

Hall effects on MHD flow past an accelerated plate

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Abstract

The simultaneous effects of rotation and Hall current on the hydromagnetic flow past an accelerated horizontal plate relative to a rotating fluid is presented. It is found that for given values of m (Hall parameter), M (Hartmann number) and an imposed rotation parameter Ω satisfying $\Omega = M^2m/(1+m^2)$, the transverse motion (transverse to the main flow) disappears and the fluid moves in the direction of the plate only. The effects of the parameters m , M and Ω on the axial and transverse velocity profiles are shown graphically, whereas the effects of the parameters on the skin-friction components are shown by tabular values.

Keywords: Hall effect, MHD flow, accelerated plate

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List of notations

A	constant acceleration
$H_z z$	component of magnetic field \mathbf{H}
H_0	applied magnetic field
$J_z z$	component of current density \mathbf{J}
μ_e	magnetic permeability
m	Hall parameter
M	Hartmann number
ν	kinematic viscosity
$\Omega_z z$	component of angular velocity
Ω	non-dimensional angular velocity
ρ	fluid density
σ	electric conductivity
t	time
T	non-dimensional time
(u, v, w)	components of velocity field \mathbf{q}
(U, V, W)	non-dimensional velocity components
(x, y, z)	Cartesian co-ordinates
Z	non-dimensional coordinate normal to the plate

1 Introduction

Stokes [1] first investigated the incompressible viscous flow past an infinite flat plate, which is being impulsively started from rest into motion in its own plane with a constant velocity, on the motion of pendulums. Rossow [2] studied the MHD flow due to an impulsive start of an infinite flat plate. It was shown by Cowling [3] that when the strength of the magnetic field is very large Ohm's law must be modified to include Hall currents. The mechanism of conduction in ionized gases in the presence of strong magnetic field is different from that in metallic substance. The electric current in ionized gases is generally carried by electrons, which undergo successive collisions with other charged or neutral particles. In the ionized gases the current is not proportional to the applied potential except when the field is very weak. In an ionized gas where the density is low and the magnetic field is very strong, the conductivity normal to

the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon, well known in the literature, is called the Hall effect. Watanbe and Pop [4] studied the effect of Hall current on the steady MHD flow over a continuously moving flat plate, when the liquid is permeated by a uniform transverse magnetic field, while Pop [5] considered the Hall effects on the MHD flow due to an impulsive start of the plate, which is valid only for small time. Kinyanjui et al.[6] studied the heat and mass transfer in unsteady free convection flow with radiation absorption past an impulsively started infinite vertical porous plate subjected to strong magnetic field including the Hall effect. Maleque and Sattar [7] investigated the steady laminar flow on a porous rotating disk with variable fluid properties taking Hall effect into account.

The study of MHD viscous flows with Hall currents has important engineering applications in problems of MHD generators, Hall accelerators as well as in flight magnetohydrodynamics. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in cosmical and geophysical fluid dynamics. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars. It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydromagnetic spin-up. This problem of spin-up in magnetohydrodynamic rotating fluids has been discussed under varied conditions by many researchers, for example Takhar et al.[8], Debnath [9], Takhar and Nath [10], and Singh [11]. In all these studies, the effect of Hall current is not considered. Recently, Hayat and Abbas [12] studied the fluctuating rotating flow of a second-grade fluid past a porous heated plate with variable suction and Hall current, while Takhar et al.[13] investigated the simultaneous effects of Hall current and free stream velocity on the magnetohydrodynamic flow over a moving plate in a rotating fluid.

In this study we have considered the magnetohydrodynamic flow past an accelerated horizontal plate in a rotating fluid in the presence of Hall current. It is well known that the effect of Coriolis force due to the Earth's rotation is found to be significant as compared to the inertia and viscous forces in the equations motion. The Coriolis force exerts

a strong influence on the hydromagnetic flow in the earth's liquid core, which plays an important role in the mean geomagnetic field. It may be noted that the flow situations studied in the present problem occur in tornadoes, or flows past rotor hubs and rotating blades in a modern helicopter used for defence purpose. In this analysis, the axial velocity (along the direction of the plate) and transverse velocity (transverse to the main flow) components are presented graphically to show the effects of the Hall parameter m , Hartmann number M and the rotation parameter Ω (representing rotational speed with which the plate and the fluid rotates in unison), while the values of skin-friction are presented in a table.

