] \\ &- \tfrac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\ \Big[exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\ &- exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})\Big], \end{aligned} \tag{11}$$

$$\begin{aligned} u \ = \ & \left(\frac{1}{Gr}+\frac{N}{a^2(1-Sc)}\right)\ erfc(\eta) + \left[\frac{t}{(Pr-1)}+\frac{Nt}{a(1-Sc)}\right] \\ & \times \left[\left(1+2\eta^2\right) erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\ exp(-\eta^2)\right] \\ & + \frac{t}{1-Pr}\left[\left(1+2\eta^2 Pr\right) erfc(\eta\sqrt{Pr}) - 2\eta\sqrt{\frac{Pr}{\pi}}\ exp(-\eta^2 Pr)\right] \\ & - \frac{N}{2a^2(1-Sc)}\Big[exp(2\,\eta\sqrt{at})\ erfc(\eta+\sqrt{at}) \\ & + exp(-2\eta\sqrt{at})\ erfc(\eta-\sqrt{at})\Big] \end{aligned}$$

$$-\frac{N(at+1)}{2a^2(1-Sc)}\Big[exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})+$$
$$+\ exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big]$$

$$+\frac{N\eta\sqrt{Sct}}{2a(1-Sc)\sqrt{K}}\ \Big[exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt})-$$
$$-\ exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})\Big]$$

$$+\frac{Nexp(at)}{2a^2(1-Sc)}\Big[exp(-2\eta\sqrt{Sc(a+K)t})\ erfc(\eta\sqrt{Sc}-\sqrt{(K+a)t})$$
$$+exp(2\eta\sqrt{Sc\ (a+K)t})\ erfc(\eta\sqrt{Sc}+\sqrt{(K+a)t})\ \Big], \qquad (12)$$

where $\eta=\dfrac{y}{2\sqrt{t}}$ and $a=\dfrac{KSc}{1-Sc}$.

The numerical values of the velocity and skin-friction are computed for different parameters like buoyancy ratio, Schmidt number, time and chemical reaction parameter. The purpose of the calculations given here is to assess the effects of the parameters Sc, K and N upon the nature of the flow and transport. The velocity profiles for different values of the Schmidt number (Sc), $Gr=1, K=2, N=0.2, Pr=0.71$ and $t=0.2$ are shown in table 1. It is clear from this table the velocity increases with increasing Schmidt number.

The effects of buoyancy ratio parameter for both aiding $(N>0)$ as well as opposing $(N<0)$, $Gr=1, K=2, Sc=0.78, Pr=0.71$ and $t=0.2$ are shown in table 2. It is observed that the velocity increases in the presence of opposing flows and decreases with aiding flows.

In table 3, the velocity profiles are shown for different values of the chemical reaction parameter(K), $Gr=1, N=0.2, Sc=0.78, Pr=0.71$ and $t=0.2$. It is observed that an decreasing values of the chemical reaction parameter leads to a fall in the velocity.

From the velocity field, the effect of mass transfer on the skin-friction is studied and is given in dimensionless form as

$$\tau=\ -\left(\frac{du}{dy}\right)_{y=0}=-\ \frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0}. \qquad (13)$$

Hence, from equation (12) and (13),

$$\tau = \frac{1}{\sqrt{\pi\, t}} \left[\frac{1}{Gr} - \frac{2t}{1+\sqrt{Pr}} + \frac{2Nt}{a(1-Sc)} \right.$$
$$- \frac{N\sqrt{\pi t Sc}}{2a(1-Sc)\sqrt{K}} erf\left(\sqrt{Kt}\right)$$
$$- \frac{N}{a^2(1-Sc)} \left(1 + \sqrt{\pi a t}\ erf(\sqrt{Kt})\right) + \frac{N}{a^2(1-Sc)} \tag{14}$$
$$- \frac{N(1+at)}{a^2(1-Sc)} \left(\sqrt{\pi K Sc t}\ erf(\sqrt{Kt}) + \sqrt{Sc}\right)$$
$$\left. + \frac{N \exp(at)}{a^2(1-Sc)} \left(\sqrt{\pi a t}\ erf(\sqrt{(K+a)t}) + \sqrt{Sc}\right)\right].$$

