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# Nonlinear mhd mixed convection flow and heat and mass transfer of first order chemical reaction over a wedge with variable viscosity in the presence of suction or injection

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**Abstract**

In the present study, an analysis is carried out to study the variable viscosity and chemical reaction effects on MHD flow, heat, and mass transfer characteristics in a viscous fluid over a porous wedge in the presence of heat radiation. The wall of the wedge is embedded in a uniform Darcian porous medium in order to allow for possible fluid wall suction or injection. The governing boundary layer equations are written into a dimensionless form by similarity transformations. The transformed coupled nonlinear ordinary differential equations are solved numerically by using the R.K.Gill and shooting methods. The effects of different parameters on the dimensionless velocity, temperature, and concentration profiles are

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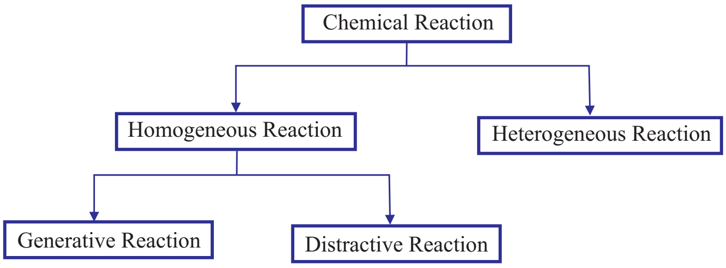
shown graphically. Comparisons with previously published works are performed and excellent agreement between the results is obtained. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.

**Nomenclature**

| | |
|---|---|
| $u, v$ | velocity components in $x$ and $y$ direction |
| $U$ | flow velocity away from the wedge |
| $g$ | acceleration due to gravity |
| $\beta^*$ | coefficient of volume expansion |
| $k_1$ | rate of chemical reaction |
| $K$ | permeability of the porous medium |
| $R$ | heat radiation parameter |
| $T$ | temperature of the fluid |
| $T_w$ | temperature of the wall |
| $T_\infty$ | temperature far away from the wall |
| $\beta$ | coefficient of thermal expansion |
| $\delta$ | heat source |
| $C$ | species concentration of the fluid |
| $C_w$ | species concentration along the wall |
| $C_\infty$ | species concentration away from the wall |
| $\rho$ | density of the fluid |
| $\sigma$ | electric conductivity of the fluid |
| $\alpha$ | thermal diffusivity |

# 1 Introduction

In many transport processes in nature and in industrial applications in which heat and mass transfer with heat radiation is a consequence of buoyancy effects caused by diffusion of heat and chemical species. the study of such processes is useful for improving a number of chemical technologies, such as polymer production and food processing. in nature, the presence of pure air or water is impossible. some foreign mass may be present either naturally or mixed with the air or water. the present trend in the field of chemical reaction with viscosity analysis is to give a mathematical model for the system to predict the reactor performance. a large amount of research work has been reported in this field. in particular, the study of chemical reaction, heat and mass transfer with heat radiation is of considerable importance in chemical and hydrometallurgical industries. chemical reaction can be codified as either heterogeneous or homogeneous processes. this depends on whether they occur at an interface or as a single phase volume reaction.




The effect of the presence of foreign mass on the free convection flow past a semi − infinite vertical plate was studied by Gebhart and Para [1]. The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated [2]. In most of cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself [3]. The effects of heat and mass transfer on laminar boundary layer flow over a wedge have been studied by many authors [4-14] in different situations. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been discussed by many authors [ 15-18] in various situations. The previous studies are based on the constant physical properties of the fluid. For most realistic fluids, the viscosity shows a rather pronounced variation with temperature. It is known that the fluid viscosity changes with temperature [19]. Then it is necessary to take into account the variation of viscosity with temperature in order to accurately predict the heat transfer rates. The effect of temperature-dependent viscosity on the mixed convection flow from vertical plate is investigated by several authors [19-22].

The aim of this work is to study the effects of variable viscosity, heat and mass transfer on nonlinear MHD mixed convection flow over a porous wedge with chemical reaction and heat radiation in the presence of suction or injection. The order of chemical reaction in this work is taken as first-order reaction. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

## 2 Mathematical analysis

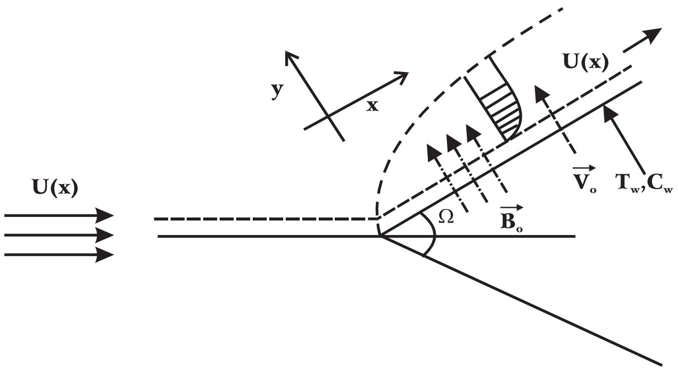



Figure 1: Flow analysis along the wall of the wedge

Let us consider a steady, laminar, hydromagnetic coupled heat and mass transfer by mixed convection flow in front of a stagnation point on a wedge plate embedded in porous medium. The fluid is assumed to be Newtonian, electrically conducting and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation ) and the concentration of species far from the wall, $C_\infty$, is infinitesimally small [2]. In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected. Let the x-axis be taken along the direction of the wedge and y-axis normal to it. An uniform transverse magnetic field of strength $B_o$ is applied parallel to the y-axis. The chemical reactions are taking place in the flow and a constant suction or injection is imposed at the wedge surface, see Fig.1. Since the magnetic Reynolds number is very small for most used in industrial applications, we assume that the induced magnetic field is negligible. For steady, two-dimensional flow under Boussinesq's approximation including variable viscosity are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+U\frac{dU}{dx}-\frac{\sigma B_0^2}{\rho}(u-U)+ \\ [g\beta(T-T_\infty)+g\beta^*(C-C_\infty)]\sin\frac{\Omega}{2}-\frac{\nu}{K}(u-U) \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q(T_\infty-T) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1C \tag{4}$$

