Theoret. Appl. Mech., Vol.34, No., pp. 289–308, Belgrade 2007

The solution of certain loss of contact between a plate and unilateral supports

Yos Sompornjaroensuk * **Kraiwood Kiattikomol** †

Abstract

This paper examines the loss of contact between a square plate and the unilateral supports under uniformly distributed load. Since the plate is rested on the unilateral supports, it will have the regions of lost contact between a plate and the supports due to the absence of restraining corner force at the plate corners. This leads to the mixed boundary conditions and these conditions are then written in the form of dual-series equations, which can further be reduced to a Fredholm integral equation by taking advantage of finite Hankel transform technique. Numerical results are given for the deflections of free edge and deflections along the middle line of the plate with different values of the Poisson's ratio. In addition, the deflection surface is also presented. From the investigation, it can be indicated that the loss of contact is decreased upon the increasing Poisson's ratio.

Keywords : plate, receding contact, mixed boundary conditions, dual-series equations, unilateral support, Hankel transform, Fredholm integral equation

*Department of Civil Engineering, King Mongkut's University of Technology Thonburi, Bangkok, 10140, Thailand and Currently Lecturer, Department of Civil Engineering, Mahanakorn University of Technology, Bangkok, 10530, Thailand. E-mail: *ysompornjaroensuk@gmail.com* or *yos@mut.ac.th*

†Department of Civil Engineering, King Mongkut's University of Technology Thonburi, Bangkok, 10140, Thailand. E-mail: *ikraomol@kmutt.ac.th*

1 Introduction

A simple bending problem of plates that analyzed in the past is the problem of uniformly loaded square plate having all edges supported by the simple support as shown in Fig. 1(a). By the meanings of this type of support condition, the plate corners in general have to be anchored by a concentrated load that called the corner force resulting from the adjacent twisting moments corresponding at that corner [1]. If there are no corner forces applied at the corners of the plate, it is found that parts of the plate near the corners will be bent away from the supports upon loading. For instance, the plate considered becomes a unilaterally supported plate.

The definition of the unilateral support was introduced by Keer and Mak [2] which can be explained as a unilateral constraint allowing only upward motion of the plate. Therefore, during bending of the plate due to uniform loading, some parts of the plate edge will separate from the supports as demonstrated in Fig. 1(b). It is now immediately seen that the boundary conditions of the plate are mixed and the solution is more difficult to solve because the shear singularity in the order of an inverse square root type exists at the points where the unilateral support changes to a free edge.

Mathematically, there are various methods used to analyze the problems of plate having mixed boundary conditions. Nowacki and Olesiak [3] and Noble [4] solved the problems of circular plate clamped on part of its boundary and simply supported on the remainder. They obtained the solutions from an integral equation in term of the unknown function. The variational method has been applied to determine the solution of these problems by Bartlett [5]. Moreover, the various cases of circular plate with combinations of clamped, simply supported and free boundary conditions loaded with uniform loading were also studied by Conway and Farnham [6] in which the method of superposition was used to formulate the problems and solved numerically by a direct point-matching approach. It is interesting to note that no attempt was made to handle the singularity of the solution analytically.

In fact, whether the boundary conditions of plate are of the mixed types, the stress singularity will be occurred at the transition points of discontinuous support. This pointed out by Williams [7], who first constructed an infinite set of the appropriate eigenfunctions to characterize the singularity orders of plate having the combinations of boundary conditions. It can be concluded from the previous works that the bending stress is unbounded for the plate with mixed boundary conditions.

Stahl and Keer [8] reconsidered the problem of uniformly loaded circular plates using the finite Hankel integral transform techniques where the moment singularity is taken into account in the analysis. Subsequently, similar method was extended by Kiattikomol and Sriswasdi [9] to treat the problem of annular plate. For the other cases of plate, Keer and Sve [10] presented the moment stress intensity factor due to the effects of crack geometry in rectangular plates under the static load where the free vibration and stability problems were carried out by Stahl and Keer [11]. A simply supported rectangular plate with an internal line support has been investigated for the free vibration and buckling problems [12]. Recently, Sompornjaroensuk and Kiattikomol [13] examined the advancing contact between the plate with different boundary conditions and an internal sagged support in which the method of analysis is identical to Dundurs et al. [14]. It is worth to notice that the singularity in the solution is in the order of square root in the shear corresponding to the nature of free contact as explained by Dundurs and Stippes [15].

Many problems of plate with mixed edge conditions and having the right-angle corners as described above, the corners of the plate are anchored which are opposed to the present paper. The problem of plate supported by the unilateral supports with no restrained corners is involved to the natural receding contact problem. This problem has been analytically investigated by Dempsey et al.[16] using a finite Fourier integral transform with including the shear singularity in the solution. For the numerical treatments, Salamon et al.[17] modeled the unilateral supports by discrete elastic springs using a finite element method where the characteristic of discrete springs as well as the spring stiffnesses can be varied from the elastic support to nearly rigid support. Another numerical method was done by Hu and Hartley [18], who utilized a direct boundary element method to study the problem of general polygonal shape of plates in which the support system consists of discrete elastic springs. However, the two latter numerical methods [17,18] do not include the singular behavior at the transition points of support condition.

