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Mobility of the compliant joints and compliant mechanisms

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Abstract

This paper deals with a mobility of the single compliant joints and entire compliant mechanisms. The compliance of the joints and the mobility of the compliant mechanism can be increased by variations of geometry as well as material type of the joints. Therefore the mobility of three kinds of compliant joints with different geometrical shapes (beam joints, film joints and notch joints) will be researched. The mobility of the compliant joints made of different material types (plastic joints, silicone joints) will be also researched. The mobility of compliant mechanisms will be analyzed by using of the Roberts-Tchebicheff mechanism which coupler point can be guided on an approximately rectilinear path.

Keywords: compliant joints, compliant mechanism, mobility

1 Introduction

Compliant or flexible-link mechanisms gain their mobility due to relative flexibility of their compliant joints [1]. There are many advantages of

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using the compliant joints in the mechanism structure: a mechanism can be built in one piece, the weight can be reduced and wear, clearance, friction, noise and need for lubrication can be eliminated. Therefore they are suitable to be applied in micromachining [3]. On the other hand, the mechanisms with compliant joints can realize relatively small displacements, that is, their mobility is limited.

There are many papers considering the structure and function of the compliant joints and compliant mechanisms. The paper [5] established basic nomenclature and classification for the components of compliant mechanisms. The paper [6] introduced a method to aid in the design of a class of compliant mechanisms wherein the flexible sections (flexural pivots) are small in length compared to the relatively rigid sections. The paper [7] presented a formal structural optimization technique called the homogenization method in order to design flexible structures (compliant mechanisms). The paper [9] introduced new ideas of technically realizable joints from nature and their integration into elastically movable structures for motion tasks in positioning and manipulating engineering.

Some papers have analyzed the influence of the geometry, as well as the material type of the compliant joints on the guiding accuracy of the compliant mechanisms [4,10,12].

The paper [8] introduced a method for determining the limit positions of compliant mechanisms for which an appropriate pseudo-rigid-body model may be created. However, there is no paper dealing with the mobility of any kind of compliant joint, that is, of any compliant mechanism.

This paper deals with the influence of the geometry, as well as the material type of the compliant joints on a mobility of the single compliant joints and entire compliant mechanisms. The aim of the paper is to determine the limits of mobility of different kinds of the compliant joints in order to aid in the design of the compliant mechanisms.

2 Influence of the geometry of compliant joints on their mobility

Figure 1 shows a rigid link with revolute joint. The link is being able to rotate without limits, that is, the angle of rotation is $\varphi = 360^o$.

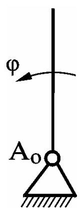

Figure 1: A rigid link with revolute joint (A_0)

Figure 2 shows the links of compliant mechanisms, consisting of relatively rigid section and relatively elastic section (compliant joint), with three different kinds of compliant joints: beam joint (Figure 2a), notch joint (Figure 2b) and film joint (Figure 2c).

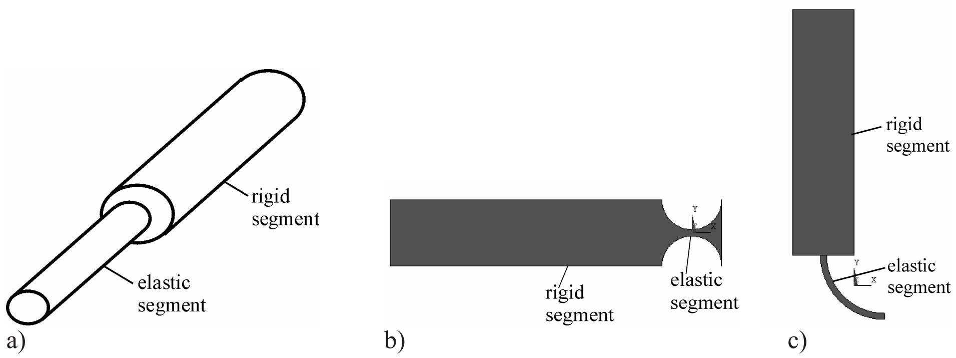

Figure 2: A link of compliant mechanism with: beam joint (a), notch joint (b) and film joint (c)

These links are moveable due to flexibility of their elastic segments. However, their mobility is limited. We will research the limits of their mobility for each presented kind of compliant joint by using of ANSYS Software. The links are assumed to be made of piacryl (modulus of elasticity $E = 3700N/mm^2$, bending strength $\sigma_{bs} = 90N/mm^2$). Maximal bending stress $\sigma_{max} < \sigma_{bs}$ determines constraint positions of the link, that is, the limits of angular displacement (mobility) and maximal bending force.