Table 1: Variation of the skin-friction components with T , M , m and Ω

T	M	m	Ω	τ_x	τ_y
0.2	0.0	0.0	0.4	0.50506	0.02690
0.2	0.5	0.5	0.4	0.51821	0.01986
0.2	0.5	0.5	0.1	0.51798	0.00000
0.2	0.5	1.0	0.4	0.51319	0.01832
0.2	0.5	5.0	0.4	0.50561	0.02365
0.2	1.0	0.5	0.4	0.55681	0.00000
0.2	5.0	0.5	0.4	1.11628	-0.84541
0.4	0.5	0.5	0.4	0.75239	0.05528
0.6	0.5	0.5	0.4	0.94574	0.09993

2 Mathematical analysis and solution

We consider an unsteady flow of an electrically conducting fluid past an infinite flat plate occupying the plane $z = 0$. Initially the fluid and the plate rotate in unison with a uniform angular velocity Ω_z about the z -axis normal to the plate. The x -axis is taken in the direction of the motion of the plate and y -axis lying on the plate normal to both x and z -axes. Relative to the rotating fluid, the plate is impulsively started from rest and set into motion with uniform acceleration in its own plane along the x -axis. A uniform magnetic field H_o , parallel to z -axis is imposed and the plate is electrically non-conducting. Due to the horizontal homogeneity of the problem, the flow quantities depend on z and t only, t being the

time variable. If (u, v, w) be the components of the velocity vector \mathbf{q} , then the equation of continuity $\nabla \cdot \mathbf{q} = 0$, gives $w = 0$ everywhere in the flow such that the boundary condition $w = 0$ is satisfied at the plate. The solenoidal relation for magnetic field $\nabla \cdot \mathbf{H} = 0$, gives $H_z = \text{constant} = H_o$ everywhere in the flow. Also, the pressure is uniform in the flow field. Similarly the equation of conservation of electric charge, $\nabla \cdot \mathbf{J} = 0$ gives $J_z = \text{constant}$, and this constant is zero since the plate is considered to be nonconducting. Also, the fluid far away from the plate is assumed undisturbed. Under these assumptions, in a rotating frame of reference and modification of Ohm's law (see Cowling [2] the momentum equations for the unsteady flow are now given by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega_z v - \frac{\sigma \mu_e^2 H_o^2}{\rho(1+m^2)}(u + mv) \quad (1)$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial z^2} - 2\Omega_z u + \frac{\sigma \mu_e^2 H_o^2}{\rho(1+m^2)}(mu - v) \quad (2)$$

where the second term on the right of the equations (1) and (2) are due to Coriolis force assumed small. Here u is the axial velocity (along the direction of the plate) and v is the transverse velocity (transverse to the main flow). The initial and boundary conditions are given by

$$u = 0, v = 0 \quad \text{at} \quad t \leq 0 \quad \text{for all} \quad z \quad (3)$$

$$\left. \begin{array}{l} u = At, v = 0 \text{ at } z = 0 \\ u \rightarrow 0, v \rightarrow 0 \text{ as } z \rightarrow \infty \end{array} \right\} t > 0 \quad (4)$$

where $A(> 0)$ is a constant.

