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Plane symmetric cosmological micro model in modified theory of Einstein's general relativity

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Abstract

In this paper, we have investigated an anisotropic homogeneous plane symmetric cosmological micro-model in the presence of massless scalar field in modified theory of Einstein's general relativity. Some interesting physical and geometrical aspects of the model together with singularity in the model are discussed. Further, it is shown that this theory is valid and leads to Einstein's theory as the coupling parameter $\lambda \to 0$ in micro (i.e. quantum) level in general.

Keywords: *anisotropy, modified Einstein's relativity, quantum micro-level*

1 Introduction

It is known that the Einstein's theory of gravitation serves as a basis for the construction of mathematical models of the Universe. In recent years researchers have taken some interesting attempts to generalize the theory of general relativity by incorporating Mach's Principle and other desired features which are lacking in the original theory. Thus

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Several modifications of Einstein's theory of general relativity are proposed by many cosmologists to unify gravitation and other effects in the Universe. Two self-creation theories are proposed by Barber [1] by modifying the Brans and Dicke [2] theory and the theory of general relativity. Both the modified theories create the Universe out of self-contained gravitational, scalar and matter fields. Brans [3] has pointed out that Barber's first theory is not only in disagreement with experiment but also inconsistent in general. Moreover, this theory severely violates equivalence principle and solution of the one-body problem revealing unsatisfactory characteristics of the theory. However, Barber's second theory of gravitation is a modification of Einstein's theory of general relativity to a variable G-theory and predicts local effects, which are within the observational limits. In this theory the Newtonian gravitational parameter G is not a constant but a function of time parameter 't'. Further, the scalar field does not gravitate directly but simply divides the matter tensor acting as a reciprocal gravitational constant. Moreover, this theory is capable of verification or falsification. It can be done by observing the behavior of both bodies of degenerate matter and photons. An observation of anomalous precessions in the orbits of pulsars about central masses and an accurate determination of the deflection of light and radio waves passing close to the sun would verify or falsify such a theory and determines λ. The theory predicts the same precession of the perihelia of the planets as general relativity and in that respect agrees with observation to within 1%. In the limit $\lambda \to 0$ this theory approaches the Einstein's theory in every respect. Many authors have studied Barber's second theory in various angles. Pimentel [4] has solved the Friedmann-Barber field equations under the assumptions of power law dependence of the scalar field on the scale factor. Soleng [5],[6] has generalized the work of Pimentel [4] and got solutions for the vacuum dominated, radiation dominated and dust filled universe of the flat FRW space-time. Reddy and Venkateswaralu [7] have got Bianchi type VI_0 cosmological solutions both in vacuum and in presence of perfect fluid with pressure equal to energy density. Venkateswarlu and Reddy [8] have also got spatially homogeneous and anisotropic Bianchi type-I cosmological macro models when the source of gravitational field is a perfect fluid. Shanti and Rao [9] have also got spatial homogeneous and anisotropic Bianchi type-II and III cosmological models both in vacuum and in presence of stiff fluid. Carvalho [10] has obtained a homogeneous and isotropic model of the early universe in which parameter gamma of 'gamma law' equation of state varies continuously with cosmological time. Also he has presented a unified descriptions of early universe for inflationary period and radiation dominated era. Shri Ram and Singh [11] have obtained spatially homogeneous and isotropic R-W model of the universe in the presence of perfect fluid by using 'gamma law' equation of state. Mohanty *et. al.,* [12] have obtained vacuum and Zeldovich fluid models for plane symmetric anisotropic homogeneous space-time. Recently Mohanty *et. al.* [13],[14] have obtained an anisotropic homogeneous Bianchi type-I cosmological micro model in Barber's second theory of gravitation wherein this scalar field describe the elementary particles and their interactions (Srivastav and Sinha [15]). Also they have obtained a micro and macro cosmological model in the presence of massless scalar field interacted with perfect fluid. Further Panigrahi and Sahu [16,17] have obtained plane symmetric inhomogeneous cosmological macro models and plane symmetric mesonic stiff fluid models in Barber's second theory of gravitation.

Till now, no author has studied the consistency of Barber's second theory for the anisotropic homogeneous plane symmetric in micro level. Thus in the paper we intend to construct a homogeneous anisotropic plane symmetric cosmological micro (i.e. quantum) model with minimal coupling of the scalar field and gravity. Also we intend to study some of the aspects of the Barber's second theory of gravitation in view of its consistency. In section 2, we derive the set of field equations and construct the micro model of the universe in section 3. In section 4 we study some physical and geometrical behavior of the solutions and section 5 embodies the conclusion.