The displacement calculation and stress analysis of the link with compliant beam joint have been performed for the beams with round cross-section (Figure 3). The diameters of the elastic segment and rigid segment have been denoted with d and D respectively, while the length of the elastic segment has been denoted with l. The length of the entire link with beam compliant joint has been denoted with a.

Two-dimensional Elastic Beam (Figure 4) has been used as a characteristic ANSYS element type in the calculation procedure. The element has three degrees of freedom at each node (I, J): translations in the nodal x- and y-directions and rotation about the nodal z-axis.

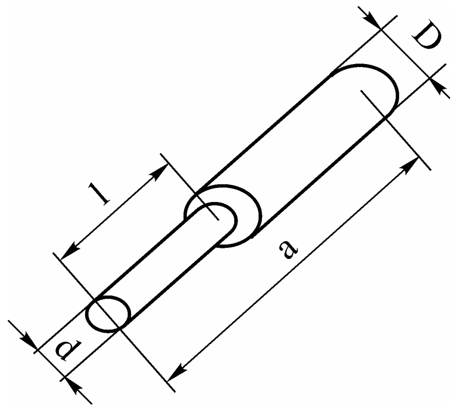

Figure 3: The parameters of the link with compliant beam joint

Figure 5 shows a link with compliant beam joint being stressed by the horizontal force F acting at the end point of the link. Deformed position of the link has been presented with dashed line. For maximal bending force F = 0.18N, that are determined by maximal bending stress less than bending strength ($\sigma_{max} < \sigma_{bs}$), and parameters a = 50mm, D = 10mm, d = 1mm, l/a = 0.1, we have obtained maximal angular displacement of $\Delta\varphi = 12.5^o$. For the force values $F = \pm 0.18N$ the entire angular mobility of this link is $2\Delta\varphi = 25^o$ (Table 1).

The displacement calculation and stress analysis of the link with

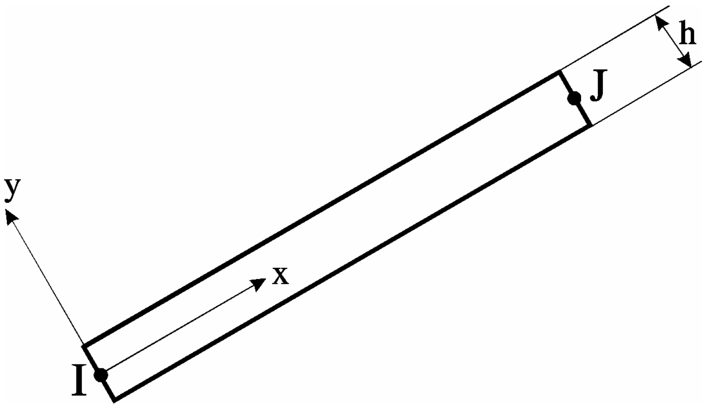

Figure 4: Two-dimensional Elastic Beam

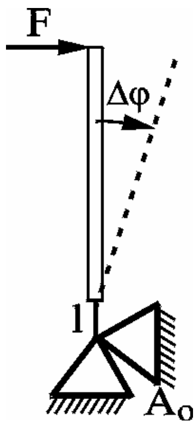

Figure 5: The mobility of the link with compliant beam joint

compliant film joint have been performed for the elements with rectangular cross-section. The widths of the elastic segment and rigid segment have been denoted with w_E and w_R respectively, while the ,,length“ of the elastic segment has been denoted with l (Figure 6). The length of the entire link with compliant film joint has been denoted with a.