The boundary conditions are,

$$\begin{aligned} &u=0,\quad v=-v_0,\quad T=T_w,\quad C=C_w \ \text{ at } \ y\ =\ 0 \\ &u=U(x), T=T_\infty, C=C_\infty \ \text{ at } \ y\to\infty \end{aligned} \tag{5}$$

By using the Rosseland diffusion approximation, [25] and [26], the radiative heat flux $q_r$ is given by $q_r=-\frac{4\sigma}{3\mu}\frac{\partial T^4}{\partial y}$ and the term $Q(T-T)$ is assumed to be the amount of heat generated absorbed per unit volume. Q is a constant, which may take on either positive or negative values. When the wall temperature $T_w$ exceeds the free stream temperature $T_\infty$, the source term represents the heat source when $Q<0$ and heat sink when $Q>0$. For the condition that $T_w<T_\infty$, the opposite relationship is true and $D$ is the effective diffusion coefficient. Assuming that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of temperature

$$T^4\cong 4T_\infty^3-3T_\infty^4 \tag{6}$$

Following the lines of Kafoussias et.al., [5], the following change of variables are introduced

$$\psi=\sqrt{\frac{2U\nu x}{1+m}}f(x,\eta) \tag{7}$$

$$\eta = \quad y\sqrt{\frac{(1+m)U}{2\nu x}} \tag{8}$$

The viscosity is assumed to be an inverse linear function of temperature given by the following [24]

$$\frac{1}{\mu} = \frac{1}{\mu_a}[1 + \chi(T - T_a] \tag{9}$$

where $\mu_a$ is the ambient fluid dynamic viscosity and $\chi$ is a thermal property of the fluid. Equ.(9) can be written as follows

$$\frac{1}{\mu} = a(T - T_r] \tag{10}$$

where $a = \frac{\chi}{\mu_a}$ and $T_r = T_a - \frac{1}{\chi}$ are constants and their values depend on the reference state and the thermal property of the fluid.

Under this consideration, the potential flow velocity can be written as

$$U(x) = Ax^m, \quad \beta_1 = \frac{2m}{1+m} \tag{11}$$

where $A$ is a constant and $\beta_1$ is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \frac{\Omega}{\pi}$ or a total angle $\Omega$ of the wedge.

The continuity equation (1) is satisfied by the stream function $\psi$ defined by

$$u = \frac{\partial \psi}{\partial y} \quad \texttt{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{12}$$

To transform Eqs.(2), (3) and (4) into a set of ordinary differential equations, we introduce the following dimensionless parameters and variables,

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{14}$$

$$Gr_1 = \frac{\nu g\beta(T_w - T_\infty)}{U^3} \qquad \text{(Grashof number)} \tag{15}$$

$$Gc_1 = \frac{\nu g\beta * (C_w - C_\infty)}{U^3} \qquad \text{(Modified Grashof number)} \tag{16}$$

$$N = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)} \qquad \text{(Buoyancy ratio)} \tag{17}$$

$$Re_x = \frac{Ux}{\nu} \qquad \text{(Reynolds number)} \tag{18}$$

$$\Pr = \frac{\nu}{\alpha} \qquad \text{(Prandtl number)} \tag{19}$$

$$Sc = \frac{\nu}{D} \qquad \text{(Schmidt number)} \tag{20}$$

$$M^2 = \frac{\sigma B_0^2}{\rho\, A} \qquad \text{(magnetic parameter)} \tag{21}$$

$$S = v_0\sqrt{\frac{(1+m)x}{2\,\nu\, U}} \qquad \text{(suction or injection parameter)} \tag{22}$$

$$\gamma = \frac{\nu\, k_1}{U^2} \qquad \text{(chemical reaction parameter)} \tag{23}$$

$$\lambda = \frac{\alpha}{KA} \qquad \text{(porous medium parameter)} \tag{24}$$

$$R = \frac{3k\nu}{16\sigma\, T_\infty^3} \qquad \text{(heat radiation parameter)} \tag{25}$$

$$\delta = \frac{Q}{A\rho\, c_p} \qquad \text{(heat source parameter)} \tag{26}$$

Now the equations (2) to (4)

$$(\theta - \theta_r\ )\frac{\partial^3 f}{\partial \eta^3} = \frac{(\theta-\theta_r)^2}{\theta_r}[-f\frac{\partial^2 f}{\partial \eta^2} - \frac{2m}{1+m}(1-(\frac{\partial f}{\partial \eta})^2) -$$

$$\frac{2}{1+m}\frac{N\phi+\theta}{1+N} Gr Re_x \sin\frac{\Omega}{2} + \frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}) + \tag{27}$$

$$\frac{2x}{m+1}\frac{\sigma B_0^2}{\rho U}(\frac{\partial f}{\partial \eta} - 1\ ) + \frac{2}{m+1}\lambda(\frac{\partial f}{\partial \eta} - 1\ )] + \frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial y^2}$$

$$\begin{aligned}(1+\frac{1}{R})\frac{\partial^2 \theta}{\partial \eta^2} &= -\Pr f\frac{\partial \theta}{\partial \eta} + \frac{2\Pr}{1+m}\theta\frac{\partial f}{\partial \eta} + \\ &\Pr\frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial \eta}) - \frac{2\ \Pr}{m+1}\delta\ \theta\end{aligned} \tag{28}$$

$$\begin{aligned}\frac{\partial^2 \phi}{\partial \eta^2} &= -Sc\ f\frac{\partial \phi}{\partial \eta} + \frac{2\ Sc\ x}{1+m}\gamma\ \phi + \frac{2\ Sc}{1+m}\phi\frac{\partial f}{\partial \eta} + \\ &\frac{2xSc}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \phi}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \phi}{\partial \eta})\end{aligned} \tag{29}$$