The numerical values of τ for $Gr = 2$ are presented in the table 4. It is observed from this table, that an decrease in the Schmidt number leads to fall in the value of skin-friction. Skin-friction increases with decreasing the chemical reaction parameter or time. It is observed that the skin-friction increases in the presence of aiding flows and decreases in the presence of opposing flows. It is observed that the skin-friction is higher in water ($Pr = 7$) than that in air ($Pr = 0.71$).

4 Conclusions

An exact analysis is performed to study the flow past an impulsively started infinite vertical isothermal plate with variable mass diffusion is found, taking into account of the homogeneous chemical reaction of first-order. The dimensionless governing equations are solved by using the Laplace-transform technique. It is observed that the velocity increases due to the presence of the foreign mass. Conclusions of the study are as follows:

1 The velocity decreases in the presence of opposing flows($N < 0$) and increases with aiding flows($N > 0$).

2 The skin-friction increases in the presence of aiding flows and decreases with opposing flows.

3 The skin-friction increases with decreasing chemical reaction parameter and decreases with increasing time.

References

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List of Tables

Table 1 Velocity for different Schmidt number

η	$Sc = 0.78$	$Sc = 0.6$	$Sc = 0.3$
0	1.0291	0.9898	0.7862
0.25	0.7609	0.7242	0.4770
0.5	0.5106	0.4704	0.1792
0.75	0.3024	0.2564	0.0764
1	0.1481	0.0961	0.0051
1.25	0.0470	0.0030	0.0001

Table 2 Velocity for different buoyancy ratio parameter

η	$N = -0.2$	$N = 0$	$N = 0.2$	$N = 0.4$	$N = 0.8$
0	0.9709	1	1.0291	1.0582	1.1163
0.25	0.7828	0.7719	0.7609	0.75	0.7281
0.5	0.5941	0.5523	0.5106	0.4689	0.3855
0.75	0.4321	0.3673	0.3024	0.2375	0.1078
1	0.3079	0.2280	0.1481	0.0682	0.0521
1.25	0.2189	0.1330	0.0470	0.0021	0.0006

Table 3 Velocity for different chemical reaction parameter

η	$K = 0.2$	$K = 2$
0	1.0291	0.8157
0.25	0.7609	0.4209
0.5	0.5106	0.0996
0.75	0.3024	0.0001
1	0.1481	0
1.25	0.0470	0

Table 4 Skin-friction

t	K	Sc	N	$Pr = 0.71$	$Pr = 7$
0.2	0.2	0.6	0.2	1.2324	1.3679
0.2	2	0.3	0.2	0.4167	0.5521
0.2	2	0.6	0.2	0.4699	0.6053
0.4	2	0.6	0.2	0.3136	0.5052
0.2	2	0.78	0.2	0.5648	0.7003
0.2	2	0.6	2	1.4867	1.6222
0.2	2	0.6	0	0.3569	0.4924
0.2	2	0.6	- 0.2	0.2439	0.3794
0.2	2	0.6	- 2	- 0.7729	0.6374

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Efekti hemijske reakcije na pokretnu izotermsku vertikalnu ploču sa promenljivom difuzijom mase

UDK 536.7

U radu je prikazano jedno egzaktno rešenje tečenja preko impulsno pokrenute beskonačne ploče sa promenljivom difuzijom mase uzimajući u obzir homogenu hemijsku reakciju prvog reda. Bezdimenzione jednačine problema su rešene tehnikom Laplasove transformacije. Brzina i skin-trenje su proučeni za različite parametre kao što je parametar hemijske reakcije, Šmitov broj i parametar odnosa potiska. Primećeno je da brzina raste sa opadanjem parametra hemijske reakcije, a raste sa porastom parametra odnosa potiska.