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Variable viscosity and thermophoresis effects on Darcy mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction

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Abstract

An analysis is presented to investigate the effect of thermophoresis particle deposition and variable viscosity on Darcy mixed convective heat and mass transfer of a viscous, incompressible fluid past a porous wedge in the presence of chemical reaction. The wall of the wedge is embedded in a uniform Darcian porous medium in order to allow for possible fluid wall suction or injection. The viscosity of the fluid is assumed to be a inverse linear function of temperature. The results are analyzed for the effect of different physical parameters, such as variable viscosity, magnetic, chemical reaction and thermophoresis parameters, on the flow, the heat and mass transfer characteristics.

Keywords: variable viscosity, thermophoresis particle deposition, chemical reaction, Darcy flow.

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1 Introduction

Thermophoresis is a phenomenon which causes small particles to be driven away from a hot surface and towards a cold one. The force experienced by a small aerosol particle in the presence of a temperature gradient is known as the **thermophoretic force**. Motion of particles under such a force is known as **thermophoresis**. Thermophoresis is an important mechanism of micro-particle transport due to a temperature gradient in the surrounding medium and has found numerous applications, especially in the field of aerosol technology. The effects of thermophoresis particle deposition with chemical reaction on the mixed convection flows are also important in the context of space technology and processes involving high temperatures.

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and/or endothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. In the application of pigments, or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in exploiting deposition mechanisms to improve efficiency. In the light of these various applications, thermophoretic deposition of radioactive particles is considered to be one of the important factors causing accidents in nuclear reactors. Thermophoresis in laminar flow over a horizontal flat plate has been studied theoretically by Goren (1977). Thermophoresis in natural convection with variable properties for a laminar flow over a cold vertical flat plate has been studied by Jayaraj et al.(1999). Selim et al. (2003) analyzed the effect of surface mass flux on mixed convective flow past a heated vertical flat permeable plate with thermophoresis. Recently, Chamkha and pop (2004) investigated the effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium. Pantokratoras, (2004), Strauss and Schubert, (1977) and Lai and Kulacki (1990) considered the variable viscosity effect for mixed convection flow along a vertical plate embedded in saturated porous medium. The variable viscosity effects on non-Darcy, free or mixed convection flow on a horizontal surface in a saturated porous medium are studied by Kumari (2001). In particular, the study of heat and mass transfer with thermophoresis and chemical reaction is of considerable importance in chemical and hydrometallurgical industries.

Effects of heat and mass transfer on mixed convection flow in the presence of suction/injection have been studied by many authors in different situations. But so far no attempt has been made to analyze the effect of thermophoresis particle deposition on Darcy mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction and hence we have considered the problem of this kind. The order of chemical reaction in this work is taken as first-order reaction. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2 Mathematical analysis

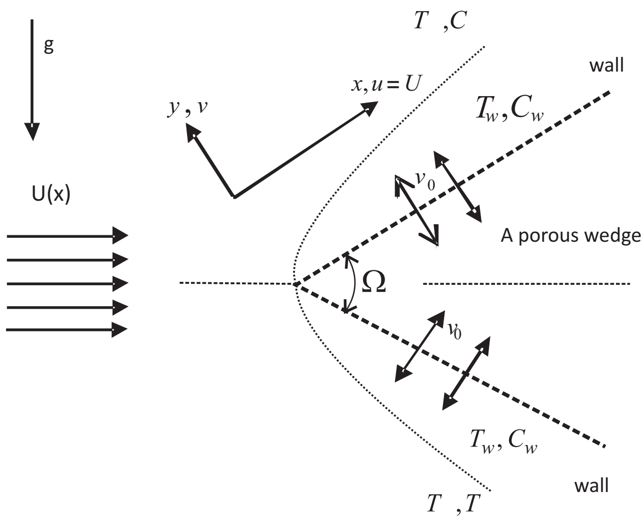


Figure 1: Flow analysis along the wall of the wedge

A two-dimensional steady mixed convective heat and mass transfer flow of a viscous, incompressible fluid over a porous wedge embedded in a porous medium is considered. The fluid is assumed to be Newtonian and its property variations due to temperature are limited to density and viscosity. The density variation and the effect of the buoyancy force is taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall, C_∞, is infinitesimally small. The flow is assumed to be in the x-direction, which is taken along the wall of the wedge and y-axis is taken to be normal to the wall. The chemical reaction is taking place in the flow and the effect of thermophoresis are being taken into account to help in the understanding of the mass deposition variation on the surface. Fluid suction or injection is imposed at the wedge surface, see Fig.1.