The boundary conditions can be written as

$$\begin{aligned}\eta = 0 \quad &: \quad \frac{\partial f}{\partial \eta}=0, \quad \frac{f}{2}(1+\frac{x}{U}\frac{dU}{dx}) + x\frac{\partial f}{\partial x} = -v_0\sqrt{\frac{(1+m)x}{2\nu U}}, \\ &\qquad \theta = 1, \quad \phi=1 \\ \eta \to \infty \quad &: \quad \frac{\partial f}{\partial \eta} = 1, \quad \theta = 0, \quad \phi = 0\end{aligned} \tag{30}$$

where $v_0$ is the velocity of suction if $v_0 < 0$ and injection if $v_0 > 0$ and $Gr = Gr_1 + Gc_1$.

The equations (26) to (28) and boundary conditions (29) can be written a

$$
\begin{aligned}
&\frac{\partial^3 f}{\partial \eta^3}+\frac{(\theta-\theta_r)}{\theta_r}[(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial \xi})\frac{\partial^2 f}{\partial \eta^2}-\frac{1-m}{1+m}\xi\frac{\partial^2 f}{\partial \xi\,\partial \eta}\frac{\partial f}{\partial \eta}-\\
&\frac{2}{1+m}M^2\xi^2(\frac{\partial f}{\partial \eta}-1)+\frac{2m}{1+m}(1-(\frac{\partial f}{\partial \eta})^2+\frac{2}{1+m}\frac{N\phi+\theta}{1+N}\\
&GrRe_x\sin\frac{\Omega}{2}\;-\;\frac{2}{m+1}\lambda(\frac{\partial f}{\partial \eta}-1\,)]-\frac{2}{1+m}\frac{1}{\theta-\theta_r}\frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial y^2}=0
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
(1+\tfrac{1}{R})\frac{\partial^2 \theta}{\partial \eta^2}+\Pr(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial \xi})\frac{\partial \theta}{\partial \eta}\;-\frac{2\Pr}{1+m}\theta\frac{\partial f}{\partial \eta}-\\
\frac{1-m}{1+m}\xi\frac{\partial \theta}{\partial \xi}\frac{\partial f}{\partial \eta}+\frac{2\ \Pr}{m+1}\delta\,\theta=0
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
&\frac{\partial^2 \phi}{\partial \eta^2}+Sc\,f\,\frac{\partial \phi}{\partial \eta}-\frac{2Sc}{1+m}\xi^2\gamma\,\phi+\\
&Sc\frac{1+m}{1-m}(\frac{\partial \phi}{\partial \eta}\xi\frac{\partial f}{\partial \xi}-\frac{\partial f}{\partial \eta}\xi\frac{\partial \phi}{\partial \xi})-\frac{2Sc}{1+m}\phi\frac{\partial f}{\partial \eta}=0
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
\eta=0\quad &:\frac{\partial f}{\partial \eta}=0,\quad \frac{(1+m)f}{2}+\frac{1-m}{2}\xi\frac{\partial f}{\partial \xi}=-S,\\
&\theta=1,\quad \phi=1\\
\eta\to\infty &:\frac{\partial f}{\partial \eta}=1,\quad \theta=0,\quad \phi=0
\end{aligned}
\tag{34}
$$

where $S$ is the suction parameter if $S>0$ and injection if $S<0$ and $\xi k\,x^{\frac{1-m}{2}}$ [5], is the dimensionless distance along the wedge ($\xi>0$). In this system of equations $f(\xi,\eta)$ is the dimensionless stream function; $\theta(\xi,\eta)$ be the dimensionless temperature ; $\phi(\xi,\eta)$ be the dimensionless concentration ; $Pr$, the Prandtl number, $Re_x$, Reynolds number etc. which are defined in (13) to (25). The parameter $\xi$ indicates the dimensionless distance along the wedge ($\xi > 0$). It is obvious that to retain the $\xi$ -derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wise location through the $\xi$- derivatives, a locally autonomous solution, at any given stream wise location can not be obtained. In such a case, an implicit marching numerical solution scheme is usually applied proceeding the solution in the $\xi$ -direction, i.e., calculating unknown profiles at $\xi_{\iota+1}$ when the same profiles at $\xi_\iota$ are known. The process starts at $\xi = 0$ and the solution proceeds from $\xi_\iota$ to $\xi_{\iota+1}$ but such a procedure is time consuming.