2 Field equations

We assume the space time described by a metric of the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2 \tag{1}$$

where A and B are functions of cosmic time $'t'$.

The Einstein-Barber-micro field equations in second self creation theory are

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T^v_{ij} \tag{2}$$

and

$$\Box\phi = \frac{8\pi}{3}\lambda T^v \tag{3}$$

where $'\phi'$ is the Barber's scalar, T^v_{ij} is the energy momentum tensor for massless scalar field, $\Box\phi$ is the invariant D'Alembertian, T^v is the trace of the energy momentum tensor T^v_{ij} and λ is a coupling constant to be determined from experiment where $|\lambda| < 1/10$. In the limit $\lambda \to 0$,this theory approaches the Einstein's theory in every respect. Due to the nature of the space time Barber's scalar ϕ is a function of $'t'$.

In order to study the cosmological effects in microscopic level the energy momentum tensor T^v_{ij} (Singh and Deo, [18]) for a micro (i.e. quantum) matter field representing massless scalar field distribution is given by

$$T^v_{ij} = v_i v_j - \frac{1}{2}g_{ij}v_k v^k \tag{4}$$

together with

$$g^{ij}v_{ij} = \sigma. \tag{5}$$

Here the scalar field v and the source density σ are both functions of cosmic time t only. The semicolon (;) denotes the covariant differentiation. Using equation (4), the set of field equations (2) and (3) for the space time (1) reduces to the following explicit forms :

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} = -4\pi\phi^{-1}v_4^2, \tag{6}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} = -4\pi\phi^{-1}v_4^2, \tag{7}$$

$$\frac{A_4^2}{A^2} + \frac{2A_4B_4}{AB} = 4\pi\phi^{-1}v_4^2, \tag{8}$$

$$\phi_{44} + \phi_4\left[\frac{2A_4}{A} + \frac{B_4}{B}\right] = \frac{-8\pi\lambda v_4^2}{3}. \tag{9}$$

Hereafter, the index 4 after a field variable denotes ordinary differentiation with respect to time t.

3 Solutions and the micro model

Adding two times of equation (6) with eqn. (7) and three times of eqn. (8), one can get

$$\frac{(A^2B)_{44}}{A^2B} = 0 \tag{10}$$

which is equivalent to

$$(A^2B)_{44} = 0. \tag{11}$$

On integration, eqn. (11) yields

$$A^2B = \alpha_1 t + \alpha_2 \tag{12}$$

where $\alpha_1(\neq 0)$ and α_2 are arbitrary constants of integration.

Now transforming the time co-ordinate $'t'$ by putting $\alpha_1 t + \alpha_2 = T$, equations (12) can be reduced to the form

$$A^2B = T. \tag{13}$$

From eqn. (13) we can write the explicit form for A and B as

$$A = T^{n_1}, \tag{14}$$

$$B = T^{n_2} \tag{15}$$

where $n_i, i = 1, 2$ are two real constants satisfying the relation $2n_1 + n_2 = 1$.

With the help of eqn. (13), eqn. (9) can be transformed to the form.

$$\phi_{TT} + \frac{\phi_T}{T} + p_1^2 v_T^2 = 0 \tag{16}$$

where we put $p_1^2 = \frac{8\pi\lambda}{3}, |\lambda| < \frac{1}{10}$ and $\frac{d\phi}{dT} = \phi_T$. Using Eqn. (14)and (15)in eqn. (8), we obtain

$$v_T^2 = \frac{[n_1^2 + 2n_1 n_2]\phi}{4\pi T^2}. \tag{17}$$

Now using the value of v_T from eqn. (17) in equation (16) , we get

$$T^2\phi_{TT} + T\phi_T + p^2\phi = 0 \tag{18}$$

where $p^2 = \frac{2\lambda}{3}(n_1^2 + 2n_1 n_2)$ and $n_1^2 + 2n_1 n_2 > 0$.

On integration, eqn (18) yields two basic solutions for ϕ i.e.

$$\phi_1 = \cos(p \ln T), \tag{19}$$

$$\phi_2 = \sin(p \ln T) \tag{20}$$

where $p = \pm\sqrt{\frac{2\lambda}{3}(n_1^2 + 2n_1 n_2)}$ and $0 < \lambda < 10^{-1}$. The second value of ϕ given in eqn. (20) is not accepted as it leads to unphysical situation.

Integrating eqn. (17), we obtain

$$v = p_2 \int \frac{\sqrt{\phi}}{T} dT + \alpha_3 \tag{21}$$

where α_3 is a constant of integration and $p_2 = \sqrt{\frac{n_1^2+2n_1n_2}{4\pi}}$.