Now introducing the dimensionless quantities

$$\left. \begin{array}{l} U = \frac{u}{(A\nu)^{1/3}}, V = \frac{v}{(A\nu)^{1/3}}, Z = z(A/\nu^2)^{1/3} \\ T = t(A^2/\nu)^{1/3}, \Omega = \Omega_z(\nu/A^2)^{1/3}, M^2 = \frac{\sigma \mu_e^2 H_o^2 \nu^{1/3}}{2\rho A^{2/3}} \end{array} \right\} \quad (5)$$

in Eqs. (1)-(2) and boundary conditions (3)-(4) we have,

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Z^2} + 2V \left(\Omega - \frac{M^2 m}{1+m^2} \right) - 2 \frac{M^2 U}{1+m^2} \quad (6)$$

$$\frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial Z^2} - 2U \left(\Omega - \frac{M^2 m}{1+m^2} \right) - 2 \frac{M^2 V}{1+m^2} \quad (7)$$

together with the boundary conditions

$$U = 0, V = 0 \quad \text{at } T \leq 0 \quad \text{for all } Z \quad (8)$$

$$\left. \begin{array}{l} U = T, V = 0 \text{ at } Z = 0 \\ U \rightarrow 0, V \rightarrow 0 \text{ as } Z \rightarrow \infty \end{array} \right\} T > 0 \quad (9)$$

Now equations (6)-(7) and boundary conditions (8)-(9) can be combined to give

$$\frac{\partial F}{\partial T} = \frac{\partial^2 F}{\partial Z^2} - 2F \left\{ \frac{M^2}{1+m^2} + i \left(\Omega - \frac{M^2 m}{1+m^2} \right) \right\} \quad (10)$$

with boundary conditions

$$F = 0 \quad \text{at } T \leq 0 \quad \text{for all } Z \quad (11)$$

$$\left. \begin{array}{l} F = T \text{ at } Z = 0 \\ F \rightarrow 0 \text{ as } Z \rightarrow \infty \end{array} \right\} T > 0 \quad (12)$$

where $F = U + iV$.

Taking Laplace transform of (10) along with the initial and boundary conditions (11) and (12) give rise to

$$\bar{F} = \frac{1}{s^2} \exp(-Z\sqrt{s+a}) \quad (13)$$

where

$$a = 2 \frac{M^2}{1+m^2} + 2i \left(\Omega - \frac{M^2 m}{1+m^2} \right) \quad (14)$$

and (Z, s) is the Laplace transform of $F(Z, T)$.

Applying Hetnarski's [15] algorithm, we have obtained the inverse Laplace transform of \bar{F} from (13) to obtain $F = U + iV$ as

$$F = \left[\left\{ \frac{T}{2} - \frac{Z}{4\sqrt{a}} \right\} \exp(-Z\sqrt{a}) \operatorname{erfc} \left(\frac{Z}{2\sqrt{T}} - \sqrt{aT} \right) + \left\{ \frac{T}{2} + \frac{Z}{4\sqrt{a}} \right\} \exp(Z\sqrt{a}) \operatorname{erfc} \left(\frac{Z}{2\sqrt{T}} + \sqrt{aT} \right) \right] \quad (15)$$

Interestingly from Eq. (10) it is observed that when $\Omega = M^2 m / (1 + m^2)$, the contribution of V into $F = U + iV$ disappears, so that the

transverse component of velocity, $V = 0$ everywhere in the flow field and thus the flow is reduced to unidirectional flow which is along the direction of the plate only.

In order to get a clear understanding of the flow field, we have carried out numerical calculation of Eq. (15) by separating F into real and imaginary parts to obtain the axial and transverse velocity components U and V . Since the arguments of the complementary error functions $erfc$ are complex we have separated the functions into real and imaginary parts (see Abramowitz and Stegun [16]).

The dimensionless skin friction at the plate $Z = 0$ is derived from Eq. (15) as

$$-\left(\frac{dF}{dZ}\right)_{Z=0} = \sqrt{\frac{T}{\pi}} \exp(-aT) + \frac{erf(\sqrt{aT})}{2\sqrt{a}}(1 + 2aT) \quad (16)$$

Separating $-(dW/dZ)_{Z=0}$ into real and imaginary parts, the dimensionless skin-friction components $\tau_x (= -(dU/dZ)_{Z=0})$ and $\tau_y (= -(dV/dZ)_{Z=0})$ can be computed. It is observed from (16) that, the components of skin-friction are oscillating one and increase unboundedly for an increase in time, which is due to the plate being an accelerated one in contrast to a stationary plate.