However, when the terms involving $\frac{\partial f}{\partial \xi}$, $\frac{\partial \theta}{\partial \xi}$ and $\frac{\partial \phi}{\partial \xi}$ and their $\eta$ derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions $f, \theta$ and $\phi$ with $\xi$ as a parameter and the computational task is simplified. Furthermore a locally autonomous solution for any given $\xi$ can be obtained because the stream wise coupling is severed. So, following the lines of [5], R.K.Gill, [23] and Shooting numerical solution scheme are utilized for obtaining the solution of the problem. Now, due to the above mentioned factors, the equations (30) to (33) are changed to

$$f''' + \frac{\theta - \theta_r}{\theta_r} f\, f'' + \frac{2m}{1+m}\frac{\theta - \theta_r}{\theta_r}(1 - f'^2) + \frac{2}{1+m}\frac{\theta - \theta_r}{\theta_r}\frac{N\phi + \theta}{1+N} Gr Re_x$$

$$\sin\frac{\Omega}{2} - \frac{\theta - \theta_r}{\theta_r}\frac{2}{1+m}(M^2\xi^2 + \lambda)(f' - 1) - \frac{2}{1+m}\frac{1}{\theta - \theta_r}\theta'\, f'' = 0 \tag{35}$$

$$(1 + \frac{1}{R})\theta'' + \Pr f\, \theta' - \frac{2\Pr}{1+m}\theta\, f' + \frac{2\ \Pr}{m+1}\delta\, \theta = 0 \tag{36}$$

$$\phi'' + Sc\, f\, \phi' - \frac{2Sc}{1+m} f'\phi - \frac{2\ Sc}{1+m}\xi^2\gamma\, \phi = 0 \tag{37}$$

with boundary conditions

$$\begin{aligned} &\eta = 0: \quad f(0) = -\frac{2}{1+m}S, \quad f'(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ &\eta \to \infty: \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \end{aligned} \tag{38}$$

# 3 Numerical solution and discussion

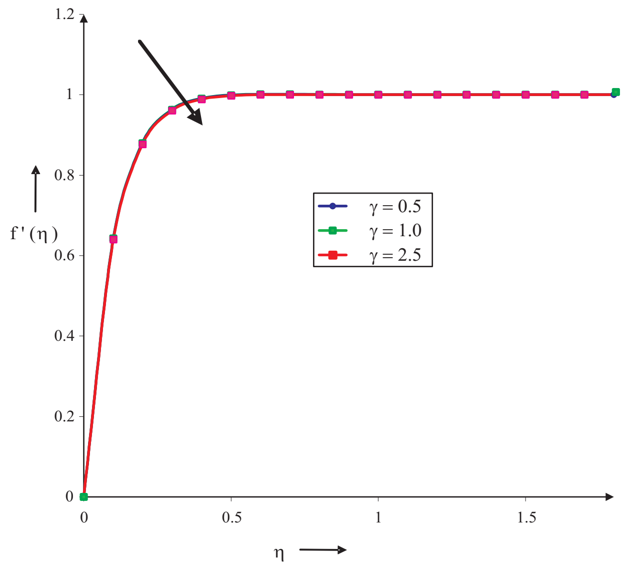
Equations (34) to (36) with boundary condition (37) were solved numerically using Runge Kutta Gill and shooting methods, [23]. The computations have been carried out for various values of variable viscosity, $\theta_r$, chemical reaction, $\gamma$, magnetic parameter, $M^2$, heat radiation, $R$, heat source, $\delta$ and porous medium,$\lambda$. In order to validate our method, we have compared steady state results of skin friction, $f''(0)$ and rate of heat transfer $-\theta'(0)$ for various values of $\theta_r$ (Table.1) with those of [24] and found them in excellent agreement.

| Pantoskratoras, [24] | | | Present work | | |
|---|---|---|---|---|---|
| $\theta_r$ | $f''(0)$ | $-\theta'(0)$ | $f''(0)$ | $-\theta'(0)$ | Pr |
| 2.0 | 0.2642 | -0.7341 | 0.2651 | -0.7345 | 20.00 |
| 4.0 | 0.3785 | -0.7888 | 0.3791 | -0.78921 | 13.33 |
| 6.0 | 0.4145 | -0.8042 | 0.4167 | -0.8061 | 12.00 |
| 8.0 | 0.4322 | -0.8112 | 0.4341 | -0.8127 | 11.43 |
| 10 | 0.4426 | -0.8162 | 0.4452 | -0.8171 | 11.11 |

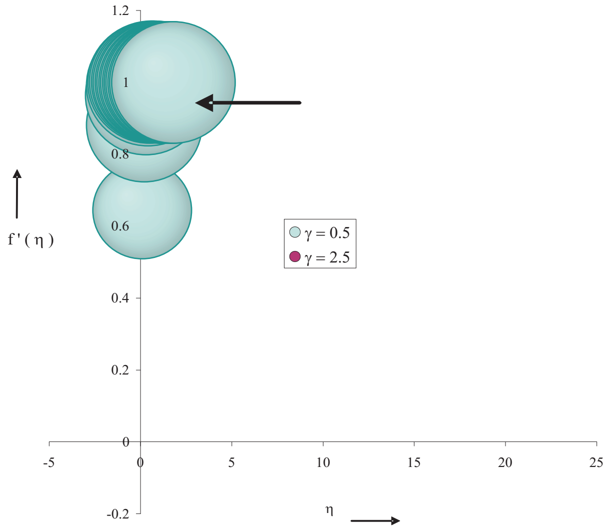
Table 1: Comparison of the values of $f''(0)$ and $-\theta'(0)$ for various values of $\theta_r$ with $\lambda = 0$, $\Omega = 30^0$, $N = 0$, $m = 0.0909$, $Sc = 0$, $M^2 = 0$ and $\delta = \gamma = S = 0$.