Therefore, the aim of this paper is to analytically examine the loss of contact between a square plate and the unilateral supports. The solution is set up by using the L?vy-N?dai approach [1] and the problem is formulated through the dual-series equations that followed to the previous work [16]. Most importantly, the correct singularity that satisfied the nature of free contact problems [15] is also considered in the analysis. Based on the method of finite Hankel transform techniques [9,10,13,14], an integral equation of the Fredholm-type governing the problem solution can be derived. The results are given for the deflections of the free edge and deflections of the middle line of the plate with different values of Poisson's ratio. The validation of the present results is compared with results obtained by other analytical [16] and numerical [18] techniques.

2 Problem formulation

To simplify the analysis, the scaled square plate configuration is shown in Fig. 1 whereas the actual length of plate is a and its scaled by the factor π/a. Therefore, the equation governing the deflection $w(x,y)$ of the plate under the uniform load (q) in the transformed coordinates (x,y) is given by

$$\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2 \partial y^2 + \partial^4 w/\partial y^4 = (a/\pi)^4 (q/D), \tag{1}$$

where $D = Eh^3/12(1-\nu^2)$ =flexural rigidity of the plate, both E and ν are, the material properties, called the Young's modulus and the Poisson's ratio, respectively, and h =plate thickness.

Using the notations given in Dempsey et al.[16] and due to the two-fold symmetry in the problem, the deflection function for the unilaterally supported square plate as shown in Fig. 1(b) can be expressed in the form of Levy-Nadai approach [1]

$$w(x,y) = (qa^4/2D) \sum_{m=1,3,5,\ldots}^{\infty} [W_m(x,y) + W_m(y,x)] + W_c, \tag{2}$$

where the last term of the equation shown above (W_c) is defined as the deflection of the corner at $x = y = 0$ and

$$W_m(x, y) = [4/(\pi m)^5 + Y_m(x)] \sin(my), \tag{3}$$

$$Y_m(u) = A_m \cosh(mu) + B_m mu \sinh(mu) + C_m \sinh(mu) + D_m mu \cosh(mu), \tag{4}$$

in which A_m, B_m, C_m, and D_m are the unknown constants to be determined.

It is notable that the first and the second terms of equation (3) represent the particular and complementary solutions of equation (1), respectively. Because of the symmetry of the lateral load and deflection function, the boundary conditions need to be considered only on the upper left quadrant of the plate as shown in Fig. 1(b), thus, the boundary conditions are:

$$\partial w/\partial y = 0 : 0 \le x \le \pi/2 \ ; \ y = \pi/2, \tag{5}$$

$$V_y = 0 : 0 \le x \le \pi/2 \ ; \ y = \pi/2, \tag{6}$$

$$M_y = 0 : 0 \le x \le \pi/2 \ ; \ y = 0, \tag{7}$$

$$w = \partial w/\partial x = 0 : e < x \le \pi/2 \ ; \ y = 0, \tag{8}$$

$$V_y = 0 : 0 \le x < e \ ; \ y = 0, \tag{9}$$

$$w = W_c : x = 0 \ ; \ y = 0, \tag{10}$$

$$R = 0 : x = 0 \ ; \ y = 0. \tag{11}$$

where equations (8) and (9) are the mixed boundary conditions of the problem that will be used to formulate the dual-series equations.

The bending moment (M_y), supplemented shearing force (V_y), and corner force (R) corresponding to the coordinates (x, y) of the scaled plate can be expressed as follows [1]:

$$M_y = -D(\pi/a)^2 (\partial^2 w/\partial y^2 + \nu \partial^2 w/\partial x^2), \tag{12}$$

$$V_y = -D(\pi/a)^3 [\partial^3 w/\partial y^3 + (2 + \nu)\partial^3 w/\partial x^2 \partial y], \tag{13}$$

$$R = 2D(1 - \nu)(\pi/a)^2 \partial^2 w/\partial x \partial y. \tag{14}$$

It can be remarked that the corner forces are considered as positive if they act on the plate in the downward direction in order to prevent the plate corners from rising up during bending.

