Theoret. Appl. Mech., Vol.36, No.1, pp. 1–27, Belgrade 2009

MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium

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Abstract

This paper presents an analytical study of the transient hydromagnetic natural convection flow past a vertical plate embedded in a porous medium, taking account of the presence of mass diffusion and fluctuating temperature about time at the plate. The governing equations are solved in closed form by the Laplace-transform technique. The results are obtained for temperature, velocity, penetration distance, Nusselt number and skin-friction. The effects of various parameters are discussed on the flow variables and presented by graphs.

Keywords: Natural convection, heat and mass transfer, magnetohydrodynamic flow, porous medium.

1 Introduction

The buoyancy force induced by density differences in a fluid causes natural convection. Natural convection flows are frequently encountered in physical and engineering problems such as chemical catalytic reactors, nuclear waste materials etc. Transient free convection is important in

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many practical applications, such as furnaces, electronic components, solar collectors, thermal regulation process, security of energy systems etc. When a conductive fluid moves through a magnetic field, an ionized gas is electrically conductive, the fluid may be influenced by the magnetic field. Magnetohydrodynamic free convection heat transfer flow is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, liquid metal fluids and MHD power generation systems etc. Transport processes in porous media are encountered in a broad range of scientific and engineering problems associated with the fibre and granular insulation materials, packed-bed chemical reactors and transpiration cooling. Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation and underground energy transport. The change in wall temperature causing the free convection flow, could be a sudden or a periodic one, leading to a variation in the flow. In nuclear engineering, cooling of medium is more important from safety point of view and during this cooling process the plate temperature starts oscillating about a non-zero constant mean temperature. Further, oscillatory flow has applications in industrial and aerospace engineering. In the literature, extensive research work performed to examine the effect of natural convection on flow past a plate. Examples of this include, Vedhanayagam et. al. [1], Martynenko et. al. [2], Kolar et. al. [3], Ramanaiah et. al. [4], Camargo et. al. [5] and Li et. al. [6]. Transient free convection flow past an isothermal vertical plate was first reported by Siegel [7] using an integral method. The experimental confirmation of these results discussed by Goldstein et. al. [8]. Another review of transient natural convection presented by Raithby et. al. [9] wherein a large number of papers on this topic were referred. In this review, the meaning of transient convection has been explained systematically. They have defined the conduction regime and the steady-state regime and that which lies between these two regimes as the transient regime. In reference to transient convection Gebhart et. al. [10] introduced the idea of leading edge effect in their book. Other studies deal with transient natural convection are by Harris et. al. [11], Das et. al. [12] and Saeid [13]. Simultaneous heat and mass transfer in laminar free convection boundary layer flows over surface can be found in monograph by Gebhart et. al.

[10] and in papers by Khair et. al. [14], Lin et. al. [15] and Mongruel e.t al. [16].

Fewer studies have been carried out to investigate the magnetohydrodynamic free convection flow. The transient natural convection flow from a plate in the presence of magnetic current first studied by Gupta [17]. Recently, Aldoss et. al. [18] investigated MHD transient free convection flow over a surface by finite difference method.

The studies of convective heat transfer in porous media have been more concerned in the past, with steady state conditions [19,20]. Meanwhile, recent engineering developments have led also to an increasing interest in accurate investigations of the transient processes in these media. Transient free convection flow past a plate embedded in a porous medium pioneered by Cheng et. al. [21]. Mass diffusion effect on transient convection flow past a surface eludicated by Jang et. al. [22], Cheng et. al. [23] and Pop et. al. [24]. A detailed review of the subject including exhaustive list of references can be found in the papers by Bradean et. al. [25] and Pop et. al. [26]. Recently, Chaudhary et. al. [27,28] analyzed free convection effects on flow past a moving vertical plate embedded in porous medium by Laplace-transform technique.

Hence, Based on the above mentioned investigations and applications, the objective of this paper is to study magnetohydrodynamic transient heat and mass transfer flow by free convection past a vertical plate, when the temperature of the plate oscillates in time about a constant mean temperature and the plate is embedded in a porous medium. The present analysis may be regarded as an extension of the work of Das et.al. [12] to include the effects of mass transfer, magnetic field and porous medium. The present investigation may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, underground water in river beds, filteration and water purification processes. This study of flow past a vertical surface can be utilized as the basis of many scientific and engineering applications, including earth science, nuclear engineering and metallurgy.In nuclear engineering ,it finds its applications for the design of the blanket of liquid metal around a thermonuclear fusion-fission hybrid reactor.In metallurgy, it can be applied during the solidification process.The results of the problem are also of great interest in geophysics, in the study of interaction of geomagnetic field with the fluid in the geothermal region.

2 Mathematical analysis

We consider a two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical plate that is embedded in a porous medium. The x'-axis is taken along the infinite plate and y'-axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time $t' > 0$, the plate temperature is raised to T'_w and a periodic temperature is assumed to be superimposed on this mean constant temperature of the plate and the concentration level at the plate is raised to C'_w. A magnetic field of uniform strength is applied perpendicular to the plate and the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected (Cowling [29]). There is no applied electric field. Viscous and Darcy resistance term is taken into account with the constant permeability porous medium.The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equations.We regard the porous medium as an assembled of small identical spherical particles fixed in space, following Yamamoto et.al. [30] .Under these conditions and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \tag{1}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{2}$$

$$\frac{\partial u'}{\partial t'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta_c(T' - T'_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'} \tag{3}$$

with following initial and boundary conditions:

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty\ \ for\ \ all\ \ y',\ t' \leq 0$$

$$u' = 0, T' = T'_w + \in (T'_w - T'_\infty)\, cos\omega' t',\ C' = C'_w\ \ at\ \ y' = 0,\ t' > 0$$

$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty\ \ as\ \ y' \to \infty, t' > 0, \tag{4}$$

where B_0 is magnetic field component along y'-axis, C' is concentration at any point in the flow field, C'_w is concentration at the plate, C'_∞ is concentration at the free stream, D is mass diffusivity, C_p is specific heat at constant pressure, g is gravitational acceleration, T' is temperature of the fluid near the plate, T'_w is the plate temperature, T'_∞ is temperature of the fluid far away from the plate, β is coefficient of volume expansion, β_c is concentration expansion coefficient, ρ is density, $\in$ is amplitude (constant), κ is thermal conductivity of fluid, ν is kinematic viscosity.

The second term of R.H.S. of the momentum equation (3) denotes buoyancy effects, the third term is the MHD term, the fourth term is bulk matrix linear resistance, that is Darcy term. The heat due to viscous dissipation is neglected for small velocities in equation (1). Also, Darcy dissipation term is neglected because it is the same order-of- magnitude as the viscous dissipation term.

The temperature distribution is independent of the flow and heat transfer is by conduction alone. This is true for fluids in initial stage due to the absence of convective heat transfer or at small Grashof number flow ($Gr \leq 1$).

We introduce the non-dimensional variables

$$t = \frac{t'}{t_R},\ y = \frac{y'}{L_R},\ u = \frac{u'}{U_R},\ \omega = \omega' t_R,\ K = \frac{{U_R}^2 K'}{\nu^2}$$

$$Pr = \frac{\mu C_p}{\kappa},\ M = \frac{\sigma B_0^2 \nu}{\rho U_R^2},\ Sc = \frac{\nu}{D},\ \theta = \frac{T' - T'_\infty}{T'_w - {T'}_\infty},$$

$$\phi = \frac{C' - {C'}_\infty}{{C'}_w - {C'}_\infty},\ Gm = \frac{\nu g \beta_c ({C'}_w - {C'}_\infty)}{U_R^3},\ \Delta T = T'_w - {T'}_\infty,$$

$$U_R = (\nu g \beta \Delta t)^{1/3},\ L_R = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},\ t_R = (g\beta\Delta T)^{-2/3}\nu^{1/3}, \qquad (5)$$

where K is permeability parameter, Pr is Prandtl number, Gm is modified Grashof number, M is magnetic parameter, Sc is Schmidt number, t is time in dimensionless coordinate, L_R is reference length, t_R is reference time, u is dimensionless velocity component, U_R is reference velocity, μ is viscosity of fluid, θ is dimensionless temperature, ϕ is dimensionless concentration, ω is frequency of oscillation.