Two-dimensional-eight-node Structural Solid (Figure 7) has been used as a characteristic ANSYS element type in the calculation procedure. The element is defined by eight nodes (I, M, J, N, K, O, L, P) having two degrees of freedom at each node: translation in the nodal x- and y-directions.

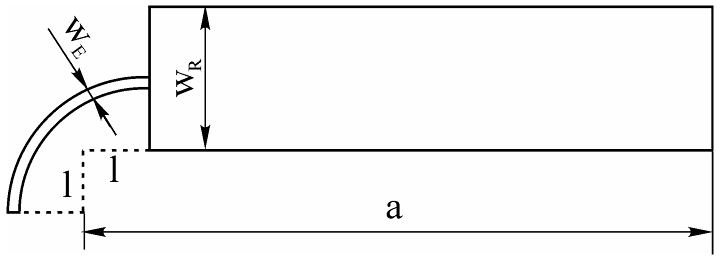

Figure 6: The parameters of the link with compliant film joint

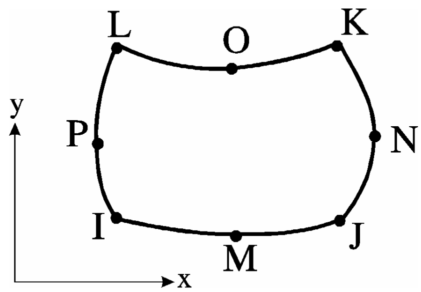

Figure 7: Two-dimensional-eight-node Structural Solid

Figure 8 shows an undeformed and a deformed position of the link with compliant film joint being stressed by the vertical force F acting at the end point of the link. For parameters $a = 50mm$, $w_R = 10mm$, $w_E = 1mm$, material thickness $\delta = 4mm$, $l/a = 0.1$ and maximal bending force $F = 1.25N$ determined by maximal bending stress less than bending strength ($\sigma_{max} < \sigma_{bs}$), we have obtained maximal angular displacement of $\Delta\varphi = 38.7^o$. For the force values $F = \pm 1.25N$ entire angular mobility of this link is $2\Delta\varphi = 77.4^o$ (Table 1).

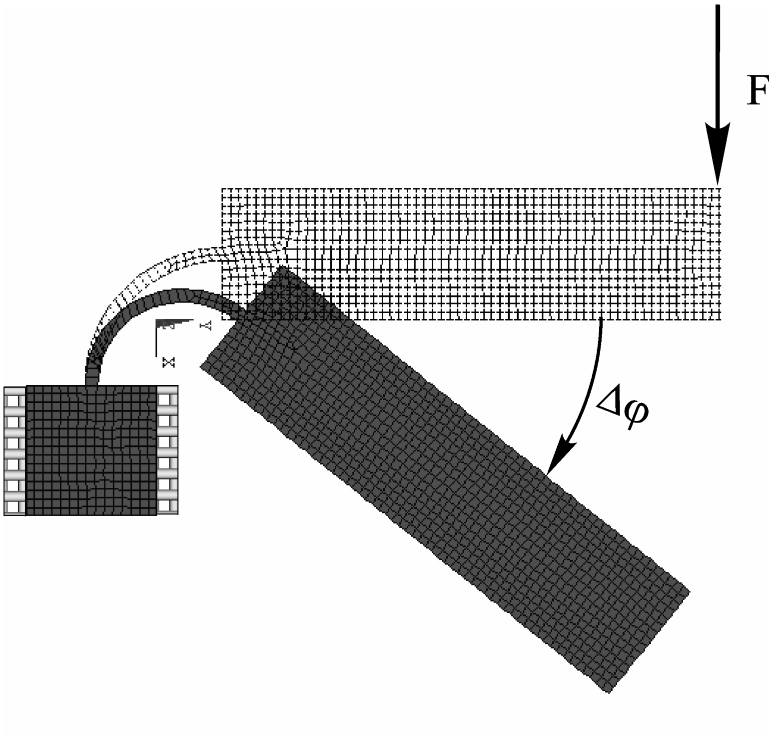

Figure 8: The mobility of the link with compliant film joint

The displacement calculation and stress analysis of the link with compliant notch joint have been performed for the elements with rectangular cross-section. The widths of the elastic segment and rigid segment have been denoted with w_E and w_R respectively (Figure 9a). The length of the entire link with compliant notch joint has been denoted with a. Two-dimensional-eight-node Structural Solid (Figure 7) has been used as a characteristic ANSYS element type in the calculation procedure.