Substituting equation (2) into equations (5) to (7) and with using the definition of equations (12) and (13) leads to the relations of unknown constants A_m, B_m, and C_m in term of D_m as follows:

$$A_m = 4\nu\eta' / (\pi m)^5 + 2D_m \eta' \coth\beta, \tag{15}$$

$$B_m = -D_m \coth\beta, \tag{16}$$

$$C_m = -4\nu\eta' \tanh\beta / (\pi m)^5 - D_m[2\eta' + \beta(\tanh\beta - \coth\beta)], \tag{17}$$

where

$$\eta' = 1/(1-\nu) \ , \ \beta = m\pi/2, \tag{18}$$

It is seen that the problem is then reduced to the determination of a single constant D_m, which can be determined from the boundary conditions along the plate edge at $y = 0$ as provided in equations (8) and (9). It is more convenient to use equations (8b) and (9) to be mixed with respect to the slope and the shear for the permission of dual-series equations to be cast into the proper form for solution [16] and then, they can be written as

$$\sum_{m=1,3,5,\ldots}^{\infty} mP_m \cos(mx) = 0 \ ; \ e < x \leq \pi/2, \tag{19}$$

$$\begin{aligned}
&\sum_{m=1,3,5,\ldots}^{\infty} \{m^3 P_m(1 + F_m^{(1)}) \sin(mx) + m^3 P_m[F_m^{(2)} \sinh(mx) - 2\eta \cosh(mx) \\
&\qquad + F_m^{(3)} mx \cosh(mx) - \eta mx \sinh(mx)]\} \\
&= \sum_{m=1,3,5,\ldots}^{\infty} [F_m^{(4)} \sin(mx) + F_m^{(5)} + F_m^{(6)} \sinh(mx) - F_m^{(5)} \cosh(mx) \\
&\qquad + F_m^{(7)} mx \cosh(mx) - F_m^{(8)} mx \sinh(mx)] \ ; \ 0 \leq x < e,
\end{aligned} \tag{20}$$

where

$$P_m = 2/(\pi m)^5 + D_m \coth\beta, \tag{21}$$

$$1 + F_m^{(1)} = \tanh\beta - \eta\beta \mathrm{sech}^2\beta, \tag{22}$$

$$F_m^{(2)} = \eta(2\tanh\beta + \beta\mathrm{sech}^2\beta), \tag{23}$$

$$F_m^{(3)} = \eta\tanh\beta, \tag{24}$$

$$F_m^{(4)} = (2/\pi^5 m^2)[(3-\nu)\tanh\beta/(3+\nu) - \eta\beta\mathrm{sech}^2\beta], \tag{25}$$

$$F_m^{(5)} = 4/[(3+\nu)\pi^5 m^2], \tag{26}$$

$$F_m^{(6)} = (2/\pi^5 m^2)[2\tanh\beta/(3+\nu) + \eta\beta\mathrm{sech}^2\beta], \tag{27}$$

$$F_m^{(7)} = 2\eta\tanh\beta/\pi^5 m^2, \tag{28}$$

$$F_m^{(8)} = 2\eta/\pi^5 m^2, \tag{29}$$

and

$$\eta = (1-\nu)/(3+\nu). \tag{30}$$

As treated in [16], the dual series equations that presented in equations (19) and (20) were further be reduced to a single integral equation by assuming the unknown function P_m as given in equation (21) in the form of finite Fourier transform where P_m was assumed to has the order of an inverse square root singularity in the shear at the transition points from unilateral support to free edge similar to Keer and Mak [2]. Moreover, this assumed P_m was also automatically satisfied the first dual series equations of equation (19) and then, equation (20) can finally be expressed in the form of a Cauchy-type singular integral equation of the first kind.

It is remarkable that in the numerical analysis of the mentioned work [16], some approximations were required to evaluate the kernel of singular integral equation. Therefore, this paper aims to present an alternate method to avoidance of this problem. For this purpose, the solution technique similar to those used by Kiattikomol et al.[19] and Kiattikomol and Porn-anupapkul [20] is applied to the present work except that, the order of singularity is assumed to be an inverse square root in the shear [2,13,14,16] instead of the moment [7].

By introducing the function P_m in the form of a finite Hankel integral transform as

$$m^2 P_m = \int_0^e t\phi(t) J_1(mt)dt \; ; \; m = 1, 3, 5, ..., \tag{31}$$

while $\phi(\cdot)$, $J_n(\cdot)$ are the unknown auxiliary function and Bessel function of the first kind and order n, respectively. This type of integral transform has been widely used to analytical study the problems in elasticity theory or in mathematical physics which can be found in the scattering technical literature [21-25].

It can easily be shown that equation (31) automatically satisfies equation (19). Verification is made by substitution of P_m that given above into equation (19) and with the help of certain identity presented in [12]

$$\sum_{m=1,3,5,...}^{\infty} m^{-1} J_1(mt)\cos(mx) = (1/2t)(t^2 - x^2)^{1/2} H(t - x) \ ; \ x + t < \pi, \tag{32}$$

where $H(\cdot)$ is the Heaviside unit step function. Thus the result yields

$$\int_0^e t\phi(t)[(1/2t)(t^2 - x^2)^{1/2} H(t - x)]dt = 0 \ ; \ e < x \leq \pi/2. \tag{33}$$

It can clearly be seen that since t and e are always less that x, the left-hand side of equation (33) is automatically vanished.