The Equations (1) – (4) reduce to following non-dimensional form:

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \qquad (6)$$

$$Sc\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \tag{7}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta + Gm\phi - \left(M + \frac{1}{K}\right)u \tag{8}$$

with the following initial and boundary conditions:

$$u = 0,\ \ \theta = 0,\ \ \phi = 0\ \ for\ all\ \ y, t \leq 0 \tag{9}$$

$$\left.\begin{array}{ll} u = 0, \theta = 1 + \in cos\,\omega t,\ \phi = 1\ \ at\ y = 0, t > 0 \\ u \to 0,\ \theta \to 0,\ \phi \to 0 \quad as\ y \to \infty,\ t > 0 \end{array}\right\} \tag{10}$$

where ωt is phase angle.

On taking Laplace-transform of Eqs. (6) to (8) and Eq.(10), we get

$$\frac{d^2\bar{\theta}}{dy^2} - p\,Pr\,\bar{\theta} = 0 \tag{11}$$

$$\frac{d^2\bar{\phi}}{dy^2} - pSc\,\bar{\phi} = 0 \tag{12}$$

$$\frac{d^2\bar{u}}{dy^2} - (p + M')\bar{u} = -\bar{\theta}(y, p) \tag{13}$$

$$\left.\begin{array}{l} \bar{u} = 0,\ \bar{\theta} = \frac{1}{p} + \frac{\in p}{p^2+\omega^2}\ \ at\ y = 0,\ t > 0 \\ \bar{u} \to 0,\ \bar{\theta} \to 0\ \ as\ y \to \infty,\ t > 0 \end{array}\right\}, \tag{14}$$

where p is the Laplace transformation parameter and $M' = M + \frac{1}{K}$

On Solving Eqs.(11-13) with the help of Eq.(14),we get

$$\bar{\theta}(y, p) = \frac{exp(-y\sqrt{pPr})}{p} + \frac{\in pexp(-y\sqrt{pPr})}{p^2 + \omega^2} \tag{15}$$

$$\bar{\phi}(y, p) = \frac{exp(-y\sqrt{pSc})}{p} \tag{16}$$

$$\bar{u}(y, p) = \frac{exp(-y\sqrt{p + M'})}{p(Pr - 1)(p - c)} - \frac{exp(-y\sqrt{pPr})}{p(Pr - 1)(p - c)}$$
$$+\frac{\in p\,exp(-y\sqrt{p + M'})}{(Pr - 1)(p^2 + \omega^2)(p - c)} - \frac{\in p\,exp(-y\sqrt{p\,Pr})}{(Pr - 1)(p^2 + \omega^2)(p - c)}$$

$$+\frac{Gm}{p(Sc-1)\left(p-\dfrac{M'}{Sc-1}\right)}\left\{exp(-y\sqrt{(p+M')})-exp(-y\sqrt{pSc})\right\} \tag{17}$$

Inverting Eqs. (15) to (17), we get

$$\theta = erfc(\eta\sqrt{Pr}) + \frac{\in}{2}g(\eta\sqrt{Pr}, i\omega) + g(\eta\sqrt{Pr}, -i\omega) \tag{18}$$

$$\phi = erfc(\eta\sqrt{Sc}) \tag{19}$$

For Pr $\neq$ 1 and Sc $\neq$ 1

$$u = -\left(\frac{1+Gm}{M'}\right)exp(-M't)g(\eta, M') + \frac{1}{M'}erfc(\eta\sqrt{Pr})$$

$$-\frac{\in\ exp(-M't)}{2(Pr-1)(c^2+\omega^2)}(c-i\omega)g(\eta, M'-i\omega) + (c+i\omega)g(\eta, M'+i\omega)$$

$$+\frac{\in}{2(Pr-1)(c^2+\omega^2)}(c-i\omega)g(\eta\sqrt{Pr}, -i\omega) + (c+i\omega)g(\eta\sqrt{Pr}, i\omega)$$

$$+\left\{\frac{1}{M'}+\frac{c\in}{(Pr-1)(c^2+\omega^2)}\right\}exp(-M't)g(\eta, e) - g(\eta\sqrt{Pr}, c)$$

$$+\frac{Gm}{M'}erfc(\eta\sqrt{Sc})$$

$$+\frac{Gm}{M'}\left\{exp(-M't)g\left(\eta, \frac{M'Sc}{Sc-1}\right) - g\left(\eta\sqrt{Sc}, \frac{M'}{Sc-1}\right)\right\} \tag{20}$$

where $\eta = \frac{y}{2\sqrt{t}}$, $c = \frac{M'}{Pr-1}$, $e = \frac{M'Pr}{Pr-1}$, for Pr = 1 and Sc = 1

$$u = -\left(\frac{1+Gm}{M'}\right)exp(-M't)g(\eta, M') + \left(\frac{1+Gm}{M'}\right)erfc(\eta) \tag{21}$$

Initially, the heat is transferred through the plate by conduction. But a little later stage, convection currents start flowing near the plate. Hence, it is essential to know the position of a point on the plate where conduction mechanism changes to convection mechanism. The distance of this point of transition from conduction to convection is given by

$$X_p = \int_0^t u(y,t)dt$$

or in terms of the Laplace transform and its inverse,

$$X_p = \ L^{-1} \left[\frac{1}{p} \, L\{u(y,t)\} \right],$$

where p is Laplace transform parameter.

$$X_p = \ L^{-1} \left[\frac{\{\bar{u}(y,p)\}}{p} \right]$$

$$X_p = L^{-1} \left\{ \frac{exp(-y\sqrt{p+M'})}{p^2(Pr-1)(p-c)} \right\} + L^{-1} \left\{ \frac{exp(-y\sqrt{pPr})}{p^2(Pr-1)(p-c)} \right\}$$

$$+L^{-1} \left\{ \frac{\in exp(-y\sqrt{p+M'})}{(Pr-1)(p^2+\omega^2)(p-c)} \right\} - L^{-1} \left\{ \frac{\in exp(-y\sqrt{pPr})}{(Pr-1)(p^2+\omega^2)(p-c)} \right\}$$

$$+L^{-1} \left[\frac{Gm}{p^2(Sc-1)\left(p - \frac{M'}{Sc-1}\right)} \left\{ exp(-y\sqrt{(p+M')}) - exp(-y\sqrt{pSc}) \right\} \right] \tag{22}$$

On solving Eq.(22), we have or Pr $\neq$ 1 and Sc $\neq$ 1

$$X_p = \frac{\eta(1+Gm)}{2M'} \sqrt{\frac{t}{M'}} \left\{ exp(-2\eta\sqrt{M't}) erfc(\eta - \sqrt{M't}) \right.$$

$$\left. - exp(2\eta\sqrt{M't}) erfc(\eta + \sqrt{M't}) \right\} + \frac{t}{M'} \left\{ (1+2\eta^2 Pr) erfc(\eta\sqrt{Pr}) \right.$$

$$\left. -2\eta\sqrt{\frac{Pr}{\pi}} exp(-\eta^2 Pr) \right\}$$

$$+\frac{Gmt}{M'} \left\{ (1+2\eta^2 Sc) erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\pi} exp(-\eta^2 Sc) \right\}$$

$$+\frac{1}{M'c} erfc(\eta\sqrt{Pr}) + \frac{(Sc-1)Gm}{M'^2} erfc(\eta\sqrt{Sc})$$

$$-\left(\frac{(Gm+1)t}{M'} + \frac{1}{M'c} + \frac{Gm(Sc-1)}{M'^2} \right) exp(-M't) g(\eta, M')$$