Figure 9b shows an undeformed and a deformed position of the link with compliant notch joint being stressed by the vertical force F acting at the end point of the link. For parameters $a = 50mm$, $w_R = 10mm$, $w_E = 1mm$, material thickness $\delta = 4mm$ and maximal bending force $F = 1.24N$ determined by maximal bending stress less than bending

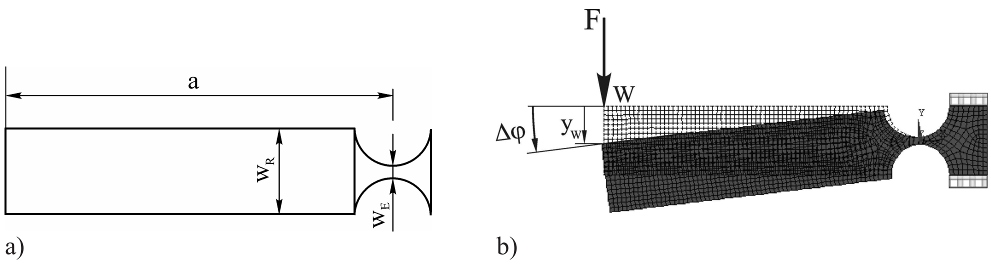

Figure 9: The parameters and mobility of the link with compliant notch joint

strength ($\sigma_{max} < \sigma_{bs}$), we have obtained maximal angular displacement of $\Delta\varphi = 6.9^o$. For the force values $F = \pm 1.24N$ entire mobility of this link is $2\Delta\varphi = 13.8^o$ (Table 1).

Compliant joint type	±F[N]	$2\Delta\varphi[^o]$
beam	0.18	25
film	1.25	77.4
notch	1.24	13.8

Table 1: Maximal bending force F and mobility $2\Delta\varphi$ of the links with compliant joints

3 Experimental determination of mobility of the link with a compliant notch joint

We have experimentally analyzed the bending strength of a sample of the link with compliant notch joint (Figure 9b) [4,12]. The sample was made of piacryl. We have loaded the sample at the point W with the different weight mass m (Figure 10), that is, with the different bending moments M = mgx, where $x = 40mm$ is the length of the sample. The vertical displacement of the point W (Δy_W) has been measured by an inductive displacement transducer (Figure 10).

We have also made an ANSYS model of the sample and numerically obtained the values of the displacement Δy_W and maximal bending

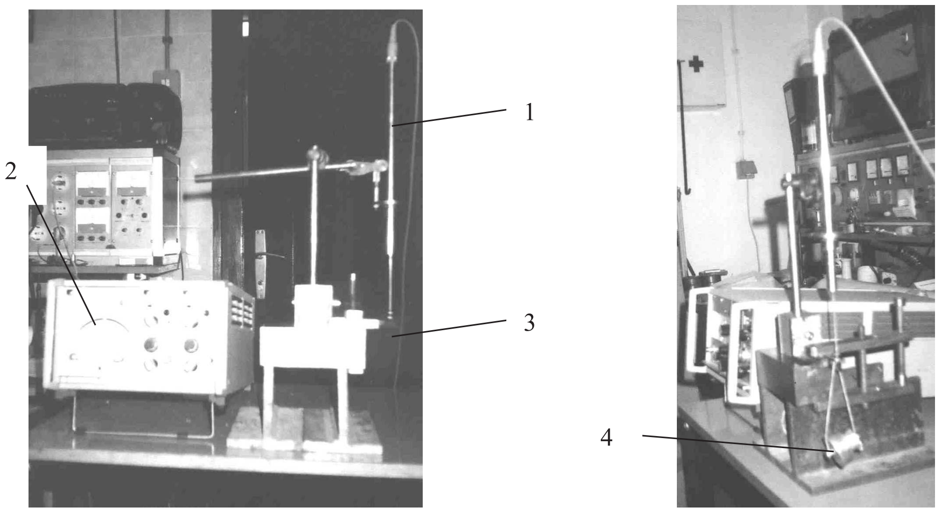

Figure 10: Experimental rig for determining of the bending strength of the sample of the link with the compliant notch joint (1 - inductive displacement transducer, 2 - amplifier, 3 - sample, 4 - weight)