The velocity, temperature and concentration profiles obtained in the dimensionless form are presented in Figures.2-8 for $Pr = 0.71$ which represents air at temperature $20^0C$ and $Sc = 0.62$ which corresponds to water vapor that represents a diffusion chemical species of most common interest in air. Grashof number for heat transfer is chosen to be $Gr = 1.0$, since the these values corresponds to a cooling problem, and Reynolds number $Rex = 3.0$. The values of $\gamma$ is chosen to be 0.5,1.0 and 3.0. The value of $\theta_r$ (for air $\theta_r > 0$) is chosen to be 1.0,3.0 and 5.0 and the value of suction, $S$ is chosen to be 3.0. The value of strength of the magnetic field is chosen to be 1.0,3.0 and 5.0. The value of the heat radiation is chosen to be 0.5, 1.0 and 3.0 and heat source, $\delta$ is chosen to be 0.5.

The dimensionless velocity profiles for different values of chemical reaction are plotted in Fig.2 and Mod.2. Due to the uniform viscosity, $\theta_r = 0.1$, it is clear that the velocity and concentration of the fluid decrease with increase of chemical reaction while the temperature is not

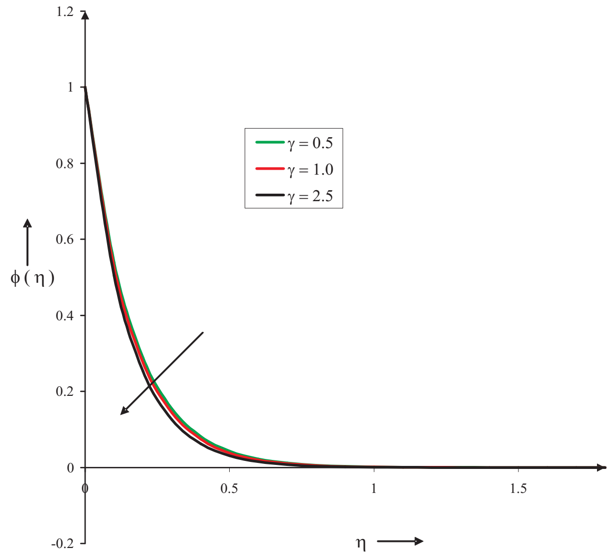



(a) (Fig.2): Influence of chemical reaction over the velocity profiles

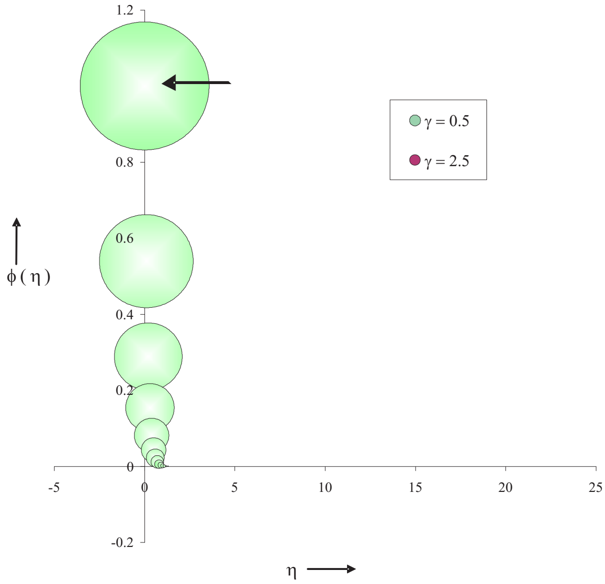



(b) (Mod.2): chemical reaction over the velocity profiles

Figure 2: with $Pr = 0.71$, $Sc = 0.62$, $m = 0.0909$, $\gamma = 1.0$, $N = R = 1.0$, $\lambda = 0.1$, $S = 3.0$, $M^2 = 0.5$, $Gr = 1.0$, $Rex = 3.0$, $\xi = 1.0$, $\theta_r = 0.1$ and $\Omega = 30^0$




(a) (Fig.3): Effects of chemical reaction over the concentration profiles




(b) (Mod.3): Effects of chemical reaction over the concentration profiles

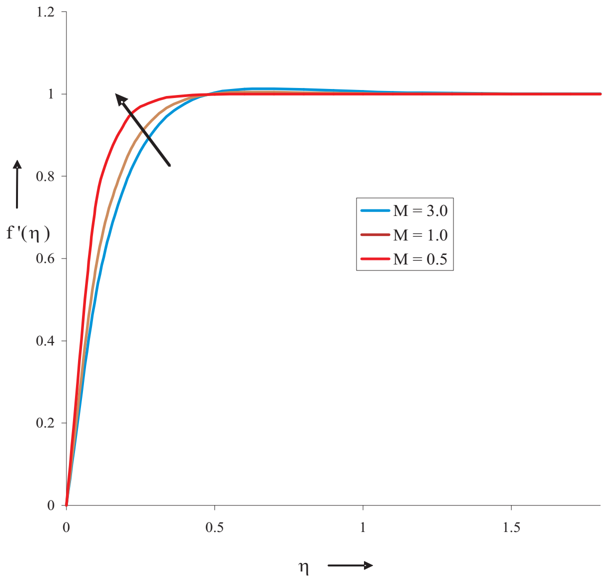
Figure 3: with $Pr = 0.71$, $Sc = 0.62$, $m = 0.0909$, with $Pr = 0.71$, $Sc = 0.62$, $m = 0.0909$, $M^2 = 1.0$, $N = R = 1.0$, $\lambda = 0.1$, $S = 3.0$, $\theta_r = 0.5$, $Gr = 1.0$, $Rex = 3.0$, $\xi = 1.0$ and $\Omega = 30^0$

significant with increase of chemical reaction parameter and these are displayed through Figs.2 and 3 and Mods.2 and 3 respectively.