$$-\frac{i \in exp(-M't)}{2\omega(Pr-1)(c^2+\omega^2)}\{(c-i\omega)g(\eta, M'-i\omega)-(c+i\omega)g(\eta, M'+i\omega)\}$$
$$+\frac{i \in}{2\omega(Pr-1)(c^2+\omega^2)}\{(c-i\omega)g(\eta\sqrt{Pr}, -i\omega)-(c+i\omega)g(\eta\sqrt{Pr}, i\omega)\}$$
$$+\left\{\frac{1}{M'c}+\frac{\in}{(Pr-1)(c^2+\omega^2)}\right\}\{exp(-M't)g(\eta, e)-g(\eta\sqrt{Pr}, c)\}$$
$$+\frac{Gm(Sc-1)}{M'^2}\left\{exp(-M't)g\left(\eta, \frac{M'}{Sc}Sc-1\right)-g\left(\eta\sqrt{Sc}, \frac{M'}{Sc-1}\right)\right\} \quad (23)$$

for Pr = 1 and Sc = 1

$$X_p=\frac{\eta(1+Gm)}{2M'}\sqrt{\frac{t}{M'}}\left\{exp(-2\eta\sqrt{M't})erfc(\eta-\sqrt{M't})\right.$$
$$\left.-exp(2\eta\sqrt{M't})erfc(\eta+\sqrt{M't})\right\}+\left\{\frac{(1+Gm)t}{M'}\right\}\left\{(1+2\eta^2)erfc(\eta)\right.$$
$$\left.-2\eta\sqrt{\frac{1}{\pi}}exp(-\eta^2)\right\}-\left\{\frac{(1+Gm)t}{M'}\right\}exp(-M't)g(\eta, M'), \quad (24)$$

where

$$g(a,b)=\frac{exp(bt)}{2}\{exp(2a\sqrt{bt})erfc(a+\sqrt{bt})+exp(-2a\sqrt{bt})erfc(a-\sqrt{bt})\},$$

where

$$a=\eta \ or \ \eta\sqrt{Pr} \ or \ \eta\sqrt{Sc} \ or \ and \ b=M' \ or \ i\omega \ or \ -i\omega \ or \ M'+i\omega \ or \ M'-i\omega$$
$$e \ or \ c \ or \ \frac{M'Sc}{Sc-1} \ or \ \frac{M'}{Sc-1}.$$

We have extend the problem of Das et. al.[12].Now,on setting M=0, $K \to \infty$, Gm=0 and taking the limit M'→0 our expressions for the velocity and the penetration distance are comparable with those of Das et. al. [12]. Further, our graphs for the velocity and the penetration distance are not comparable with those of Das et. al. [12]. Since in the numerical calculations for the velocity and the penetration distance they assigned the values to ωt, t, and ω separately, and the value given to ω does not match with the values of ωt and t, taken altogether, which is not the appropriate way to fix these material parameters. In our analysis, we assigned the values to ωt and t, after that from these values, the value of ω is set. Hence, our numerical results are not comparable with those of Das et.al.[12].

In expressions, erfc (x_1+iy_1) is complementary error function of complex argument which can be calculated in terms of tabulated functions (Abramowitz et.al. [31]). The table given in Abramowitz et.al. [31] do not give erfc (x_1+iy_1) directly but an auxiliary function $W_1(x_1+iy_1)$ which is defined as

$$erfc(x_1+iy_1) = W_1(-y_1+ix_1)exp\{-(x_1+iy_1)^2\}$$

Some properties of $W_1(x_1+iy_1)$ are

$$W_1(-x_1+iy_1) = W_2(x_1+iy_1)$$

$$W_1(x_1-iy_1) = 2exp\{-(x_1-iy_1)^2\} - W_2(x_1+iy_1)$$

where $W_2(x_1+iy_1)$is complex conjugate of $W_1(x_1+iy_1)$.

SKIN-FRICTION: In non-dimensional form, the skin-friction is given by

$$\tau \;=\; -\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{25}$$

For Pr $\neq$ 1 and Sc $\neq$ 1

$$\tau \;=\; \frac{1}{M'}\sqrt{\frac{Pr}{\pi t}} - \frac{1}{M'}f(M') \;-\; \frac{Gm}{\sqrt{M'}}\,erf\sqrt{M't}$$

$$+\;\frac{\in\; exp(-i\omega t)\,(c-i\omega)}{2(Pr-1)\,(c^2+\omega^2)}\{\sqrt{Pr}\,f(-i\omega)$$

$$-\,f(M'-i\omega)\} + \frac{\in\; exp(i\omega t)\,(c+i\omega)}{2(Pr-1)\,(c^2+\omega^2)}\{\sqrt{Pr}\,f(i\omega) \;-\; f(M'+i\omega)\}$$

$$+\left\{\frac{1}{M'} + \frac{c\in}{(Pr-1)\,(c^2+\omega^2)}\right\}$$

$$\left\{\frac{exp(-M't)}{\sqrt{\pi t}} - \sqrt{\frac{Pr}{\pi t}} + \sqrt{\frac{M'Pr}{Pr-1}}\,exp\left(\frac{M't}{Pr-1}\right)\right.$$

$$\left(erf\sqrt{\frac{M'Prt}{Pr-1}} - erf\sqrt{\frac{M't}{Pr-1}} \right) \Bigg\} \tag{26}$$

for Pr = 1 and Sc = 1

$$\tau = -\left(\frac{1}{M'}\right) f(M') - \frac{Gm}{\sqrt{M'}}\, erf\,\sqrt{M't} + \left\{\frac{1}{M'}\right\} \left\{\frac{exp(-M't)}{\sqrt{\pi t}}\right\} \tag{27}$$

NUSSELT NUMBER: From temperature field, the rate of heat transfer in non-dimensional form is expressed as

$$Nu = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0}$$

$$Nu = \sqrt{\frac{Pr}{\pi t}} + \frac{\in \sqrt{Pr}}{2}\{exp(-i\omega t) f(-i\omega) + exp(i\omega t) f(i\omega)\} \tag{28}$$

where

$$f(d) = \sqrt{d}\; erf\sqrt{dt} + \frac{exp(-dt)}{\sqrt{\pi t}}$$

$$d = M' \;\; or \;\; -i\omega \;\; or \;\; i\omega \;\; or \;\; M' - i\omega \;\; or \;\; M' + i\omega.$$

3 Discussion

The convection flows driven by combinations of diffusion effects are very important in many applications.The foregoing formulations may be analyzed to indicate the nature of interaction of the various contributions to buoyancy.In order to gain physical insight into the problem, the value of $\in$ is chosen 1.0. The values of Prandtl number are chosen 0.71,1,7 which represent air, electrolytic solution and water respectively at 20^oC temperature and 1 atmospheric pressure and the values of Schmidt number are chosen to represent the presence of species by hydrogen (0.22), water vapour (0.60), ammonia (0.78) and carbon dioxide (0.96) at 25^oC temperature and 1 atmospheric pressure. Figure 1 reveals the transient temperature profiles against η (distance from the plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically. The magnitude of temperature for air (Pr = 0.71) is greater than that of water (Pr = 7). This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness. The temperature falls with an increase in the phase angle ωt for both air and water, also it is noted that it falls slowly when the plate is isothermal $\left(\omega t = \frac{\pi}{2}\right)$ in comparison to the other values of ωt. Figure 2 concerns with the effect of Sc on the concentration. It is noted that the concentration at all points in the flow field decreases exponentially with η and tends to zero as $\eta \to \infty$. A comparison of curves in the figure shows a decrease in concentration with an increase in Schmidt number. Physically it is true, since the increase of Sc means decrease of molecular diffusivity . That results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Sc and lower for large values of Sc.

Figure 3 represents the velocity profiles due to the variations in ωt, Sc and Pr. It is evident from the figure that the velocity increases and attains its maximum value in the vicinity of the plate and then tends to zero as $\eta \to \infty$. The velocity for Pr = 0.71 is higher than that of Pr = 7. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Further,the velocity decreases with an increase in ωt for both air and water when hydrogen gas is presented in the flow.Moreover,the velocity is marginally affected by the variations in the phase angle. The velocity decreases owing to an increase in the value of Sc when the plate is isothermal for both Pr = 0.71 and Pr = 7. Figure 4 reveals the effects of M, K, Pr on the velocity profiles. It is obvious from the figure that the velocity near the plate exceeds at the plate i.e. the velocity overshoot occurs. It is observed that an increase in the value of M leads to fall in the velocity. It is because that the application of transverse magnetic field will result a resistitive type force (Lorentz force) similar to drag force which tends to resist the flow and thus reducing its velocity. The presence of a porous medium increases the resistance to flow resulting in decrease in the flow velocity. This behaviour is depicted by the decrease in the velocity as K decreases for both air and water. The magnitude of velocity for air is higher than that of water. Figure 5 illustrates the influences of t, Gm and Pr on the velocity. It is obvious from the figure that the maximum velocity attains in the vicinity of the plate then decreases to zero as $\eta \to \infty$. It is noted that the velocity increases with increasing time t for both air and water.