stress σ_{max}. Experimental results have been compared with the results obtained by ANSYS Software (Figure 11).

The experimental analyze shows that for:

- $\sigma_{max} = 0..65N/mm^2$ $(M = 0..150Nmm)$ material piacryl shows elastic properties and the experimental results fit to the numerical results;

- $\sigma_{max} = 65..90N/mm^2$ $(M = 150..230Nmm)$ the material shows hyper elastic properties and the experimental results are a little bit greater than the numerical results;

- $\sigma_{max} > 90N/mm^2$ $(M > 230Nmm)$ the material begins to be plastically deformed, that is, creeping of material appears and the experimental results are considerably greater than the numerical results.

The comparison of experimentally and numerically obtained results proves that ANSYS Software can be used in displacement and stress calculation of the links with compliant joint and compliant mechanisms in the elastic and hyper elastic area of the bending strain.

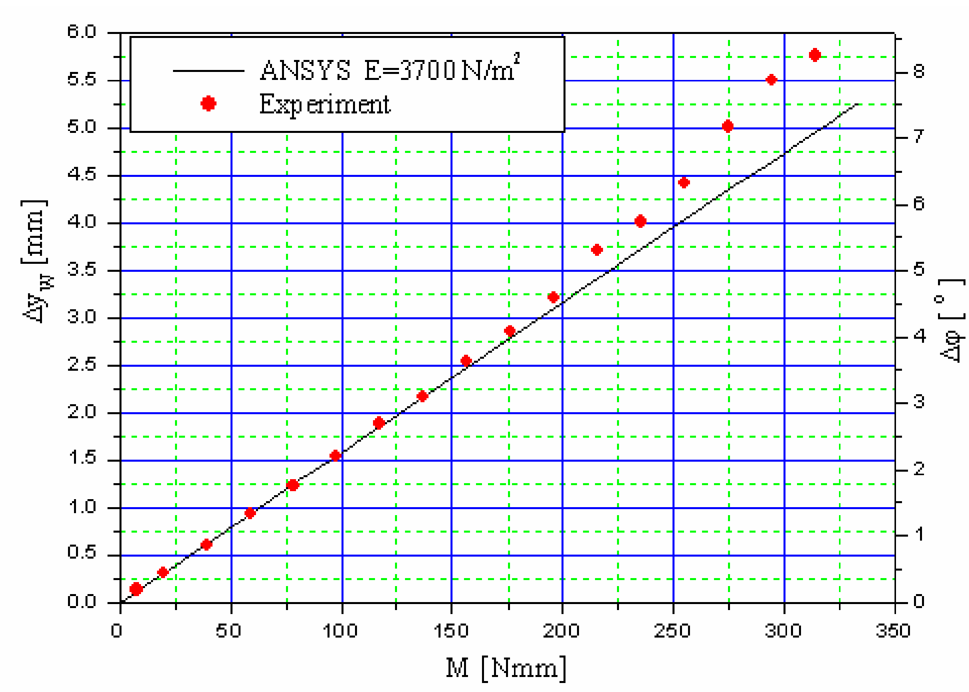

Figure 11: Experimentally and numerically obtained results of the displacement Δy_W

4 Influence of the material type of compliant joints on their mobility

The compliance of the joints and the mobility of the compliant mechanism can be also increased by alteration of the material type of the joints. If the compliant joints have been made of silicone (Figure 12) (modulus of elasticity $E_2 = 1.3 N/mm^2$, bending strength $\sigma_{bs} = 7.9 N/mm^2$ [11]), while the links of the mechanism have been made of some other material with greater rigidity (modulus of elasticity E_1 on Figure 13), maximal permissible bending stresses will be considerably decreased and at the same time the mobility of entire compliant mechanism will be considerably increased.