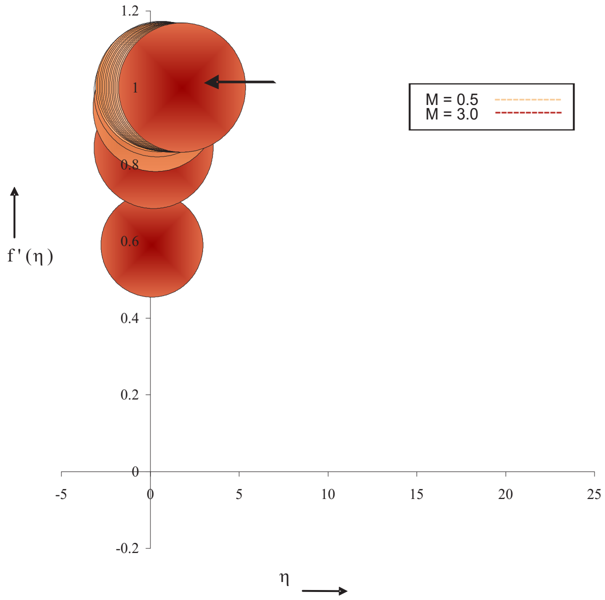
Figure 4 and Model 4 display the influence of the magnetic effects on velocity profiles. It is clear that the velocity increases with increase of the strength of the magnetic effect, while the temperature and concentration are not significant with increase of magnetic effects. As the strength of the magnetic effect increases, the Lorentz force, which oppose the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the mixed convection flow. The dimensionless temperature profiles for different values of heat radiation are plotted in Fig.5and Mod.5. Due to the uniform viscosity, $\theta_r = 0.1$, it is clear that the temperature of the fluid decreases with increase of heat radiation while the velocity and concentration are not significant with increase of heat radiation effects.

The effects of the viscosity parameter $\theta_r$ on velocity and temperature profiles are shown through Figures 6 and 7 and Models 6 and 7. It is seen that the velocity increases with increase of viscosity parameter while the thermal boundary layer thickness decreases as the viscosity increases. So, the increase of viscosity accelerates the fluid motion and reduces the temperature of the fluid along the wall. Also, it is observed that the concentration of the fluid is almost not affected with increase of the viscosity.

The effects of the strength of the magnetic field on skin friction,$f''(0)$ for both suction/injection is shown in Figure 8 and Model 8. In the case of suction, it is seen that the skin friction decreases and for injection, increases with increase of the strength of the magnetic field. It is seen that increasing the magnetic effect is to increase the velocity in the boundary layer and thus decrease the skin friction at wall, but the opposite trend is true when the magnetic strength is increased. All these physical behavior are due to the combined effect of the strength of magnetic field and viscosity at the wall of the wedge.

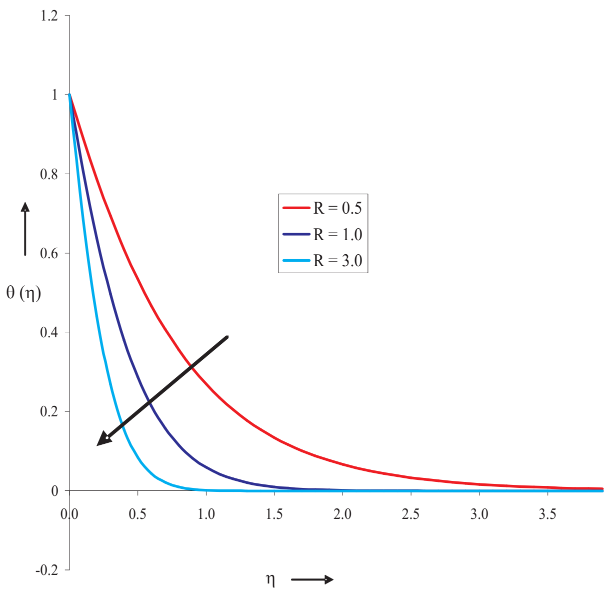



(a) (Fig.4): Influence of magnetic effects over the velocity profiles

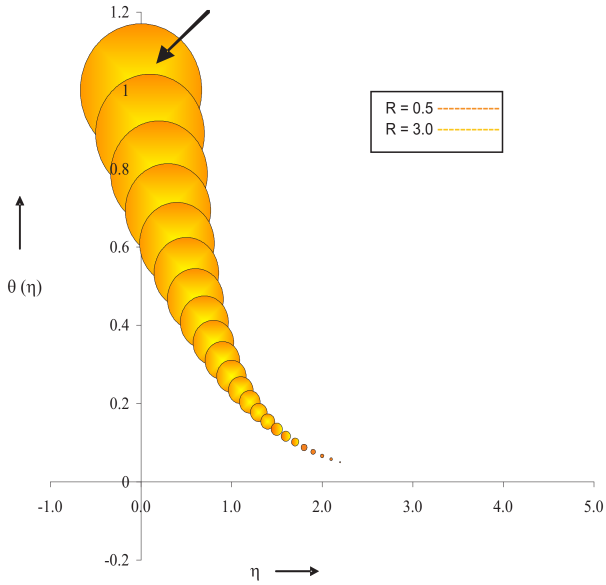



(b) (Mod.4): Magnetic effects over the velocity profiles

Figure 4: with $Pr = 0.71$, $Sc = 0.62, m = 0.0909$, $\gamma = 1.0$, $N = R = 1.0$, $\lambda = 0.1, S = 3.0$, $\theta_r = 0.5, Gr = 1.0$, $Rex = 3.0, \xi = 1.0$ and $\Omega = 30^0$