3 Results

First, the effects of rotation parameter Ω on the variation of axial velocity U and transverse velocity V in the presence of Hall parameter ($m = 0.5$) and Hartmann number ($M = 0.5$) are presented in Figures 1 and 2 respectively for time $T = 2.0$ against the Z axis.

It is observed that the axial velocity decreases as Ω increases from 0.1 to 0.8, while the transverse velocity V increases. The negative sign for V in this figure 2 indicates that this component is transverse to the main flow direction in the clockwise sense. The curve corresponding to $\Omega = 0.1$ in Figure 2 merges with the axis of Z , which means that for this given value of Ω the transverse component vanishes indicating a unidirectional flow and the flow is in the direction of the plate only. As referred earlier for $m = 0.5, M = 0.5, \Omega = M^2m/(1 + m^2) = 0.1$, and thus for this imposed rotational value, the transverse velocity is zero everywhere in

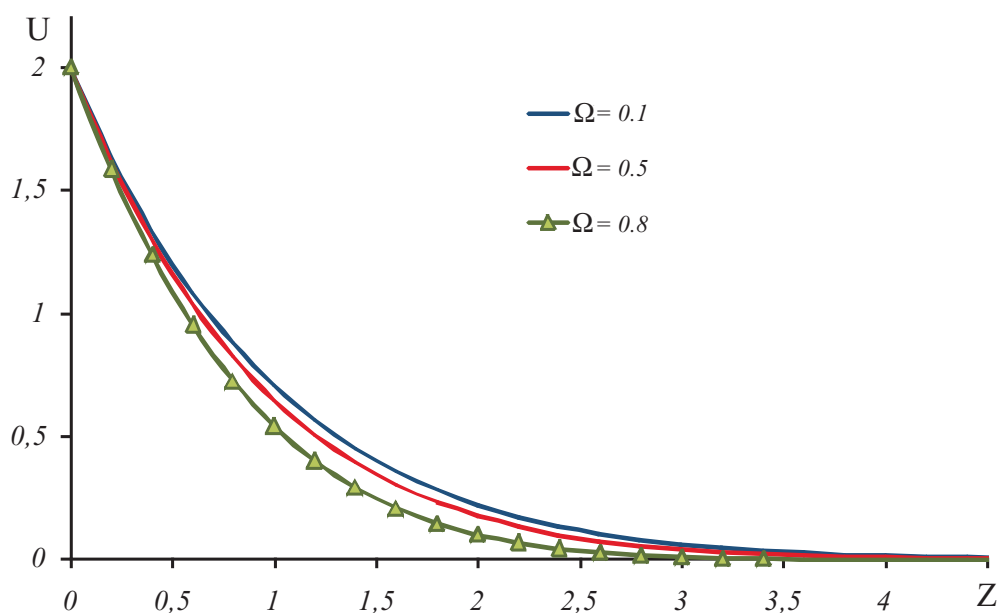


Figure 1: Axial velocity profiles for several values of Ω with $M = 0.5$, $m = 0.5$, $T = 2.0$

the flow field. Thus we conclude that for $\Omega = M^2m/(1 + m^2)$, for given m and M , the axial velocity is maximum, and decreases as Ω increases. But reverse trend is seen for transverse velocity with minimum for $\Omega = M^2m/(1 + m^2)$ and increases as Ω increases. Therefore, the introduction of Hall current on the MHD flow past an accelerated plate reduces the flow to a unidirectional flow when $\Omega = M^2m/(1 + m^2)$ which is an added realism over the study made by Deka et al.[14].

Figures 3 and 4 are drawn to show the effect of Hall parameter on the fluid velocities with the assigned values of $M = 0.5$, $\Omega = 0.1$ and $T = 2.0$.

Here also, the curve for $m = 0.5$ in case of transverse velocity shown in Figure 4 coincides with the Z axis, indicating the absence of transverse component of velocity. It is observed that due to an increase in the Hall parameter there is rise in both the axial and transverse velocity components. The effects of Magnetic field parameter on the velocity components U and V are shown in Figures 5 and 6 respectively.