Further, the magnitude of velocity leads to an increase with an increase in Gm. It is due to the fact that an increase in the value of modified Grashof number has the tendency to increase the mass buoyancy effect. It is also found that the effect of time on the velocity is more dominant than other parameters.

Effects of variations in ωt, Sc and Pr on the penetration distance are presented in figure 6. It is clear from the figure that the penetration near the plate increases owing to the presence of the foreign gases such as hydrogen and water vapour. Further,we noticed that it decreases with an increase in the value of Sc for an isothermal plate. The penetration distance decreases on increasing ωt when hydrogen gas is presented in the flow for both Pr = 0.71 and Pr = 7. Like the velocity, the penetration is marginally affected by the variations in the phase angle. Figure 7 shows the effects of the variations in M, K, Pr on the penetration .It is noted that the penetration falls owing to an increase in the magnetic parameter for both air and water. On the contrary, it increases with an increase in K. The reason for them is same as that of explained for the velocity. Figure 8 concerns with the penetration against η for the various values of t ,Gm and Pr. It is concluded from the figure that it increases with increase in t and Gm.On the other hand, it decreases with an increase in Pr.Again, the reason for it is same as that of explained for the velocity.

Figure 9 depicts the Nusselt number against time. It is found that the rate of heat transfer falls with increasing ωt. Nusselt number for Pr = 7 is higher than that of Pr = 0.71. The reason is that smaller values of Pr are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Prandtl number. Hence, the rate of heat transfer is reduced. Figure 10 reveals the skin-friction against time t for various values of parameters M, K, Gm, Sc and Pr. It is noticed that the skin friction decreases with an increase in permeability parameter, modified Grashof number and Schmidt number while it increases with an increase in magnetic parameter for both air and water. The magnitude of the skin-friction for water is greater than air.