As described previously, the form of P_m that given in equation (31) is also required to produce an inverse square root singularity in the shear at the tips of unilateral support. To verify the correct order of singularity, it is convenient to rewrite the expression of the shear force distribution on the unilateral support in the new form. Thus equation (20) may be expressed as

$$V_y(e < x \leq \pi/2, 0) \sim -\frac{d}{dx} \sum_{m=1,3,5,...}^{\infty} [m^2 P_m \cos(mx)$$

$$+ m^2 P_m F_m^{(1)} \cos(mx) + ... \tag{34}$$

Substituting equation (31) into the first term on the right-hand side of equation (34) and utilizing the identity [12]

$$\sum_{m=1,3,5,...}^{\infty} J_1(mt)\cos(mx) = 1/2t - (x/2t)(x^2 - t^2)^{-1/2} H(x - t)$$

$$+\int_0^\infty [\exp(\pi s)+1]^{-1} I_1(ts)\cosh(xs)ds \ ; \ x+t<\pi, \tag{35}$$

where $I_n(\cdot)$ is the modified Bessel function of the first kind and order n, therefore, the singular part in the vicinity of the tip of support can be found to be

$$V_y(e+\varepsilon,0) \sim -(e/2)\phi(e)(2e\varepsilon)^{-1/2} + O(\varepsilon^{1/2}). \tag{36}$$

This revealed that there is a singularity in shear of order $O(\varepsilon^{-1/2})$ as expected. It can be noted that equation (36) is obtained by substituting $x = e+\varepsilon$ into equation (34) where ε is the small distance measured from the singular point.

3 Fredholm integral equation

The method of reduction of equation (20) to the integral equation form can be seen in the recent work of Sompornjaroensuk and Kiattikomol [13]. With using the identity of equation (35) and the assistance of some certain identities given in [26,27], equation (20) is then reduced to the following inhomogeneous Fredholm integral equation of the second kind, with introducing $t = er, e\rho$

$$\Phi(\rho) + \int_0^1 K(\rho,r)\Phi(r)dr = f(\rho) \ ; \ 0 \le \rho \le 1, \tag{37}$$

where

$$\Phi(\rho) = \phi(e\rho) \ , \ \Phi(r) = \phi(er), \tag{38}$$

$$\begin{aligned} K(\rho,r) = 2e^2 r \sum_{m=1,3,5,\dots}^{\infty} & [-4\eta m/\pi - \eta m L_1(me\rho) - \eta m^2 e\rho L_0(me\rho) \\ & + m F_m^{(1)} J_1(me\rho) + m(F_m^{(2)} - F_m^{(3)}) I_1(me\rho) \\ & + m^2 F_m^{(3)} e\rho I_0(me\rho)] J_1(mer) \end{aligned}$$

$$-2e^2 r \int_0^\infty s[\exp(\pi s)+1]^{-1} I_1(se\rho) I_1(ser) ds, \tag{39}$$

$$f(\rho) = 2 \sum_{m=1,3,5,\dots}^{\infty} [F_m^{(4)} J_1(me\rho) + (F_m^{(6)} - F_m^{(7)}) I_1(me\rho) + m F_m^{(7)} e\rho I_0(me\rho)$$

$$+(F_m^{(8)} - F_m^{(5)}) L_1(me\rho) - m F_m^{(8)} e\rho L_0(me\rho)], \tag{40}$$

in which $L_n(\cdot)$ is the modified Struve function of order n.

4 Solution to the integral equation with zero corner force condition

To determine the unknown auxiliary function $\Phi(\rho)$, the Simpson's rule of integration can be applied for this purpose to transform equation (37) into a system of linear algebraic equations which is solved numerically for the discretized value of $\Phi(\rho)$ by using the direct method of Gaussian elimination with partial pivoting [28]. However, the correct value of $\Phi(\rho)$ is constrained with the condition of zero corner force. This condition can be obtained from equation (11). Substituting equations (2) into (14) and using the prescribed condition of equation (11), after changing the variable $t = er$ where $0 \le r \le 1$, leads to

$$e^2 \int_0^1 T\,(er)\, r\Phi(r) dr = B, \tag{41}$$

where

$$T(er) = \sum_{m=1,3,5,\dots}^{\infty} [(1+\nu)\eta' \tanh\beta - \beta \mathrm{sech}^2\beta] J_1(mer), \tag{42}$$

$$B = 2 \sum_{m=1,3,5,\dots}^{\infty} [(1/\pi^5 m^3)(\tanh\beta - \beta \mathrm{sech}^2\beta)]. \tag{43}$$

It is observed from equation (37) that the function $\Phi(\rho)$ depends on the value of noncontact length e. The method of finding the value e is done by first assuming e in equation (37) and then, the function $\Phi(\rho)$ is obtained. The correct value of e can be found by iteration until the condition of equation (41) is satisfied. Thus Fig. 2 presents the variation of $\Phi(\rho)$-values with varying of the Poisson's ratio. In the present investigation, the Poisson's ratios are taken as 0.1, 0.3, and 0.5. As a result, it is found that the noncontact length or the loss of contact length e is independent of the level of loading, but only depended on the Poisson's ratio. This corresponds to the nature of receding contact problems [15].