The fundamental equations for steady incompressible flow can be defined as follows:

Continuity equation:

$$div\,\vec{V} = 0 \tag{1}$$

Momentum equation:

$$(\vec{V}\,.\,grad\,\vec{V}) = -\frac{1}{\rho}\,grad\,p + \nu\,\nabla^2\,\vec{V} + \vec{g}\,\{\beta(T - T_\infty) + \beta^*(C - C_\infty)\} \tag{2}$$

Energy equation:

$$(\vec{V}.\,grad)T = \frac{k_e}{\rho\,c_p}\,\nabla^2 T \tag{3}$$

Species concentration equation:

$$(\vec{V}.\,grad)C = D\,\nabla^2\,C\ -\ (div\,\vec{v}_T C)\ \pm\ k_1 C \tag{4}$$

where $\vec{V}$ the velocity vector, p is is the pressure, ν is the kinematic coefficient of viscosity and $\vec{g}$ is the acceleration due to gravity.

Under these conditions, the basic governing boundary layer equation of momentum, energy and diffusion for mixed convection flow neglecting Joule's viscous dissipation under Boussinesq's approximation including variable viscosity can be simplified to the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) + U\frac{dU}{dx} - \frac{\nu}{K}(u - U) +$$

$$(g\beta(T-T_\infty)+g\beta^*(C-C_\infty))\sin\frac{\Omega}{2} \tag{6}$$

$$\rho\, c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=k_e\frac{\partial^2 T}{\partial y^2} \tag{7}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial(v_T C)}{\partial y}-k_1 C \tag{8}$$

T he boundary conditions are,

$$u=0,\quad v=-v_0,\quad T=T_w,\quad C=C_w\quad at\quad y\;=\;0 \tag{9}$$

$$u=U(x),\quad T=T_\infty,\quad C\;=C_\infty\quad at\quad y\to\infty \tag{10}$$

where D is the effective diffusion coefficient, μ is the dynamic viscosity, ρ is the fluid density, c_p is the specific heat at constant pressure, k_e is the porous medium effective thermal conductivity, K is the permeability of the porous medium, $v_T(=-k\frac{\nu}{T}\frac{\partial T}{\partial y})$ is the thermophoretic velocity, where k is the thermophoretic coefficient. The third term on the right hand side of Equ.(6) stands for the first-order (Darcy) resistance.

As in the line of Kafoussias et al., (1997), we introduce the following change of variables

$$\eta=(\sqrt{\frac{(1+m)U}{2\nu\, x}})y,\quad \psi=(\sqrt{\frac{2U\nu\, x}{1+m}})f(x,\eta),\quad \theta=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\alpha_1=\frac{T_r-T_\infty}{T_w-T_\infty}\quad and\quad \phi=\frac{C-C_\infty}{C_w-C_\infty} \tag{11}$$

It is assumed that the viscosity of the fluid is inverse linear function of temperature (Kumari, 2001; Lai and Kulacki, 1990) and it can be written as

$$\frac{1}{\mu}=\frac{1}{\mu_a}(1+\chi(T-T_a)) \tag{12}$$

where μ_a is the ambient fluid dynamic viscosity and χ is a thermal property of the fluid.

Equation (12) can be written as follows

$$\frac{1}{\mu}=a(T-T_r) \tag{13}$$

where $a = \dfrac{\chi}{\mu_a}$ and $T_r = T_a - \dfrac{1}{\chi}$ are constants and their values depend on the reference state and the thermal property of the fluid.