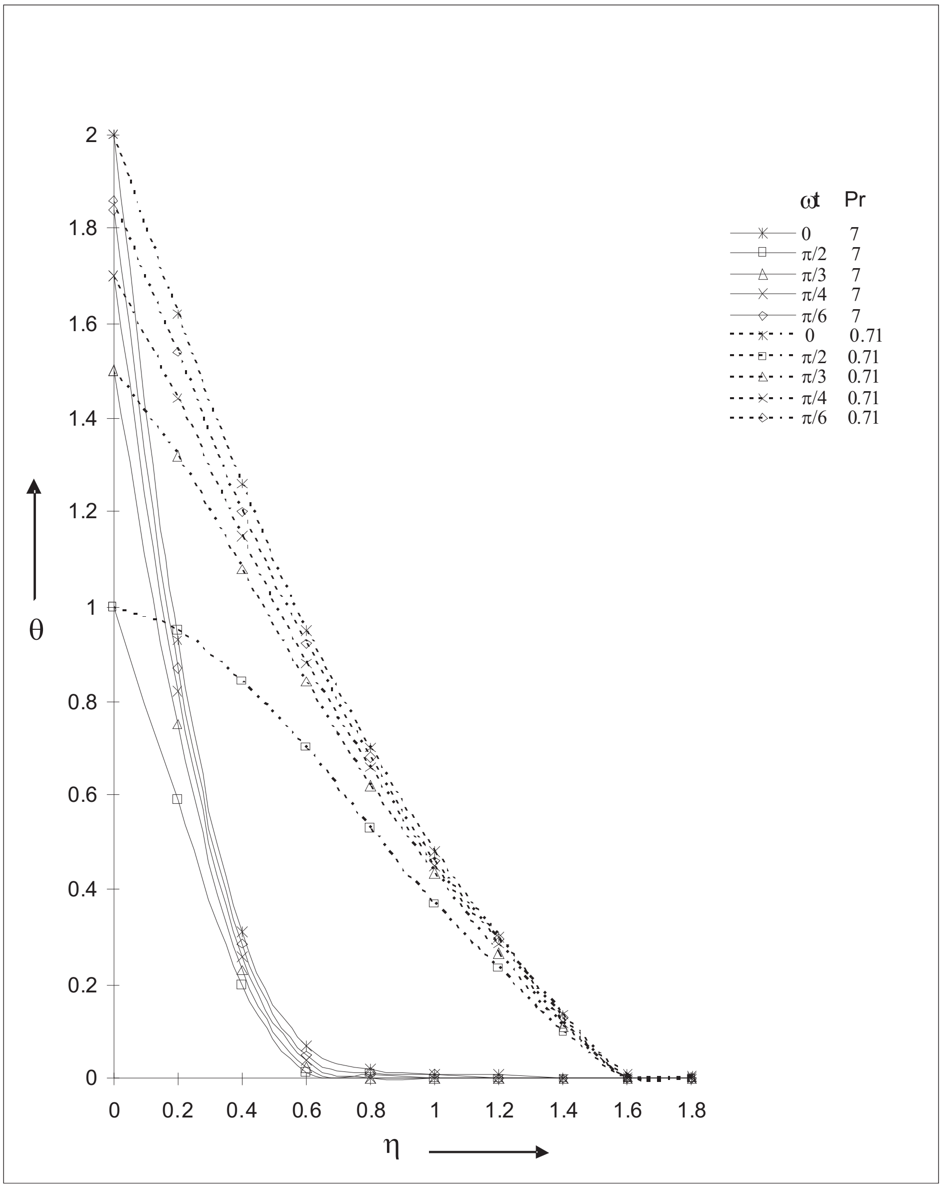

Figure 1: Transient temperature profiles

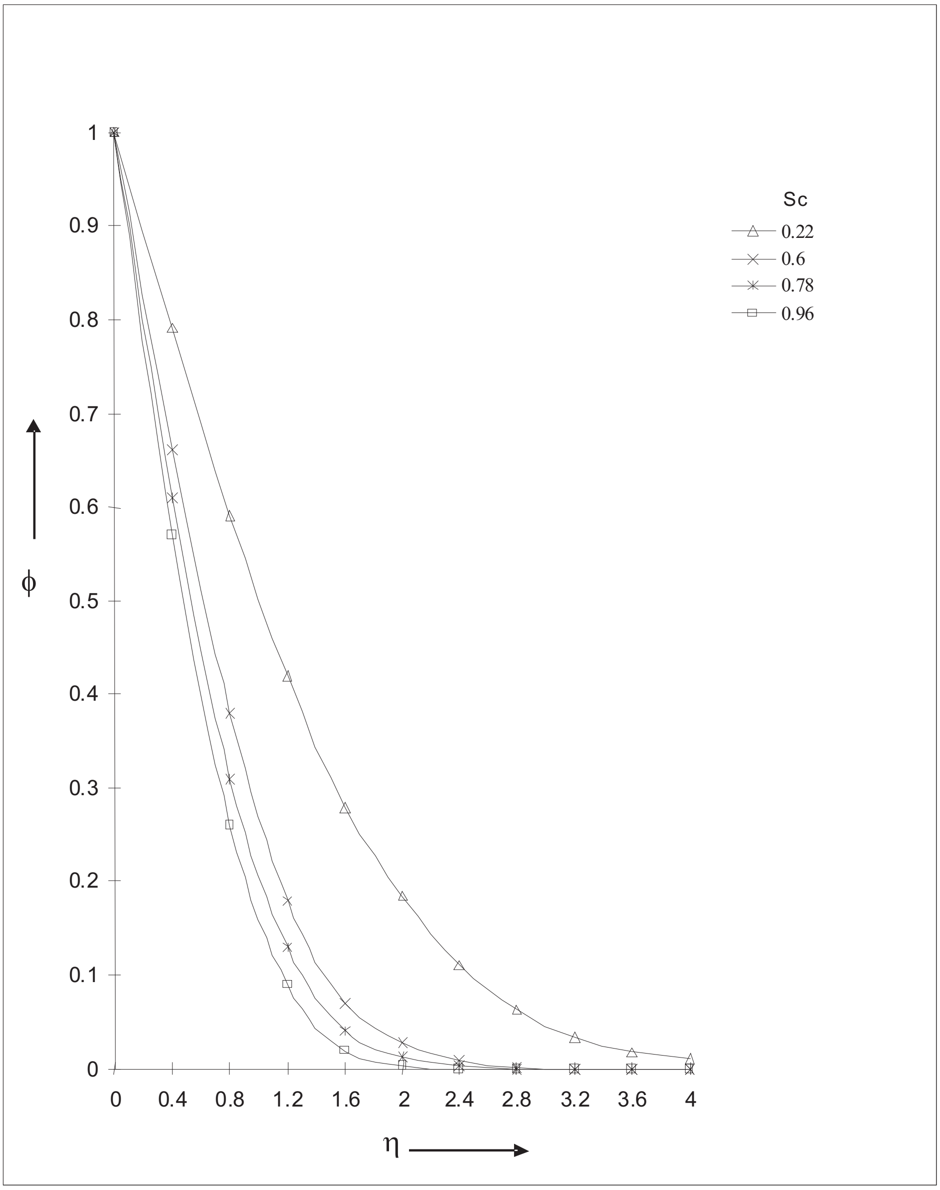

Figure 2: Concentration profiles

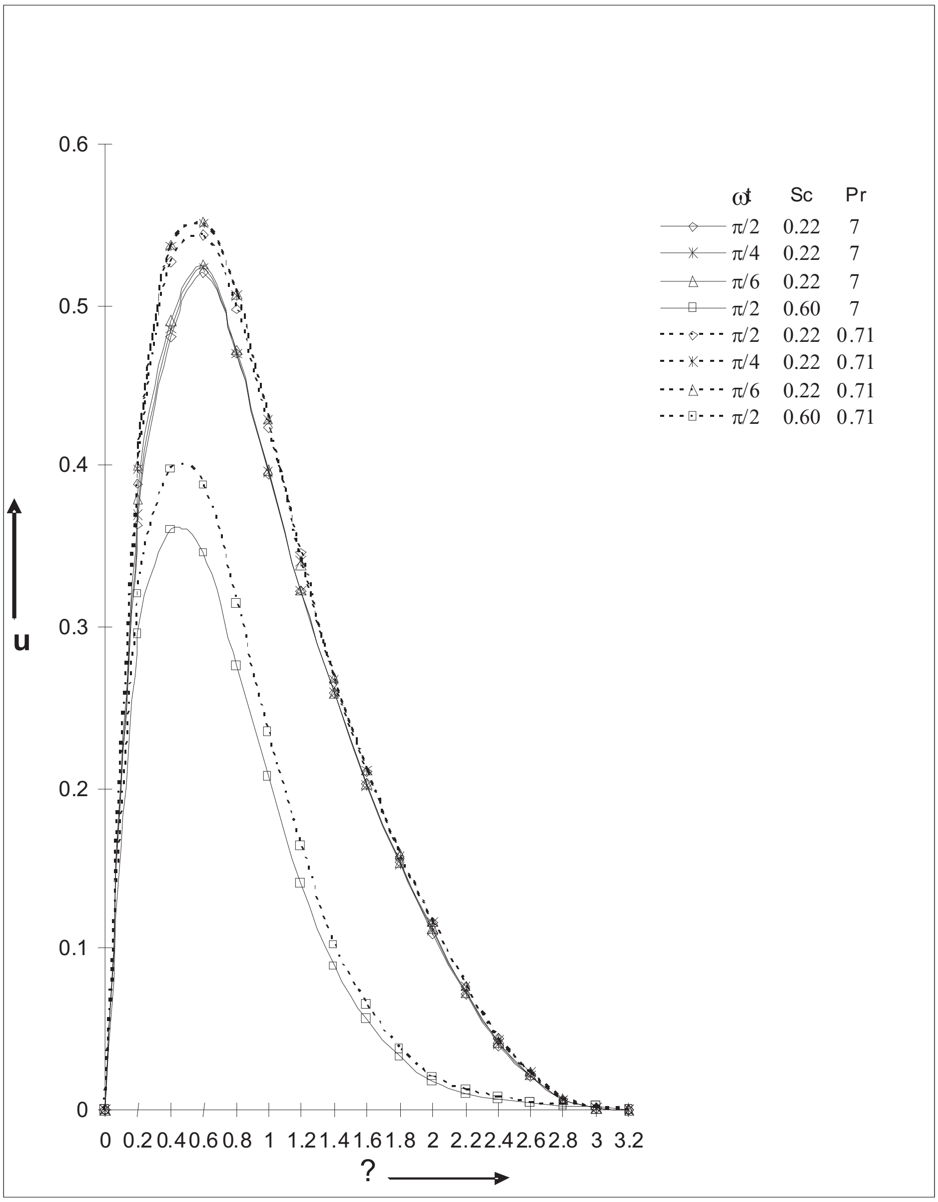

Figure 3: Velocity profiles when Gm=10, M=5, K=0.5, t =0.2

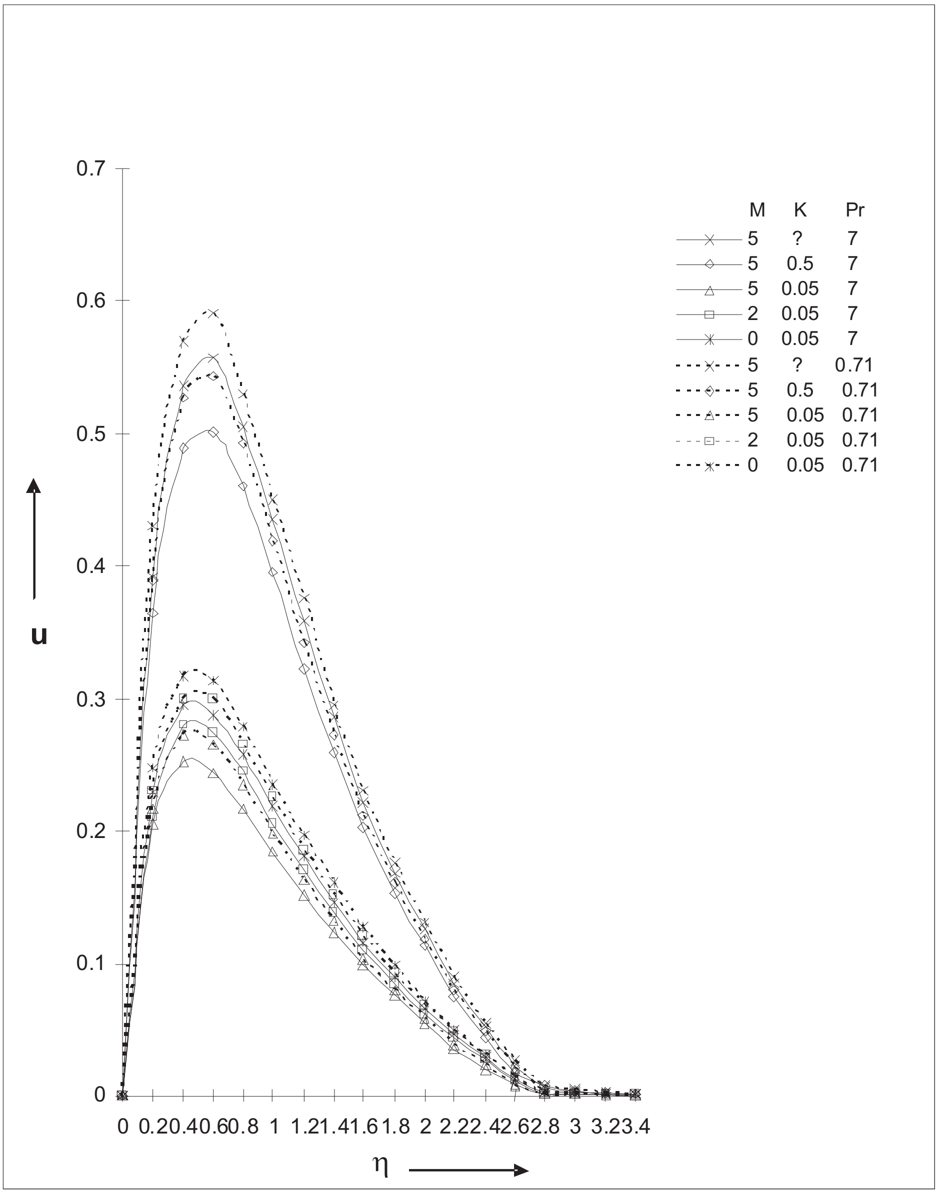

Figure 4: Velocity profiles when Gm=10, Sc= 0.22, t =0.2, ω $\mathbf{t} = \pi/\mathbf{2}$