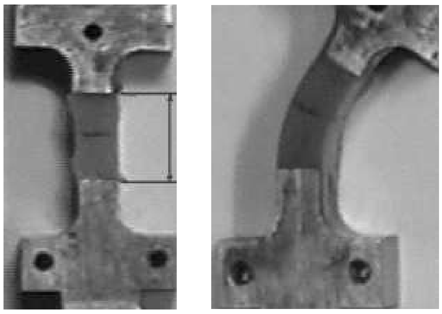

Figure 12: Laboratory model of silicone compliant joint [9]

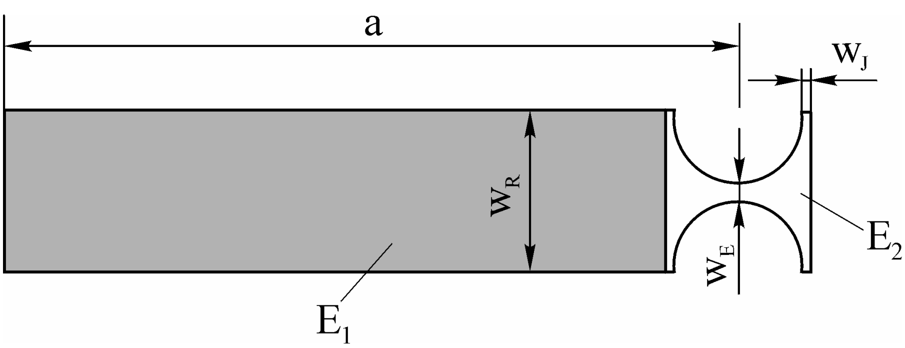

Figure 13: Compliant silicone notch joint

For parameters $a = 50mm, w_R = 10mm, w_E = 1mm, w_J = 0.5mm,$

material thickness $\delta = 4mm$ and maximal bending force F = 0.003N determined by maximal bending stress less than bending strength($\sigma_{max} < \sigma_{bs}$), we have obtained maximal angular displacement of $\Delta\varphi = 43.2^o$. For the force values $F = \pm 0.003N$ entire mobility of this link is $2\Delta\varphi = 86.4^o$. Table 2 shows the maximal bending force F and mobility $2\Delta\varphi$ of the links with compliant plastic (made of piacryl) and silicone notch joints.

Compliant joint type	±F[N]	$2\Delta\varphi[^o]$
plastic notch	1.24	13.8
silicone notch	0.003	86.4

Table 2: Maximal bending force F and mobility $2\Delta\varphi$ of the links with compliant plastic and silicone notch joints

5 Mobility of compliant mechanisms

The mobility of compliant mechanisms will be analyzed by using of the *Roberts-Tchebicheff* four-bar linkage (Figure 14), which coupler point C can be guided on an approximately rectilinear path for the values of the input crank rotation angle $\varphi = -5^o \div 43^o$. The link length ratio should be [2]:

$$\begin{aligned}
&a = \overline{A_0A}\\
&b = \overline{B_0B} = a\\
&c = \overline{AB} = 0.85a\\
&d = \overline{A_0B_0} = 2.44a\\
&\overline{AC} = \overline{BC} = 1.105a.
\end{aligned}$$

We have designed and manufactured the compliant counterparts of the *Roberts-Tchebicheff* rigid-body four-bar linkage [4]. Figure 15 shows the compliant counterpart of *Roberts-Tchebicheff* four-bar linkage with the beam joints (a), film joints (b) and notch joints (c).

These compliant mechanisms are moveable due to force acting at the point S, located in the middle of the input crank. The length of the input link is $a = \overline{A_0A} = 50mm$ while the undeformed position of the mechanism (symmetrical position) is determined by $\varphi = 37.345^o$.