Under this consideration, the potential flow velocity can be written as

$$U(x) = Ax^m, \quad \beta_1 = \frac{2m}{1+m}, \quad where \quad 0 \ < \ m \ < 1, \tag{14}$$

where A and m are constants and β_1 is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \dfrac{\Omega}{\pi}$ for a total angle Ω of the wedge.

The continuity equation (1) is satisfied by the stream function $\psi(x, y)$ and it is defined as

$$u = \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x} \tag{15}$$

The equations (6) to (8) become

$$\begin{aligned}
(\theta - \alpha_1\ )\frac{\partial^3 f}{\partial \eta^3} = \frac{(\theta - \alpha_1)^2}{\alpha_1}[-f\frac{\partial^2 f}{\partial \eta^2} - \frac{2m}{1+m}(1 - (\frac{\partial f}{\partial \eta})^2) - \\
\frac{2}{1+m}\gamma_1(\theta + N\phi)\sin\frac{\Omega}{2} + \frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}) + \\
\frac{2}{m+1}\lambda(\frac{\partial f}{\partial \eta} - 1\ )\ ] + \frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2}
\end{aligned} \tag{16}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} = -\Pr\frac{\partial \theta}{\partial \eta} + \frac{2\Pr}{1+m}\theta\frac{\partial f}{\partial \eta} + \Pr\frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial \eta}) \tag{17}$$

$$\begin{aligned}
\frac{\partial^2 \phi}{\partial \eta^2} = -Sc\ (f - \tau\frac{\partial \theta}{\partial \eta})\frac{\partial \phi}{\partial \eta} + \frac{2\ Sc}{1+m}\phi\frac{\partial f}{\partial \eta} + \frac{2xSc}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \phi}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \phi}{\partial \eta}) + \\
Sc\tau\frac{\partial^2 \theta}{\partial \eta^2}\phi + \frac{2\ Sc\ x}{1+m}\gamma\ \phi
\end{aligned} \tag{18}$$

where the Grashof number Gr_x, Local buoyancy parameter γ_1, Sustentation parameter N, Reynolds number Re_x, Modified local Reynolds number Re_k, Prandtl number Pr, Schmidt number Scs suction/injection parameter S, chemical reaction parameter γ, thermophoresis particle deposition parameter τ and porous medium parameter λ, are defined as

$$Gr_x = \frac{g\beta\ \nu(T_w - T_\infty)}{U^3}, \quad \gamma_1 = \frac{Grx}{Rex^2}, \quad Re_x = \frac{Ux}{\nu}, \quad Re_k = \frac{U\sqrt{K}}{\nu},$$

$$\Pr = \frac{\nu}{\alpha_e}, \quad Sc = \frac{\nu}{D} \mathrm{s} S = v_0 \sqrt{\frac{(1+m)x}{2\,\nu\,U}}, \tag{19}$$

$$\gamma = \frac{\nu\, k_1}{U^2} \mathrm{s}\tau = -\frac{k(T_w - T_\infty)}{T_r} \quad and \quad \lambda = \frac{\alpha_e}{KA},$$

where α_e is the effective thermal diffusivity of the porous medium ($\alpha_e = \frac{k_e}{\rho\, c_p}$).

The boundary conditions can be written as

$$\eta = 0: \frac{\partial f}{\partial \eta} = 0, \ \frac{f}{2}(1 + \frac{x}{U}\frac{dU}{dx}) + x\frac{\partial f}{\partial x} = -v_0\sqrt{\frac{(1+m)x}{2\nu U}}, \quad \theta = 1, \ \phi = 1$$

$$\eta \to \infty: \frac{\partial f}{\partial \eta} = 1, \ \theta = 0, \ \phi = 0 \tag{20}$$

where v_0 is the velocity of suction if $v_0 < 0$ and injection if $v_0 > 0$.