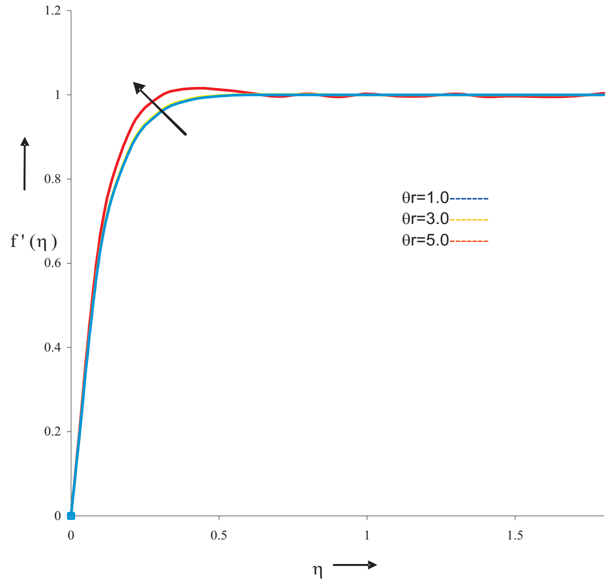



(a) (Fig.5): Heat radiation over the temperature profiles

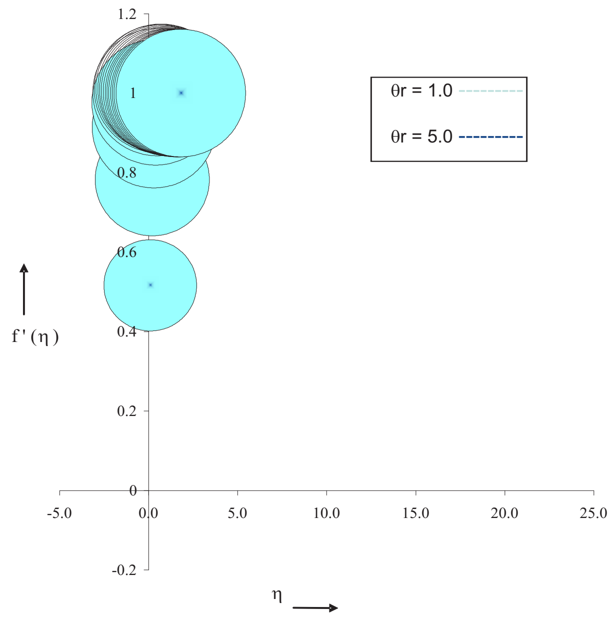



(b) (Mod.5): Heat radiation over the temperature profiles

Figure 5: with $Pr = 0.71, Sc = 0.62, m = 0.0909,\ \gamma = 1.0,\ N = 1.0,$ $\lambda = 0.1, S = 3.0,\ \theta_r = 0.5, Gr = 1.0,\ Rex = 3.0,\ \xi = 1.0$ and $\Omega = 30^0$

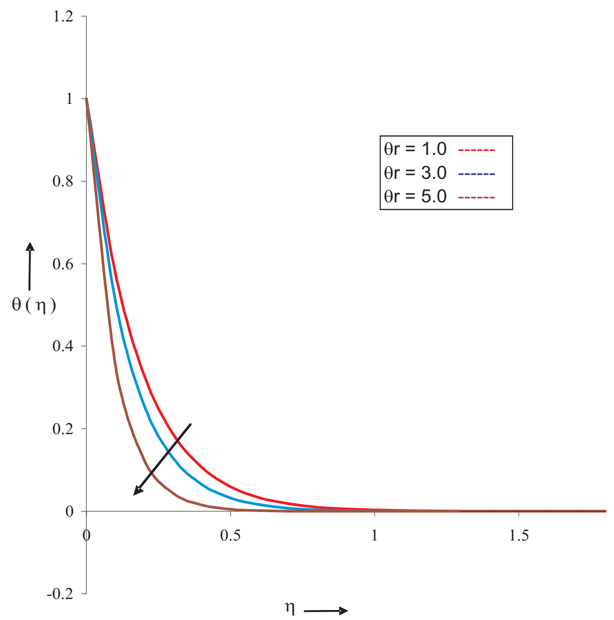



(a) (Fig.6): Influence of viscosity over the velocity profiles

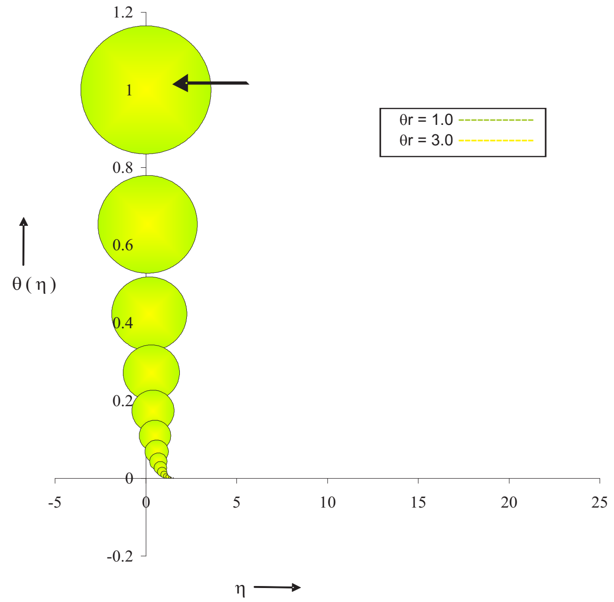



(b) (Mod.6): Influence of viscosity over the velocity profiles

Figure 6: with $Pr = 0.71, Sc = 0.62, m = 0.0909,\ \gamma = 1.0, N = R = 1.0,\lambda = 0.1, S = 3.0, M^2 = 0.5,\ Gr = 1.0, Rex = 3.0, \xi = 1.0$ and $\Omega = 30^0$