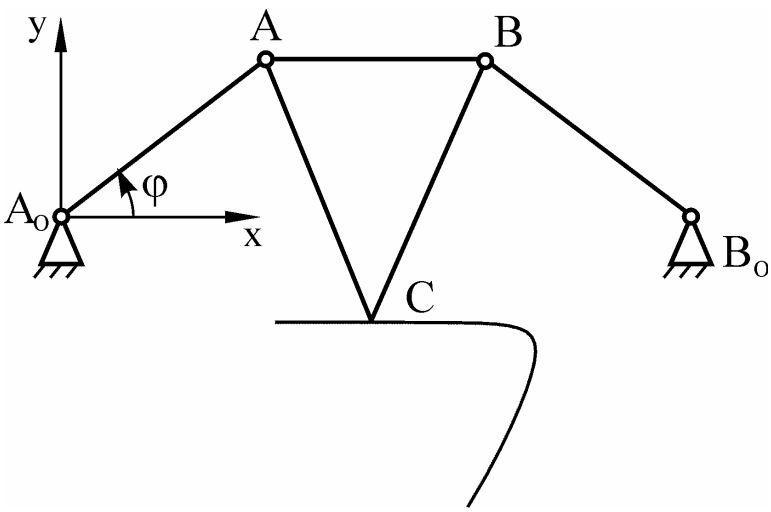

Figure 14: *Roberts-Tchebicheff* four-bar linkage for rectilinear guiding of the coupler point C

The parameters of the links and joints of these compliant mechanisms are:

- compliant mechanism with beam joints: $D = 10mm$, $d = 1mm$, $l/a = 0.1$;
- compliant mechanism with film joints: $w_R = 10mm$, $w_E = 1mm$, $\delta = 4mm$, $l/a = 0.1$;
- compliant mechanism with notch joints: $w_R = 10mm$, $w_E = 1mm$, $\delta = 4mm$.

Table 3 shows the maximal bending force F and mobility $2\Delta\varphi$ of these compliant mechanisms. These results are obtained numerically by using the ANSYS Software.

6 Conclusion

Introducing of compliant joints in the mechanism structure is desirable, because compliant mechanisms have less weight, wear, clearance, friction and noise than their rigid-body counterparts. On the other hand, the