Let $\xi = k\, x^{\frac{1-m}{2}}$ Kafoussias and Nanousis (1997), is the dimensionless distance along the wedge ($\xi > 0$)

The equations (16) to (18) and boundary conditions (20) can be written as

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{(\theta - \alpha_1)}{\alpha_1}[(f\frac{\partial^2 f}{\partial \eta^2} + \frac{1-m}{1+m}\xi(\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2} - \frac{\partial^2 f}{\partial \xi\, \partial \eta}\frac{\partial f}{\partial \eta}) +$$

$$\frac{2}{1+m}\gamma_1(\theta + N\phi)\sin\frac{\Omega}{2} - \frac{2}{m+1}\xi^2\lambda\Pr(\frac{\partial f}{\partial \eta} - 1) - \tag{21}$$

$$\frac{2m}{m+1}((\frac{\partial f}{\partial \eta})^2 - 1\,)] - \frac{2}{1+m}(\frac{1}{\theta - \alpha_1})\frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2} = 0$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + \Pr(f\frac{\partial \theta}{\partial \eta} + \frac{1-m}{1+m}\xi(\frac{\partial f}{\partial \xi}\frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \xi}\frac{\partial f}{\partial \eta}\,) - \frac{2\Pr}{1+m}\theta\frac{\partial f}{\partial \eta} = 0 \tag{22}$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + Sc\,(f - \tau\frac{\partial \theta}{\partial \eta})\,\frac{\partial \phi}{\partial \eta} + Sc\frac{1+m}{1-m}(\frac{\partial \phi}{\partial \eta}\xi\frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \eta}\xi\frac{\partial \phi}{\partial \xi})\, - \frac{2Sc}{1+m}\phi\frac{\partial f}{\partial \eta}$$

$$-Sc\tau\frac{\partial^2 \theta}{\partial \eta^2}\phi - \frac{2Sc}{1+m}\xi^2\gamma\,\phi = 0 \tag{23}$$

$$\eta = 0 : \frac{\partial f(\xi,\eta)}{\partial \eta} = 0, \quad \frac{(1+m)}{2} f(\xi,\eta) + \frac{1-m}{2}\xi \frac{\partial f(\xi,\eta)}{\partial \xi} = -S, \theta(\xi,\eta) = 1,$$

$$\phi(\xi,\eta) = 1$$

$$\eta \to \infty : \frac{\partial f(\xi,\eta)}{\partial \eta} = 1,\ \theta(\xi,\eta) = 0,\ \phi(\xi,\eta) = 0 \qquad (24)$$

where S is the suction parameter if $S > 0$ and injection if $S < 0$.

The system of equations (21) to (23) can also be written as

$$f''' + \frac{(\theta-\alpha_1)}{\alpha_1}[(ff'' + \frac{2}{1+m}\gamma_1(\theta + N\phi)\sin\frac{\Omega}{2} - \frac{2}{m+1}\xi^2\lambda\Pr(f'-1)$$

$$-\frac{2m}{m+1}(f'^2 - 1)] - \frac{2}{1+m}(\frac{1}{\theta-\alpha_1})\theta' f'' = \qquad (25)$$

$$-\frac{(\theta-\alpha_1)}{\alpha_1}\ \frac{1-m}{1+m}\xi(f''\frac{\partial f}{\partial \xi} - f'\frac{\partial f'}{\partial \xi})$$

$$\theta'' + \Pr f\theta' - \frac{2\Pr}{1+m}f'\theta = -\Pr\frac{1-m}{1+m}\xi(\theta'\frac{\partial f}{\partial \xi} - f'\frac{\partial \theta}{\partial \xi}) \qquad (26)$$

$$\phi'' + Sc\,(f - \tau\theta')\,\phi' - \frac{2Sc}{1+m}\phi\, f' - Sc\tau\theta''\phi - \frac{2Sc}{1+m}\xi^2\gamma\phi =$$

$$-Sc\frac{1-m}{1+m}\xi(\phi'\frac{\partial f}{\partial \xi} - f'\frac{\partial \phi}{\partial \xi}) \qquad (27)$$

with boundary conditions

$$f'(\xi,0) = 0, \frac{(1+m)}{2}f(\xi,0) + \frac{1-m}{2}\xi\frac{\partial f(\xi,0)}{\partial \xi} = -S, \theta(\xi,0) = 1, \phi(\xi,0) = 1$$

$$f'(\xi,\infty) = 1, \theta(\xi,\infty) = 0, \phi(\xi,\infty) = 0 \qquad (28)$$

where the prime denote partial differentiation with respect to η, whereas the boundary conditions (24) remain the same. This form of the system is the most suitable for the application of the numerical scheme described below.