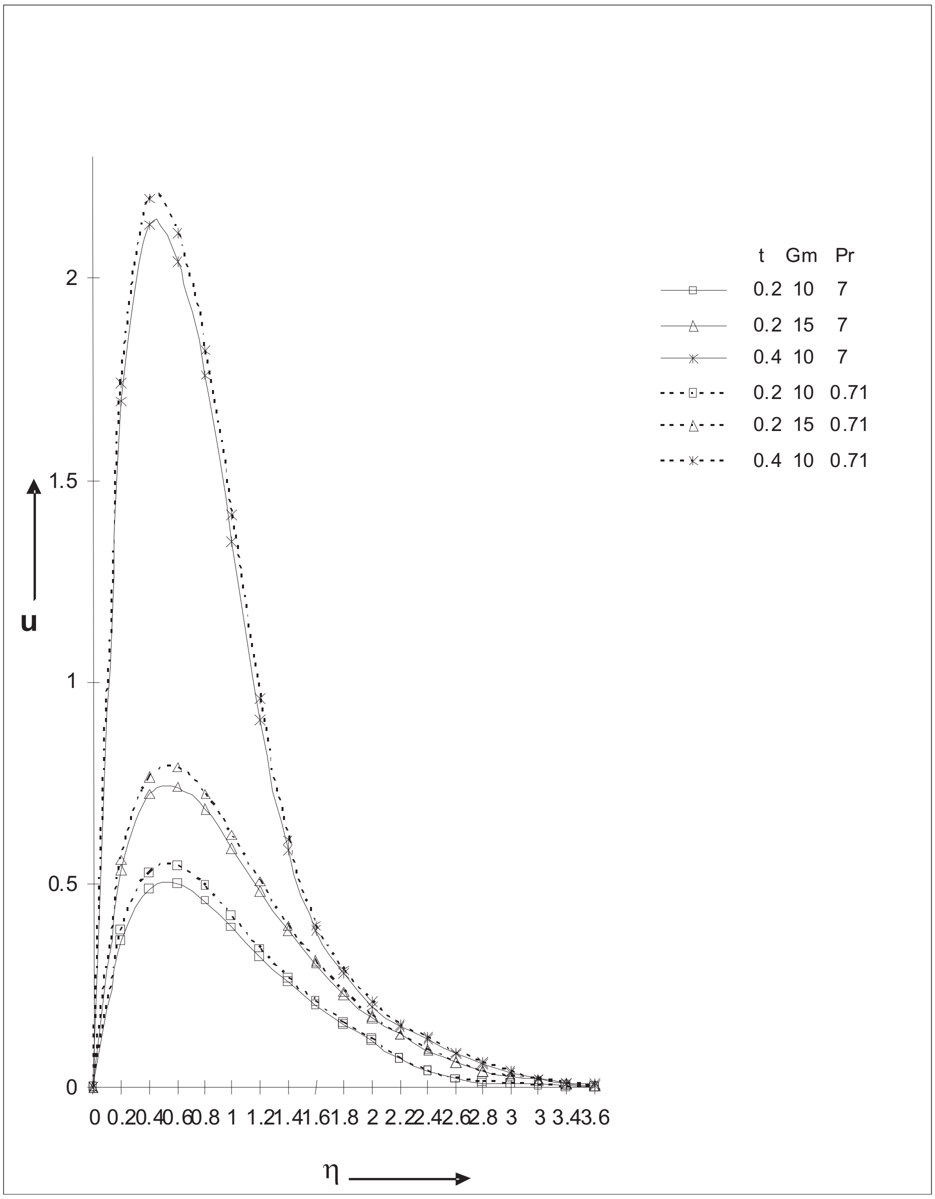

Figure 5: Velocity profiles when M=5, K=0.5, Sc=0.22, ω **t** $=\pi/\mathbf{2}$

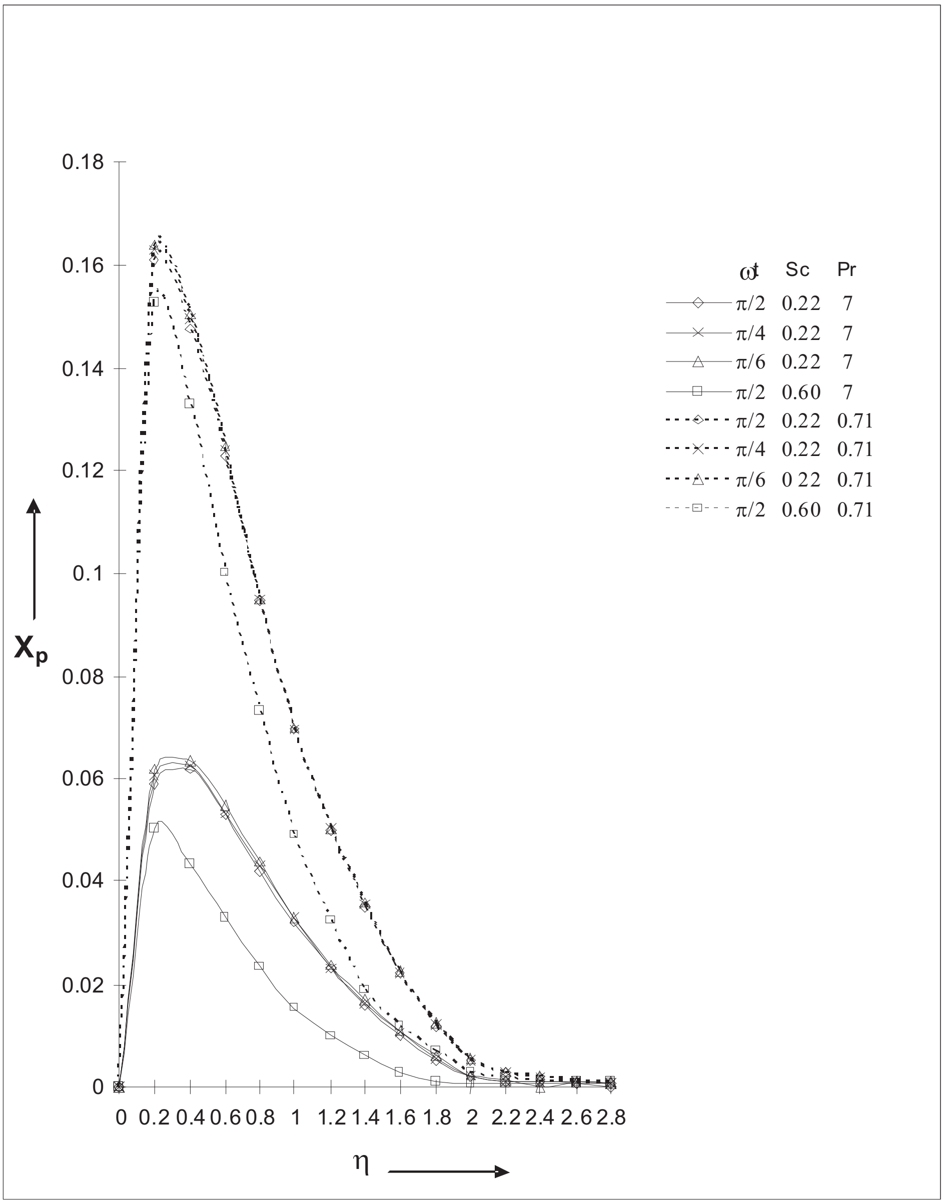

Figure 6: Penetration distances when Gm=10, M=5, K=0.5, t =0.2

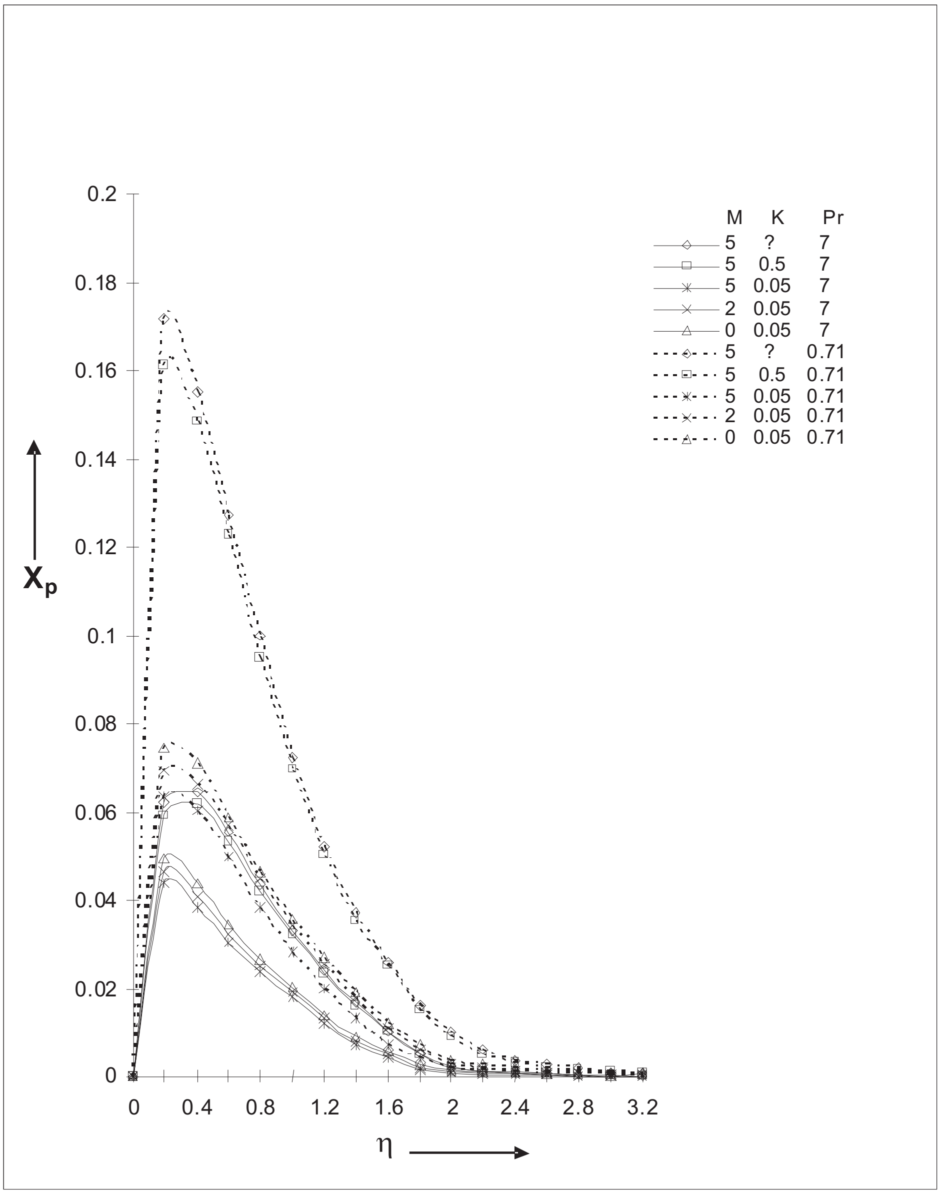