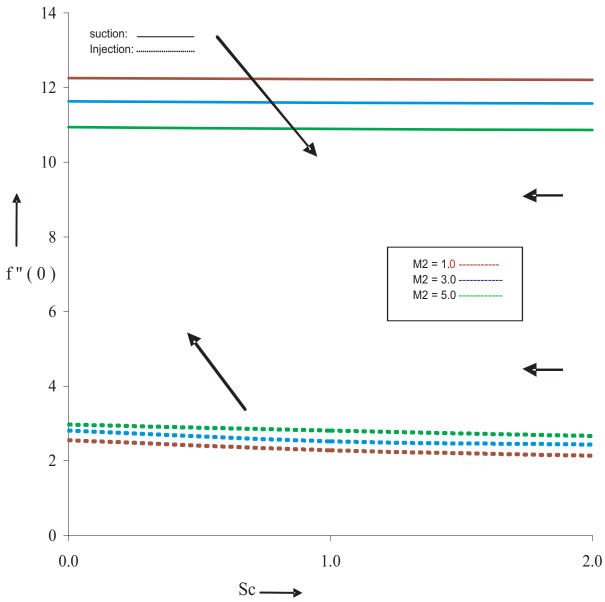



(a) (Fig.7): Effects of viscosity over the temperature profiles

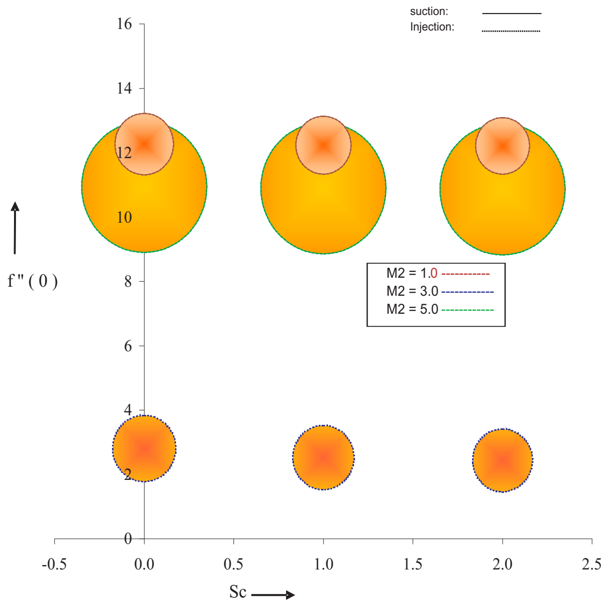



(b) (Mod.7): Influence of viscosity over the temperature profiles

Figure 7: with $Pr = 0.71, Sc = 0.62, m = 0.0909,\ \gamma = 1.0,\ N = R = 1.0, \lambda = 0.1,\ S = 3.0, M^2 = 0.5, Gr = 1.0,\ Rex = 3.0, \xi = 1.0$ and $\Omega = 30^0$




(a) (Fig.8): Magnetic field over the skin friction




(b) (Mod.8): Magnetic field over the skin friction

Figure 8: with $Pr = 0.71, Sc = 0.62, m = 0.0909,\ \gamma = 1.0, N = R = 1.0, \lambda = 0.1,\ Gr = 1.0, Rex = 3.0, \xi = 1.0,\ \theta_r = 0.54,$ and $\Omega = 30^0$

# 4 Conclusions

This paper studied the effects of variable viscosity, heat and mass transfer on nonlinear MHD mixed convection flow over a porous wedge with chemical reaction and heat radiation in the presence of suction or injection The results are presented graphically (Models in 3-D) and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. Comparisons with previously published works are performed and excellent agreement between the results are obtained.

We conclude the following from the results and discussions:

- Velocity and concentration of the fluid decrease with increase of chemical reaction. All these physical behavior are due to the combined effects of the strength of the viscosity and magnetic effects.

- Velocity of the fluid increases with increase of the strength of the magnetic effect. As the strength of the magnetic effect increases, the Lorentz force, which oppose the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the mixed convection flow.

- Increase of the viscosity accelerates the fluid motion and reduces the temperature of the fluid along the wall of wedge.

- In the presence of uniform magnetic effects, temperature decreases with increase of heat radiation.

- In the case of suction, skin friction decreases and for injection, increases with increase of the strength of the magnetic field. It is interesting to note that increasing the magnetic effect is to increase the velocity in the boundary layer and thus decrease the skin friction at wall, but the opposite trend is true when the magnetic strength is increased. All these physical behavior are due to the combined effect of the strength of magnetic field and viscosity at the wall of the wedge.

It is hoped that the present investigation may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

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# Nelinearno MHD konvekciono tečenje sa prenosom mase i toplote kao i hemijskom reakcijom prvog reda preko klina sa promenljivom viskoznošću u prisustvu ubrizgavanja ili isisavanja

Izvedena je studija uticaja promenljive viskoznosti i hemijske radijacije na MHD tečenje kao i karakteristike prenosa toplote i mase za viskozni fluid preko poroznog klina u prisustvu toplotne radijacije. Zid klina je potopljen u Darsijevu poroznu sredinu da bi se dozvolilo moguće ubrizgavanje ili isisavanje u zid. Jednačine graničnog sloja su pomoću transformacija sličnosti napisane u bezdimenzionom obliku. Transformisane spregnute nelinearne obične diferencijalne jednačine su numeički rešene korišćenjem metode R.K.Gill-a i "shooting" metode. Grafički su prikazani uticaji raznih parametara na bezdimenzionu brzinu, temperaturu i profile koncentracije. Uporedjenje sa prethodno objavljenim radovima je pokazalo odlično slaganje rezultata. Posle njihovog grafičkog prikaza došlo se do zaključka da analizirani parametri značajno utiču na polje tečenja i druge fizički važne veličine.

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