5 Deflections of unilaterally supported square plate

The remaining important physical quantity is the deflection of the corner W_c which has remained undetermined. This can be obtained by imposing $y = 0$ into equation (2) and using equations (15) to (17) together with equation (21), yields

$$w(x,0) = (qa^4\eta'/D)\sum_{m=1,3,5,\ldots}^{\infty} P_m \sin(mx) + W_c \; ; \; 0 \le x \le \pi/2. \tag{44}$$

Considering the identity that given below [12],

$$\sum_{m=1,3,5,\ldots}^{\infty} m^{-2} J_1(mt)\sin(mx)$$

$$= (1/4)[(x/t)(t^2 - x^2)^{1/2} + t\sin^{-1}(x/t)] \; ; \; x < t, \; x + t < \pi, \tag{45}$$

$$= \pi t/8 \; ; \; x \ge t, \; x + t < \pi, \tag{46}$$

after that substituting equation (31) for P_m and equation (46) into equation (44), then, the quantity of W_c is determined by applying the boundary condition that presented in equation (8a), leads to

$$W_c = -(qa^4\pi\eta' e^3/8D)\int_0^1 \rho^2\Phi(\rho)d\rho. \tag{47}$$

Subsequently, the deflection of free edge is also determined by using equations (45) and (47). Thus it can be taken in the form as

$$\frac{w(x,0)}{(qa^4/D)} = (\eta' e^3/4) \int_{\xi}^{1} \Phi(\rho)[\xi(\rho^2 - \xi^2)^{1/2}$$

$$+\rho^2 \sin^{-1}(\xi/\rho) - (\pi/2)\rho^2]d\rho, \tag{48}$$

where $\xi = x/e$ and $0 \leq \xi, \rho \leq 1$. Figure 3 presents the numerical results for the deflection of free edge that calculated from equation (48).

To determine the deflection along the middle line of the plate, the unknown constants A_m, B_m, C_m, and D_m that shown in equations (15) to (17) and (21) should be rewritten in the forms of integral representation by using equation (31), the results are then given as

$$A_m = -4/(\pi m)^5 + 2\eta'(e/m)^2 \int_0^1 \rho\Phi(\rho)J_1(me\rho)d\rho, \tag{49}$$

$$B_m = 2/(\pi m)^5 - (e/m)^2 \int_0^1 \rho\Phi(\rho)J_1(me\rho)d\rho, \tag{50}$$

$$C_m = 2[2\tanh\beta - \beta \sec h^2\beta]/(\pi m)^5 - [2\eta' \tanh\beta - \beta \sec h^2\beta]$$

$$\times(e/m)^2 \int_0^1 \rho\Phi(\rho)J_1(me\rho)d\rho, \tag{51}$$

$$D_m = -2\tanh\beta/(\pi m)^5 + \tanh\beta(e/m)^2 \int_0^1 \rho\Phi(\rho)J_1(me\rho)d\rho. \tag{52}$$

Substitution of equations (49) to (52) and equation (47) into equation (2) with setting $y = \pi/2$, hence, the deflection along the middle line of the plate $w(x, \pi/2)$ can be calculated. The numerical results are shown in Fig. 4.

6 Results and conclusions

In the preceding analysis, an integral equation that governed the problem solution is derived analytically and its numerical solution is also presented graphically as in Fig. 2. From the obtained results, it indicates that the loss of contact between a plate and the unilateral supports is not depended on the level of loading but strongly depended on the Poisson's ratio. Therefore, the values of the loss of contact length e for each case of the Poisson's ratios ν are listed as follows: $e = 0.3015$, 0.2626, 0.2188 for $\nu = 0.1$, 0.3, and 0.5, respectively. This can be concluded that the loss of contact is decreased upon the increasing of the Poisson's ratio.

In addition to the solution of integral equation, the deflections of the free edge $w(x, 0)$ and deflections along the middle line $w(x, \pi/2)$ of the plate with different values of the Poisson's ratio are, respectively, demonstrated in Figs. 3 and 4. The results are also compared with other techniques and an excellent agreement is found. As the results obtained, it can be seen that the magnitude of both deflections is increased when the Poisson's ratio is decreased. To consider the global deformation of the plate upon loading, the deflection surface that bounded by the region $0 \leq x, y \leq \pi/2$ is presented for an example in Fig. 5 only for a case of the Poisson's ratio taken as 0.3.