Figure 7: Penetration distances when Gm=10, Sc=0.22, t =0.2, $\omega\ \mathbf{t} = \pi/\mathbf{2}$

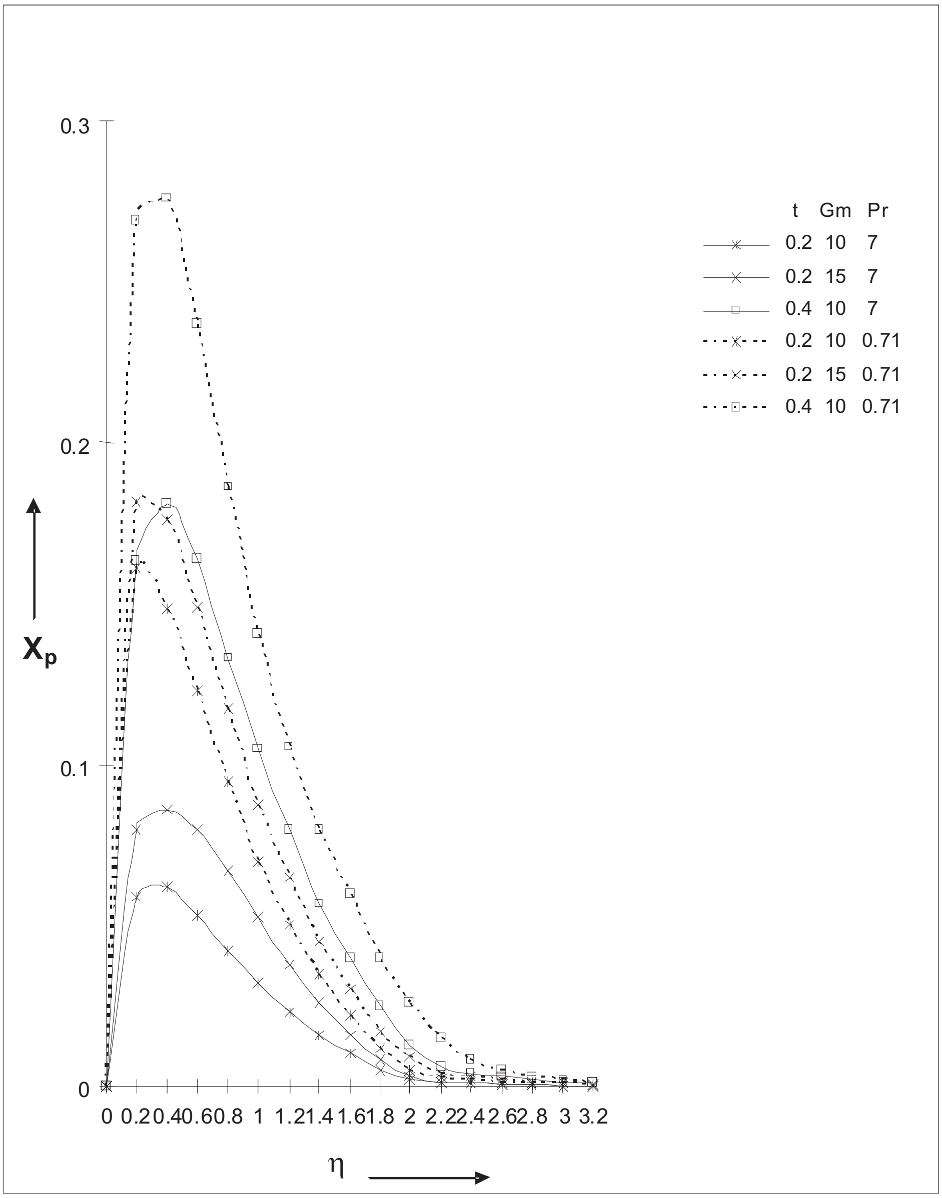

Figure 8: Penetration distances when M=5, K=0.5, Sc=0.22, ω $\mathbf{t}$ $=\pi/\mathbf{2}$