With the help of eqn. (19), eqn. (21) yields

$$v = p_2[lnT - \frac{p^2}{12}(lnT)^3 - \frac{p^4}{480}(lnT)^5 - ...] + \alpha_3. \tag{22}$$

In new coordinate system, the source density σ of the scalar field v given by eqn. (5) for the space time (1) reduces to

$$\sigma = \alpha_1^2 (v_{TT} + \frac{v_T}{T}). \tag{23}$$

With the help of eqn. (21), eqn. (23) reduces to

$$\sigma = \frac{p_2 \alpha_1^2}{2} . \frac{\phi_T}{T\sqrt{\phi}}. \tag{24}$$

Using eqn. (19) in eqn (24), we find

$$\sigma = \frac{-pp_2 \alpha_1^2}{2T^2} . \frac{\sin(p\ lnT)}{\sqrt{\cos(p\ lnT)}}. \tag{25}$$

The energy density associated with the scalar field (Anderson, [19]; Mohanty and Pradhan, [20]) is given by

$$\rho = \frac{1}{2} v_4^2, \tag{26}$$

which is equivalent to (in transformed coordinate)

$$\rho = \frac{1}{2} \alpha_1^2 v_T^2. \tag{27}$$

Now using eqn. (17) and (19) in eqn. (27), we obtain

$$\rho = \frac{\alpha_1^2 p_2^2}{2T^2} [\cos(p\ lnT)]. \tag{28}$$

Thus the anisotropic homogeneous plane symmetric cosmological micro model in Barber's second self-creation theory for the space time (1) is given by

$$dS^2 = dT^2 - T^{2n_1}(dX^2 + dY^2) - T^{2n_2} dZ^2. \tag{29}$$

4 Some physical and geometrical behavior of the solutions

The Barber's scalar ϕ, the scalar field v, the source density σ and the energy density ρ are functions of time parameter t and given by equations (19) , (22), (25) and (28) respectively.

(i) When $T \to 0$ or ∞, then the quantities ϕ, v, σ and ρ are undetermined. The metric potentials A and B tend to zero as $T \to 0$. Hence the space time collapses at $T = 0$ and admits a singularity at $t = \infty$. From (22) we see that v is a logarithmic function of time. Thus "Big-bang" of the universe can be avoided by introducing a massless scalar field.

(ii) When $T = 1$ then $\phi = 1, v =$ a constant , $\sigma = 0$ and $\rho =$a constant. For physically acceptable mesonic field we have $\rho > 0$. This situation leads to the only possibility with $T > 0$. Again for the realty of σ it is necessary that $T > 0$ and $p_2 < 0$. Also when $T = 1$ then the metric potentials A and B becomes constant and in this case the space time reduces to a flat space time. Thus from above results it is inferred that when $T = 1$ the micro model of the universe does not exist.

(iii) When the coupling parameter $\lambda \to 0$ $(T \neq 0$ or $\infty)$ then $\phi \to 1, v \to$ a constant, $\sigma \to 0$ and $\rho \to a$ constant. These results are also obtained in case (ii) and clearly show that second self creation theory leads to Einstein theory as $\lambda \to 0$. Also we conclude that both the scalar field v and Barber's scalar ϕ exhibit singularity at $T = \infty$. Since ϕ is a trigonometric function, it is periodic. For $\lambda > 0$, in the first and second quarter of the principal period ϕ_1 is decaying. However in the third and fourth quarter of the principal period ϕ_1 is growing.

(iv) Here the spatial volume is given by $V = T$. As $T \to 0, V \to 0$ and as $T \to \pm\infty, V \to \pm\infty$. These results show that the universe starts expanding with zero volume and blows up at infinite past and future. Also the model (29) admits expansion and contraction of the universe with time in different directions depending on the signature of the quantities n_i, $i = 1, 2$ involved in the model.

The scalar expansion θ is calculated as $\theta = \frac{\alpha_1}{T}$. From this relation it is evident that $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. Thus the universe is expanding with increase of time but the rate of expansion becomes slow as time increases.