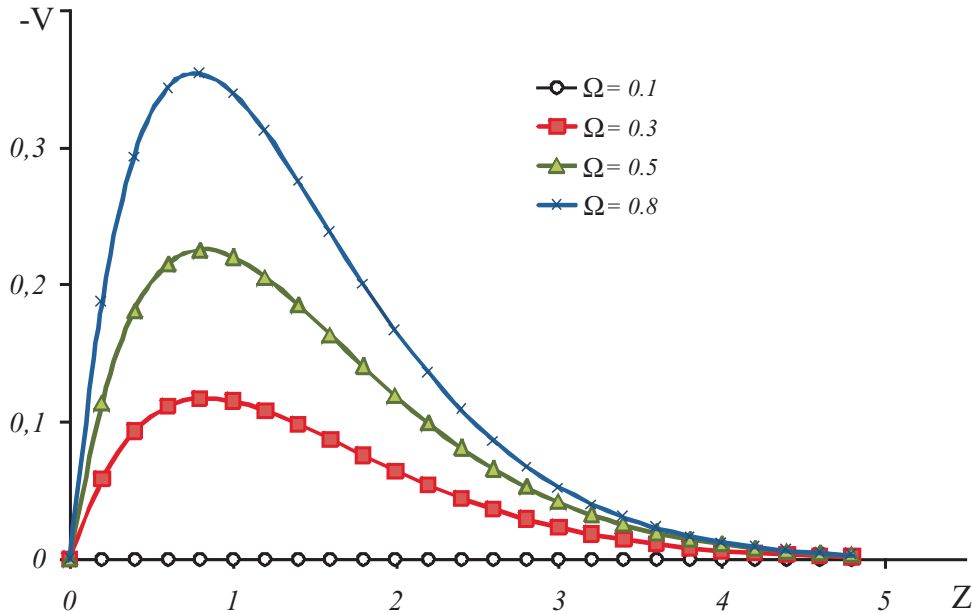


Figure 2: Transverse velocity profiles for several values of Ω with $M = 0.5$, $m = 0.5$, $T = 2.0$. The graph corresponding to $\Omega = 0.1$ satisfying the identity $\Omega = M^2m/(1+m^2)$ for $M = 0.5$ and $m = 0.5$, showing a vanishing component.

Due to an increase in the Hartmann number M , the transient axial velocity decreases while the transverse velocity increases. Interestingly as M increases beyond $M > 5$, there is sudden fall in the axial velocity, while the peak value of the transverse velocity remain same.

The effect of various parameters T , M , m and Ω on the skin-friction components τ_x , τ_y are presented in Table I. It is observed that for $T = 0.2$, $M = 0.5$, $m = 0.5$, both τ_x and τ_y increase with an increase in Ω , while the transverse component (τ_y) disappears when the identity $\Omega = M^2m/(1+m^2)$ is satisfied referred earlier as can be seen from the third and sixth sets of row in the Table. On the other hand due to an increase in m , keeping other values fixed, τ_x increases steadily, while τ_y first decreases reaches a minimum and thereafter increases for further increase in m . It is