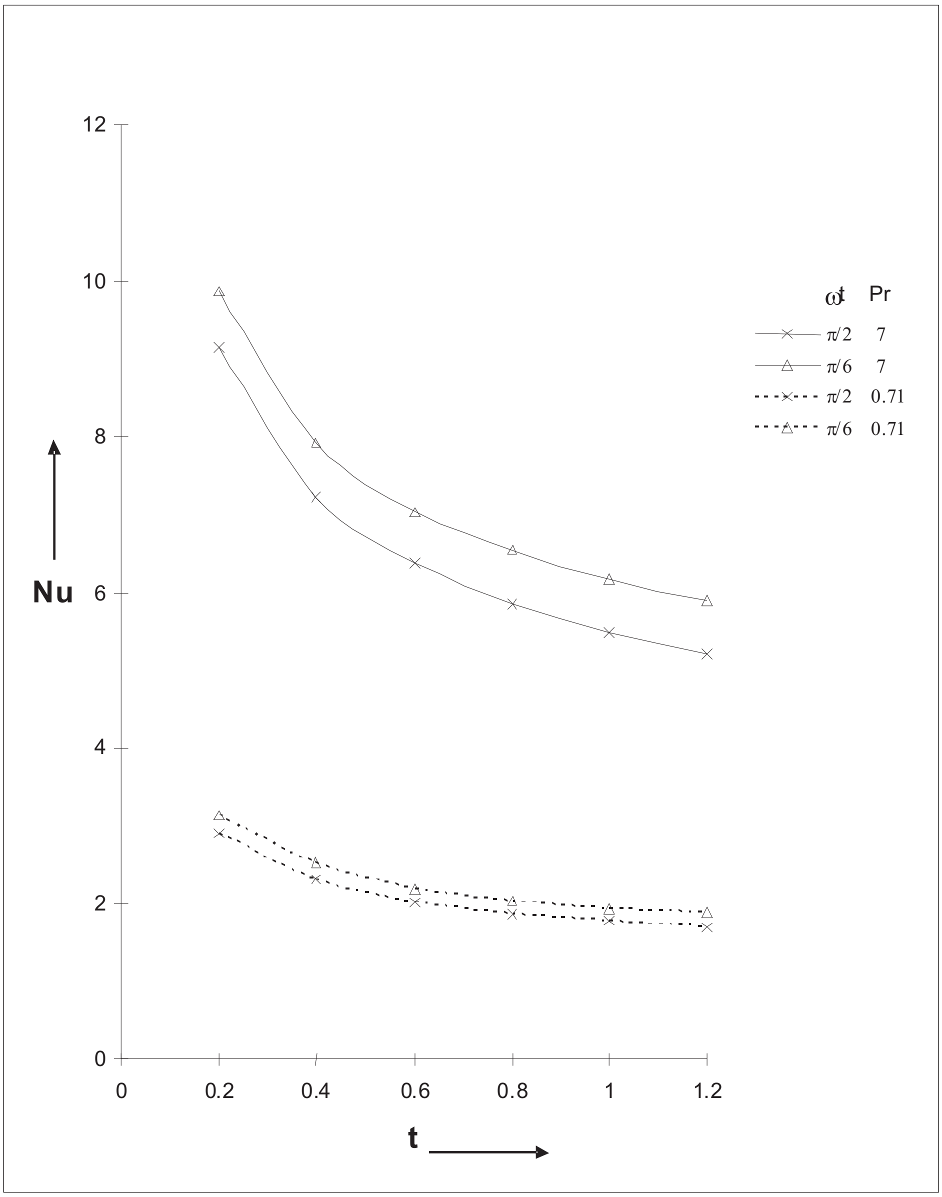

Figure 9: Nusselt number

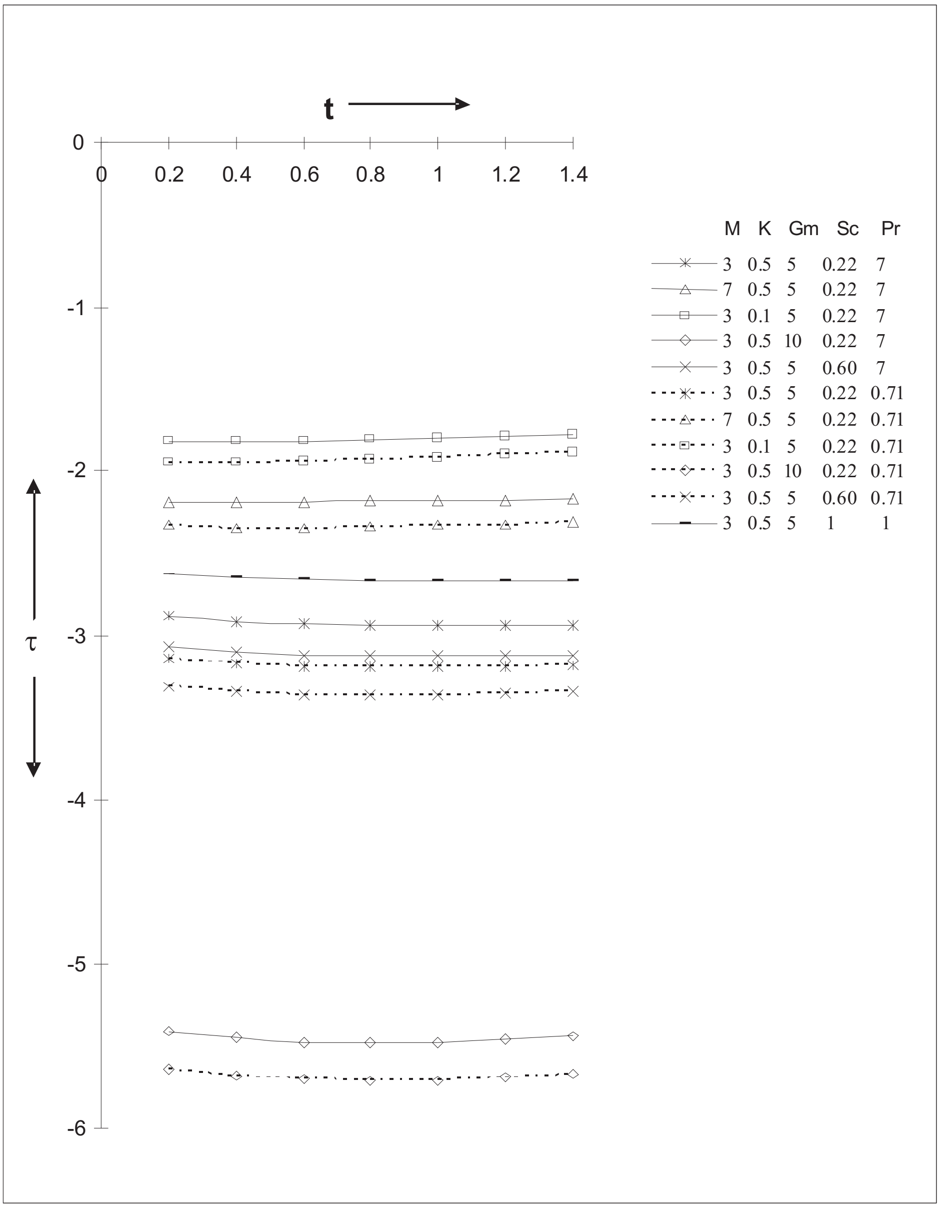

Figure 10: Skin-friction when ω $\mathbf{t}$ $=\pi/\mathbf{4}$

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Submitted on November 2007.

MHD toplotno i difuzno maseno tečenje prirodnom konvekcijom po površi unutar porozne sredine

Predstavljena je analitička studija prelazne hidromagnetske prirodne konvekcije po površi unutar porozne sredine uzimajući u obzir difuziju mase i vremensku fluktuaciju temperature na ploči. Dobijene jednačine su rešene u zatvorenom obliku tehnikom Laplasove transformacije. Dobijeni su rezultati za temperaturu, brzinu, rastojanje prodiranja, Nuseltov broj i trenje na zidu. Uticaji različitih parametara na promenljive tečenja su diskutovani igrafički prikazani.

doi:10.2298/TAM0901001C Math.Subj.Class.: 76W05