(v) An observer viewing the universe from any vantage point will find that it does not look the same in all directions then this property is known as anisotropy. The assumption of anisotropy at every point of space time requires that the metric of the space-time be other than the spherically symmetric. The shear scalar or anisotropy (Raychoudhuri, [21]) defined by

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right]$$

for the micro model (29) yields $\sigma^2 = \frac{2\alpha_1^2}{3T^2}(n_1 - n_2)^2$. This equation shows that $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$.Thus the shape of the universe changes uniformly in x and y directions only and the rate of change of the shape of the universe becomes slow with increase of time. It is observed that anisotropy $\sigma^2 = \frac{\alpha}{T^2}$ exists throughout the evolution except for $n_i = \frac{1}{3}, i = 1, 2$. It is also observed that

$$\lim_{T\to\infty} \frac{\sigma}{\theta} \approx 0.816, (n_i \neq \frac{1}{3}, i = 1, 2)$$

which confirms that the universe remains anisotropic throughout the evolution. This can be seen by considering the present day observational limits in the temperature anisotropy. But when $n_i = \frac{1}{3}, i = 1, 2$ then $\sigma^2 = 0$ and $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$. This relation confirms that the space time is isotropic. By an indirect argument Collins et.al. [22] have obtained that the upper limit for $\frac{\sigma}{\theta}$ relating to isotropy of primordial black body radiation is 10^{-3}. However it is evident from above analysis that the freedom is more in Barber's second theory.

The Weyl curvature is supposed to refer to free gravitational field. It has significant relation with anisotropy i.e. anisotropy of space time would require Weyl curvature to be non-zero. More clearly anisotropic models are conformally non-flat and hence Weyl curvature is non-zero. However, the standard FRW models are isotropic and homogeneous. Thus these are conformally flat with Weyl curvature zero. For the model (29) the non-vanishing components of Weyl curvature tensor are given by

$$\begin{aligned} w_{1212} &= \frac{\alpha_1^2}{3T^{2n_2}}[n_1 - n_2][n_2 - 1], \\ w_{1313} = w_{2323} &= \frac{-\alpha_1^2}{6T^{2n_1}}[n_1 - n_2][n_2 - 1], \\ w_{1414} = w_{2424} &= \frac{\alpha_1^2}{6T^{1+n_2}}[n_1 - n_2][n_2 - 1], \\ w_{3434} &= \frac{-\alpha_1^2}{3T^{4n_1}}[n_1 - n_2][n_2 - 1]. \end{aligned} \tag{30}$$

From above we see that Weyl curvature is non-zero except the case $n_i = \frac{1}{3}, i = 1, 2$. Thus the space time is anisotropic through out the evolution. However, the space time is isotropic when $n_i = \frac{1}{3}, i = 1, 2$. It is interesting to note that as $T \to 0$ Weyl curvature tends to ∞ and as $T \to \infty$ Weyl curvature tends to zero. Thus it is inferred that the universe had a highly anisotropic singular state at $T = 0$ but it leads to isotropic state for larger value of T.

As Weyl curvature is non-zero, the anisotropy of the universe exists. If the anisotropy of the universe is increases then it will affect the equilibrium of thermodynamic system of the universe. Ultimately the entropy in the universe will increase. If the universe will reach a maximum value of entropy then no work will be possible in the universe. As a result the destruction of the universe may occurs. As the rotation ω turns out to be zero the model is non-rotating.

5 Conclusion

In this paper we have presented the anisotropic homogeneous plane symmetric micro model in Barber's second theory. It is observed that the Big-bang of the universe at initial stage can be avoided by introducing a massless scalar field. Also it is observed that the model is expanding, shearing and non-rotating incase of $n_i \neq \frac{1}{3}, i = 1, 2$. However, the model is expanding, non shearing and non-rotating when $n_i = \frac{1}{3}, i = 1, 2$. Further, it is observed that the universe had a highly anisotropic singular state at initial stage of the evolution and approaches to isotropy for larger value of T. It is important to note that Barber's scalar $\phi \to 1$ as the coupling constant $\lambda \to 0$ ($T \neq 0$ or ∞). Also when $\lambda \to 0, p \to 0$ and consequently the massless scalar field $v_B \to v_E$ and the source density $\sigma_B \to \sigma_E$, where the physical quantities with subscripts B and E represent those in Barber and Einstein theories respectively. Thus Barber's second theory is valid in all respects and leads to Einstein's theory as $\lambda \to 0$ in micro level. The model (29) may be useful in self creation cosmology for the study of dynamics in quantum level.

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Ravanski simetrični kosmološki mikromodel u modifikonanoj Ajnštajnovoj teoriji opšte relativnosti

UDK 530.12

U ovom radu u okviru modifikonane Ajnštajnove teorije opšte relativnosti proučavamo anizotropni homogeni ravanski simetrični kosmološki mikromodel u prisustvu skalarnog polja bez mase. Diskutovani su neki interesantni fizički i geometrijski aspekti ovog modela zajedno sa njegovog singularnošću. Dalje, pokazano je u opštem slučaju da ova teorija važi i da vodi ka Ajnštajnovoj teoriji kada parametar sprezanja iščezava, $\lambda \to 0$, na mikro (tj. kvantnom) nivou.