In the conclusions of this paper, an alternative analytical method is proposed to study the behavior of square plate supported by the unilateral supports and the loss of contact is examined. Although the case of a square plate subjected to a uniformly distributed load is only considered, however, the present method can be extended to other cases of rectangular plate under the arbitrary loads but the formulation of problem is more complicated.

Based on the finite Hankel integral transform techniques, the solution of problem can be formulated and treated analytically, and can be obtained from the inhomogeneous Fredholm integral equation of the second kind in term of an unknown auxiliary function satisfying the order of an inverse square root singularity in the shear at the tips of contact between the plate and the unilateral supports. The advantages of this present method are that the singularity of the problem is isolated and the solution is determined with no approximation. A good agreement is found from the analysis when compared to the other investigators.

References

[1] S.P. Timoshenko and S. Woinowsky-Krieger, Theory of plates and shells, 2^{nd} ed., McGraw-Hill, Singapore, 1959.

[2] L.M. Keer and A.F. Mak , Loss of contact in the vicinity of a right-angle corner for a simply supported, laterally loaded plate, J. Appl. Mech. (ASME), 48, (1981), 597-600.

[3] W. Nowacki and Z. Olesiak, Vibration, buckling and bending of a circular plate clamped along part of its periphery and simply supported on the remaining part, Bull. Acad. Pol. Sci., 4, (1956), 247-258.

[4] B. Noble, The vibration and buckling of a circular plate clamped on part of its boundary and simply supported on the remainder, In Proc. Ninth Midwestern Mechanics Conference, Univ. of Wis., Madison, Wis., (1965), 141-146.

[5] C.C. Bartlett, The vibration and buckling of a circular plate clamped on part of its boundary and simply supported on the remainder, Q. Jl. Mech. Appl. Math., 16, (1963), 431-440.

[6] H.D. Conway and K.A. Farnham, Deflections of uniformly loaded circular plates with combinations of clamped, simply supported and free boundary conditions, Int. J. Mech. Sci., 9, (1967), 661-671.

[7] M.L. Williams, Surface stress singularities resulting from various boundary conditions in angular corners of plates under bending, In Proc. First U.S. National Congress of Applied Mechanics (ASME), Ill. Inst. of Tech, Chicago, Ill., 1, (1952), 325-329.

[8] B. Stahl and L.M. Keer, Bending of a uniformly loaded circular plate with mixed boundary conditions, AIAA Journal, 10, (1972), 739-743.

[9] K. Kiattikomol and V. Sriswasdi, Bending of a uniformly loaded annular plate with mixed boundary conditions, AIAA Journal, 26, (1988), 487-492.

[10] L.M. Keer and C. Sve, On the bending of cracked plates, Int. J. Solids Struct., 6, (1970), 1545-1559.

[11] B. Stahl and L.M. Keer, Vibration and stability of cracked rectangular plates, Int. J. Solids Struct., 8, (1972), 69-91.

[12] B. Stahl and L.M. Keer, Vibration and buckling of a rectangular plate with an internal support, Q. Jl. Mech. Appl. Math., 25, (1972), 467-478.

[13] Y. Sompornjaroensuk and K. Kiattikomol, Singularity of advancing contact problems of plate bending, In Proc. Tenth East Asia-Pacific Conference on Structural Engineering and Construction (EASEC-10), Bangkok, Thailand, (2006), 193-198.

[14] J. Dundurs, K. Kiattikomol and L.M. Keer, Contact between plates and sagged supports, J. Eng. Mech. Div. (ASCE), 100, (1974), 445-456.

[15] J. Dundurs and M. Stippes, Role of elastic constants in certain contact problems, J. Appl. Mech. (ASME), 37, (1970), 965-970.

[16] J.P. Dempsey, L.M. Keer, N.B. Patel and M.L. Glasser, Contact between plates and unilateral supports, J. Appl. Mech. (ASME), 51, (1984), 324-328.

[17] N.J. Salamon, T.P. Pawlak and F.F. Mahmoud, Plates in unilateral contact with simple supports: pressure loading, J. Appl. Mech. (ASME), 53, (1986), 141-145.

[18] C. Hu and G.A. Hartley, Boundary element analysis of thin plates unilaterally edge supported, Eng. Anal. Bound. Elem., 12, (1993), 47-55.

[19] K. Kiattikomol, L.M. Keer and J. Dundurs, Application of dual series to rectangular plates, J. Eng. Mech. Div. (ASCE), 100, (1974), 445-456.

[20] K. Kiattikomol, S. Porn-anupapkul and N. Thonchangya, Application of dual series equations to a square plate with varying corner supported lengths, In Proc. Recent Advances in Structural Engineering, Japan-Thai Civil Engineering Conference, Bangkok, Thailand, (1985), 115-130.

[21] G.M.L. Gradwell and K.R.P. Iyer, Unbonded contact between a circular plate and an elastic half-space, Journal of Elasticity, 4, (1974), 115-130.

[22] Y.M. Tsai, Symmetrical contact problem of a thick transversely isotropic plate, Journal of Elasticity, 16, (1986), 179-188.