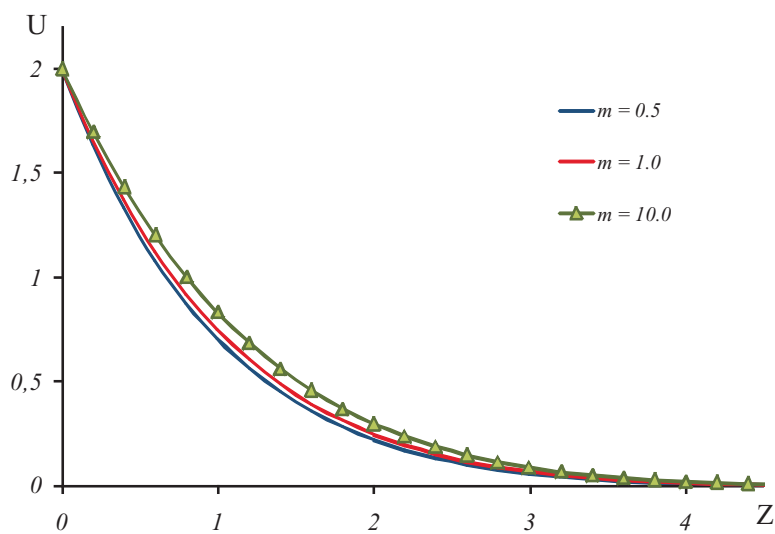


Figure 3: Axial velocity profiles for several values of m with $M = 0.5$, $T = 2.0$, $\Omega = 0.1$

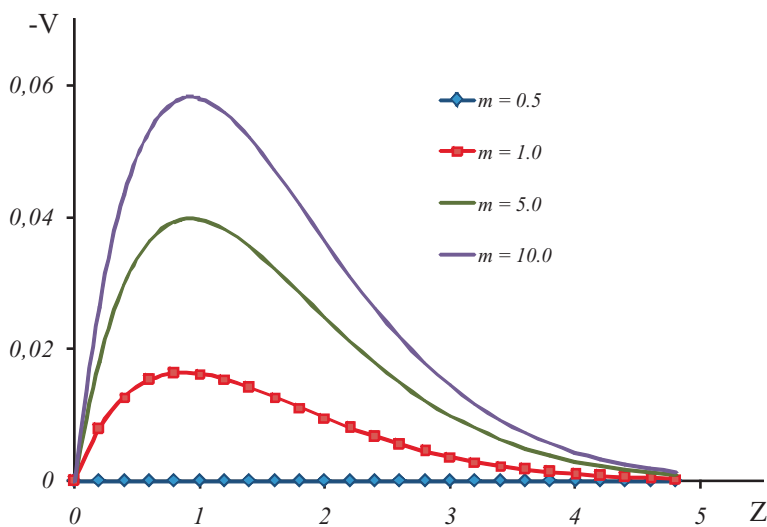


Figure 4: Transverse velocity profiles for several values of m with $M = 0.5$, $T = 2.0$, $\Omega = 0.1$. The graph corresponding to $m = 0.5$ satisfying the identity $\Omega = M^2 m / (1 + m^2)$ for $M = 0.5$, showing a vanishing component.

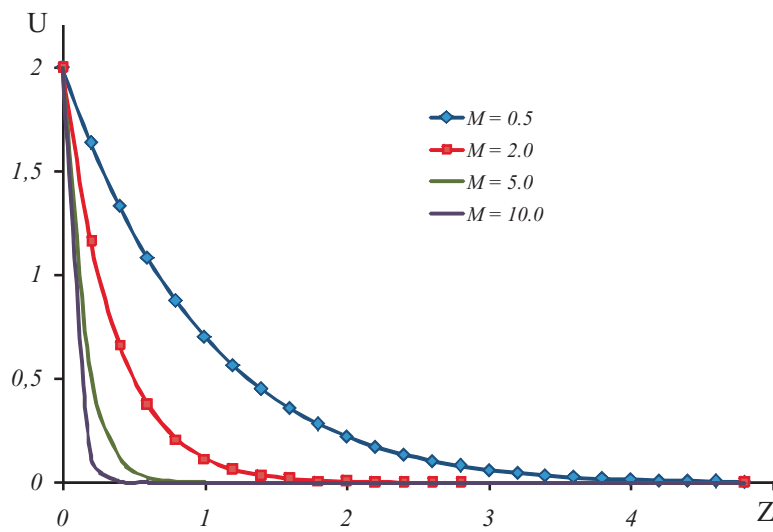


Figure 5: Axial velocity profiles for several values of M with $m = 0.5$, $T = 2.0$, $\Omega = 0.1$