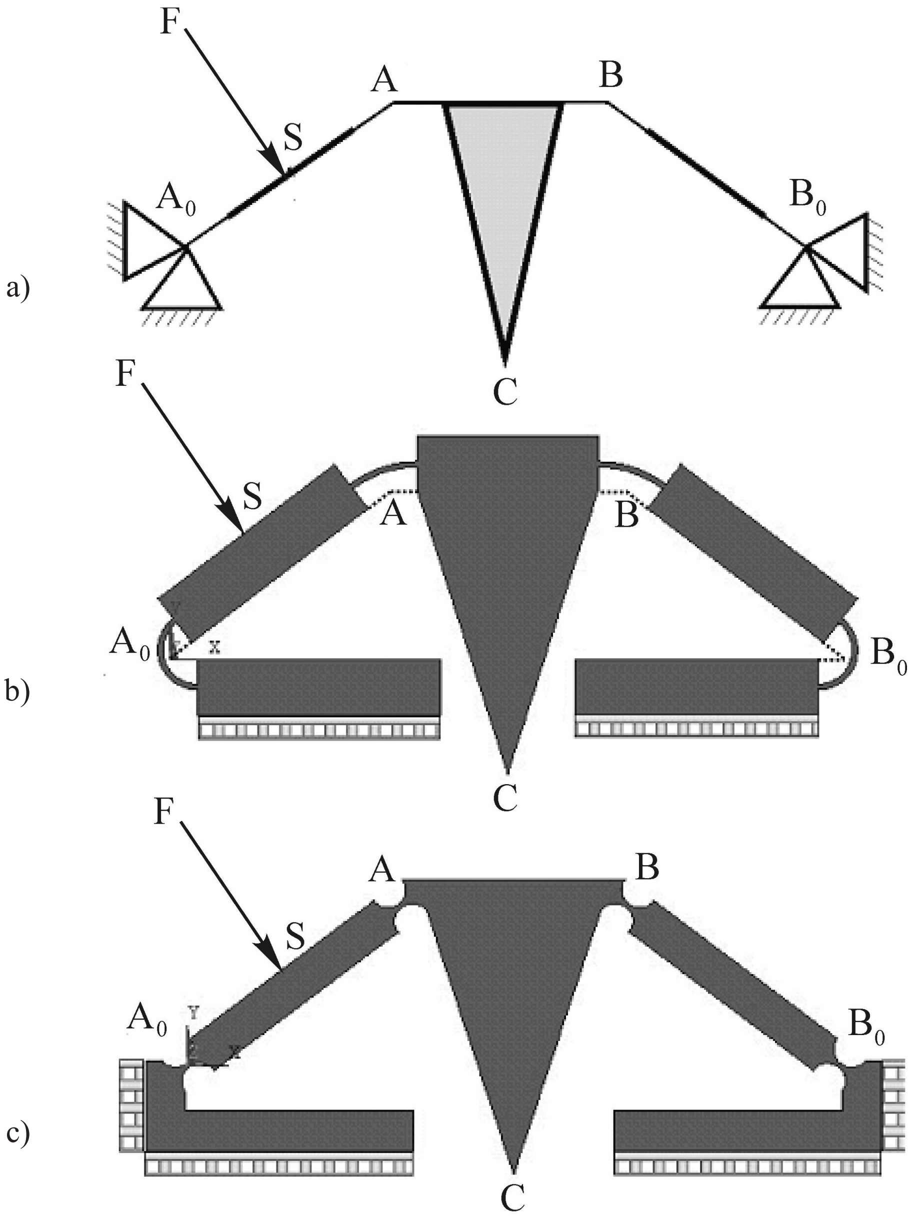

Figure 15: Compliant counterparts of *Roberts-Tchebicheff* four-bar linkage with the beam joints (a), film joints (b) and notch joints (c)

Compliant joint type	$\pm$F[N]	$2\Delta\varphi[^o]$
beam	1.45	17
film	6.10	20.6
notch	12.2	5.2

Table 3: Maximal bending force F and mobility $2\Delta\varphi$ of the compliant *Roberts-Tchebicheff* four-bar linkages

mobility of the compliant mechanisms is limited, that is, they can realize relatively small displacements.

In this paper we have analyzed the mobility of the single link with compliant beam, film and notch joint, as well as the mobility of entire compliant mechanisms (*Roberts-Tchebicheff* compliant four-bar linkages for rectilinear guiding). On the basis of the results numerically obtained by using the ANSYS Software it can be concluded that:

- the single link (as well as the entire compliant mechanism) with film joint has greater mobility than the one with beam or notch joint,
- the single link with beam joint can stand considerably smaller values of bending forces than the one with film or notch joint,
- the compliant mechanisms with silicone joints have greater mobility than the ones with plastic joints; however, the compliant mechanism with plastic joints can be built in one piece of material, and therefore it is easier to be manufactured than the compliant mechanism with silicone joints.

We have also experimentally analyzed the bending strength of a sample of the link made of piacryl with notch joint. The comparison of experimentally and numerically obtained results has proved that ANSYS Software can be used in displacement and stress calculation of the links with compliant joint and compliant mechanisms in the elastic and hyper elastic area of the bending strain.

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Pokretljivost gipkih zglobova i gipkih mehanizama

UDK 531.132.2

U radu je razmatrana pokretljivost pojedinačnih gipkih zglobova i integralnih gipkih mehanizama. Gipkost zglobova i pokretljivost gipkog mehanizma mogu se povećati promenom geometrije, kao i promenom materijala od koga je zglob izradjen. Zbog toga je u ovom radu analizirana pokretljivost tri vrste gipkih zglobova, različitih geometrijskih oblika (zglobovi u obliku štapa, u obliku filma i u obliku zareza). Razmatrana je i pokretljivost gipkih zglobova izradjenih od različitih vrsta materijala (zglobovi od plastike, zglobovi od silikona). Pokretljivost gipkih mehanizama je analizirana na primeru Roberts-Čebiševljevog mehanizma koji vodi odredjenu tačku spojke duž približno pravolinijske putanje.