[23] J. De and B. Patra, Cruciform crack in an orthotropic elastic plane, Int. J. Fracture, 53, (1992), 387-397.

[24] H. Fildis and O.S. Yahsi, The axisymmetric crack problem in a non-homogeneous interfacial region between homogeneous half-spaces, Int. J. Fracture, 78, (1996), 139-164.

[25] B. Wang, J. Han and S. Du, Dynamic response for functionally graded materials with penny-shaped cracks, Acta Mechanica Solida Sinica (AMSS), 12, (1999), 106-113.

[26] I.N. Gradshteyn and I.M. Ryzhik, Table of integrals, series, and products, 4^{th} ed., Academic Press, New York, 1956.

[27] M. Abramowitz and I.A. Stegun, Handbook of mathematical functions with formulas, graphs, and mathematical tables, Dover Publications, New York, 1964.

[28] G.H. Golub and C.F. Van Loan, Matrix computations, 2^{nd} ed., The Johns Hopkins, University Press, Baltimore, 1990.

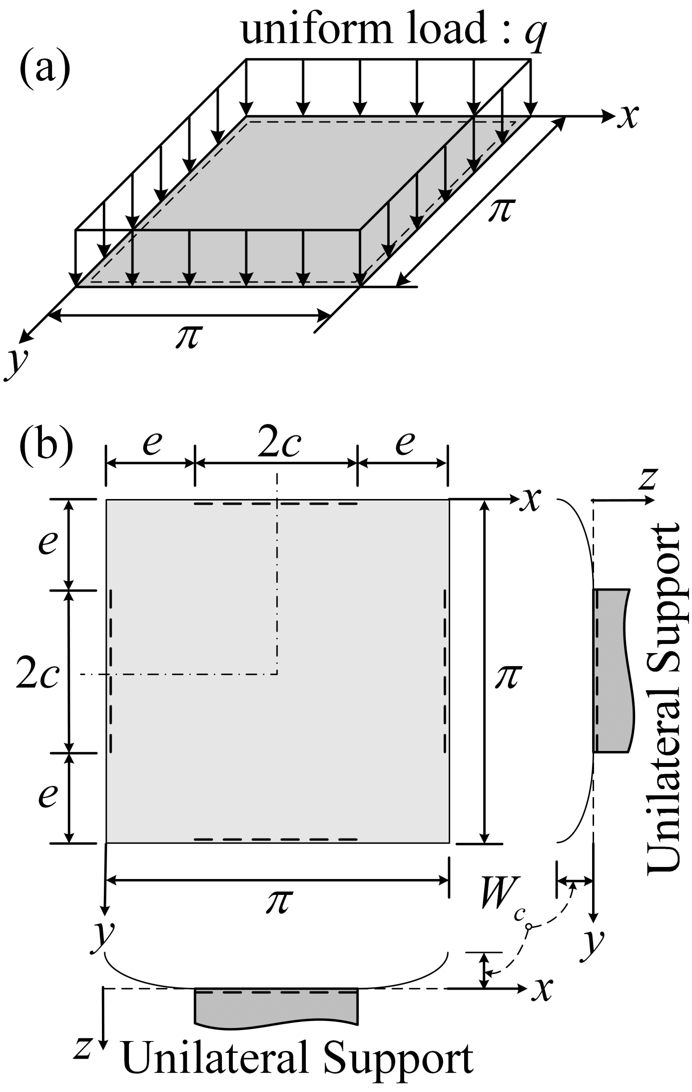

Figure 1: Uniformly loaded square plate with (a) simply supported edges and (b) unilaterally supported edges