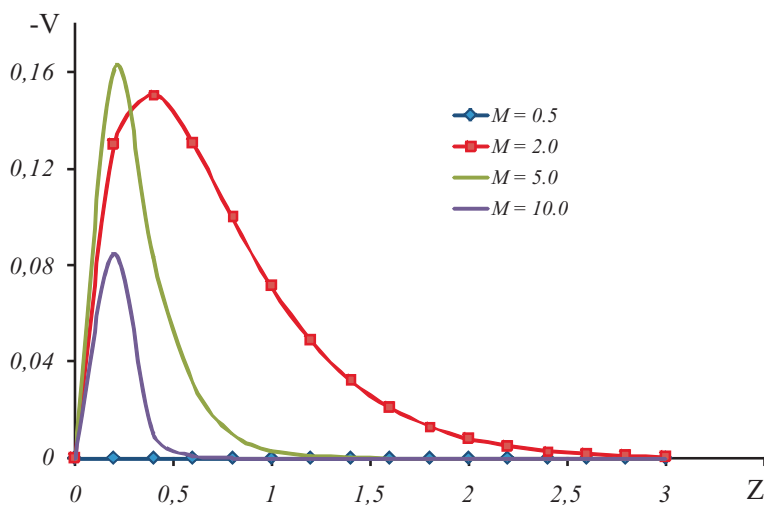


Figure 6: Transverse velocity profiles for several values of M with $m = 0.5$, $T = 2.0$, $\Omega = 0.1$. The graph corresponding to $M = 0.5$ satisfying the identity $\Omega = M^2 m / (1 + m^2)$ for $m = 0.5$, showing a vanishing component.

interesting to see that as M increases the axial component of skin-friction increases, while the transverse component decreases from a positive value to negative value, keeping other parametric values fixed. This result admits a physical interpretation. At a fixed instant, an increase in Ω causes a gradual thinning of the boundary layer develops on the plate. This results in an increase of the shear stress at the plate with increasing value of Ω . On the other hand, for fixed values of M , m and Ω , an increase in time results in an increase in the plate velocity which in turn implies a gradual thinning of the boundary layer on the plate. In addition, another prime observation in this study reveals that due to the inclusion of the magnetohydrodynamic effect (M) and Hall parameter (m), the transverse component of skin friction decreases as compared to the study made by Deka et al.[14] in absence of M and m .

4 Conclusions

The study of rotating flow of an electrically conducting fluid in the presence of Hall effect reveals the novel phenomenon of reducing the two-dimensional flow to a one-dimensional one, which do not occur in the absence of rotation. Thus, in our study it has been found that when the rotation parameter Ω with which the plate rotates in unison with the fluid equals the value $M^2m/(1+m^2)$, for given Hartmann number M and Hall parameter m , the transverse component of velocity $V = 0$ everywhere in the flow field so that the fluid moves in the direction of the plate only and thus attain a maximum axial velocity due to the absence of the flow in the transverse direction. The skin-friction component along the plate decreases steadily, while the transverse component of the skin-friction decreases, reaches a minimum for an increase in m and thereafter increases for further increase in m . Both the skin-friction components along and transverse direction of the plate increases with an increase in Ω and T which leads to the gradual thinning of the boundary layer develops on the plate. Further, it has been found that due to an increase in M , the axial skin-friction component increases, while the transverse component decreases from a positive value to a negative value.

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Submitted on October 2007.

Hall-ovi efekti na MHD tečenje preko ubrzavajuće ploče

Prikazani su istovremeni uticaji obrtanja i Hall-ove struje na hidromagnetsko tečenje preko ploče ubrzavajuće u odnosu na obrtni fluid. Pokazuje se da za date vrednosti Hall-ovog parametra m , Hartmann-ovog broja M i parametra nametnutog obrtanja Ω (određenog sa $\Omega = M^2 m / (1 + m^2)$) kretanje, poprečno u odnosu na glavni tok, iščezava te se fluid kreće samo u pravcu ploče. Uticaji parametara m , M and Ω na profile uzdužne i poprečne brzine su prikazani grafički, dok su uticaji ovih parametara na trenje na zidu dati tabelarnim vrednostima.