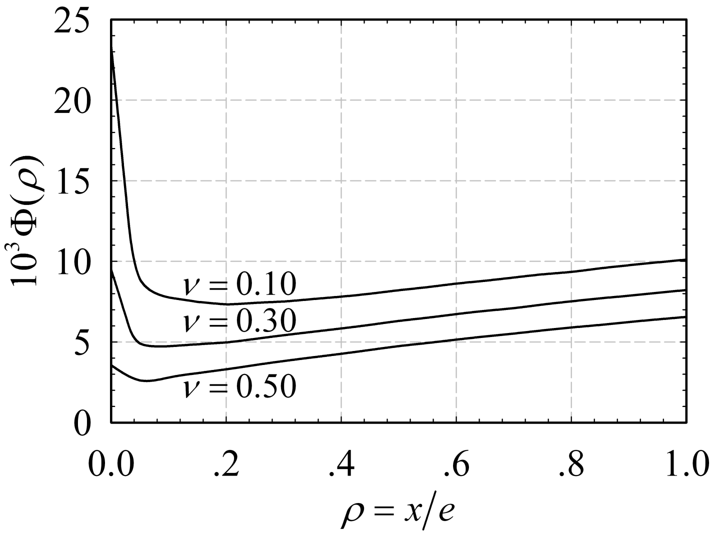

Figure 2: Auxiliary function $\Phi(\rho)$ in integral equation

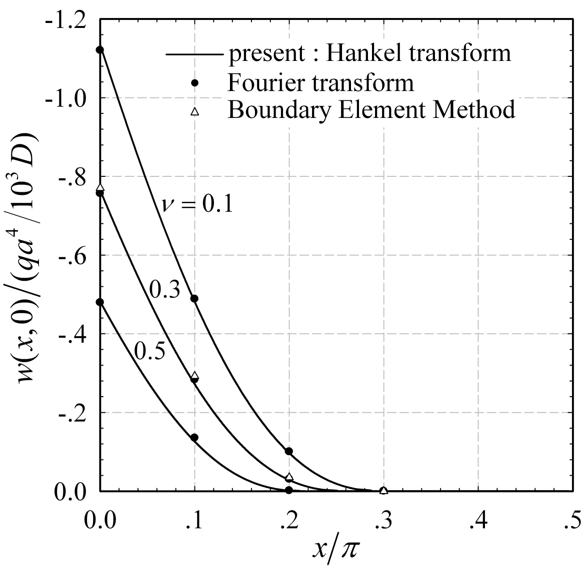

Figure 3: Deflections of free edge for unilaterally supported square plate

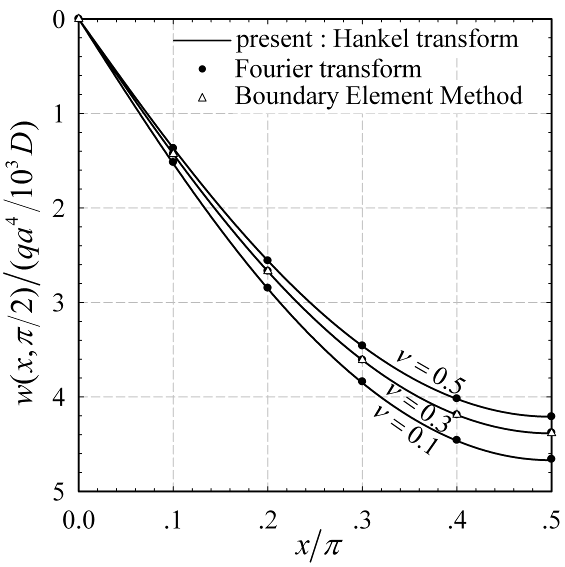

Figure 4: Deflections along the middle line for unilaterally supported square plate

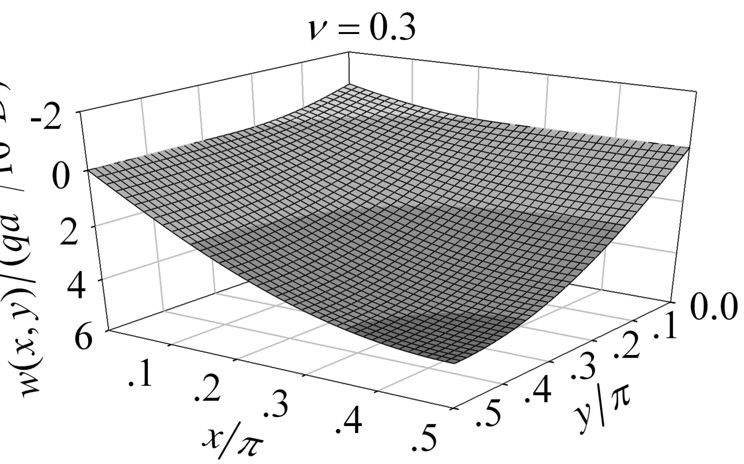

Figure 5: Deflection surface for a quarter segment ($0 \leq x, y \leq \pi/2$) of square plate ($\nu = 0.3$)

Submitted on June 2007.

Rešenje nekog gubitka kontakta medju pločom i jednostranim oloncima

Razmatra se gubitak kontakta medju kvadratnom pločom i jednostranim osloncima pri jednoliko rasporedjenom opterećenju. Pošto ploča miruje na jednostranim osloncima, imaće oblasti izgubljenog kontakta zbog odsustva ugaonih sila na uglovima ploče. Ovo vodi ka mešovitim graničnim uslovima koji su tada napisani u obliku jednačina dvostrukih redova. Ove se, pak, redukuju na Fredholmovu integralnu jednačinu uzimajući u obzir Hankelovu transformacionu tehniku. Numerički rezultati su dati za odstupanja slobodne ivice kao i odstupanja duž srednje linije ploče za različite vrednosti Poasonovog količnika. Pored toga i površ odstupanja je takodje prikazana. Može se zaključiti da se gubitak kontakta smanjuje porastom Poasonovog količnika.

doi:10.2298/TAM0704289S Math.Subj.Class.: 74K20, 47A13, 45B05