It may be observed that the equations (25) – (27) remain partial differential equations after transformation, with $\partial/\partial\xi$ terms on the right hand side. In this system of equations, it is obvious that the non-similarity aspects of the problem are embodied in the terms containing partial derivatives with respect to ξ. This problem does not admit similarity solutions. Thus, with ξ-derivatives terms retained in the system of equations, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. Formulation of the system of equations for the local nonsimilarity model with reference to the present problem will now be discussed.

At the first level of truncation, the terms accompanied by $\xi\dfrac{\partial}{\partial\xi}$ are small. This is particularly true when $\xi << 1$. Thus the terms with $\xi\dfrac{\partial}{\partial\xi}$ (on the right hand sides of equations (25) to (27) are deleted to get the following system of equations:

$$f''' + \frac{(\theta-\alpha_1)}{\alpha_1}[(ff'' + \frac{2}{1+m}\gamma_1(\theta+N\phi)\sin\frac{\Omega}{2} - \frac{2}{m+1}\xi^2\lambda\,\Pr(f'-1) - \frac{2m}{m+1}(f'^2-1)] - \frac{2}{1+m}(\frac{1}{\theta-\alpha_1})\theta' f'' = 0 \tag{29}$$

$$\theta'' + \Pr f\theta' - \frac{2\Pr}{1+m}f'\theta = 0 \tag{30}$$

$$\phi'' + Sc\,(f-\tau\theta')\,\phi' - \frac{2Sc}{1+m}\phi\, f' - Sc\tau\theta''\,\phi - \frac{2Sc}{1+m}\xi^2\gamma\,\phi = 0 \tag{31}$$

with boundary conditions

$$f'(\xi,0)=0, \frac{(1+m)}{2}f(\xi,0)+\frac{1-m}{2}\xi\frac{\partial f(\xi,0)}{\partial\xi} = -S, \theta(\xi,0)=1, \phi(\xi,0)=1$$
$$f'(\xi,\infty)=1, \theta(\xi,\infty)=0, \phi(\xi,\infty)=0 \tag{32}$$

Equations (29-31) can be regarded as a system of ordinary differential equations for the functions $f, \theta\ and\ \phi$ with ξ as a parameter for given pertinent parameters.

The major physical quantities of interest are the local skin friction coefficient; the local Nusselt number and the local Sherwood number are defined, respectively, by:

$$C_f = \frac{f''(\xi,0)}{Re_x^{\frac{1}{2}}};\ N_u = -\frac{\theta'(\xi,0)}{Re_x^{\frac{1}{2}}}\ \ and\ \ S_h = -\frac{\phi'(\xi,0)}{Re_x^{\frac{1}{2}}} \tag{33}$$

The mass diffusion equation (31) can be adjusted to meet these circumstances if one takes (i) $\gamma > 0$ for the destructive reaction,(ii) $\gamma = 0$ for no chemical reaction and (iii) $\gamma < 0$ for the generative reaction. The momentum equation (29) can also be attempt these circumstances if one takes $\gamma_1 >> 1.0$ corresponds to pure free convection, $\gamma_1 = 1.0$ corresponds to mixed convection and $\gamma_1 << 1.0$ corresponds to pure forced convection. Throughout this calculation we have considered $\gamma_1 = 1.0$ unless otherwise specified.

3 Numerical solution

The boundary layer over the wedge, subjected to a velocity of suction or injection, is described by the system of partial differential equations (25) - (27), and its boundary conditions (28). In this system of equations $f(\xi\eta)$ is the dimensionless stream function; $\theta(\xi, \eta)$ be the dimensionless temperature; $\phi(\xi, \eta)$be the dimensionless concentration; Pr, the Prandtl number; Re_x, Reynolds number etc. which are defined in (19). It is obvious that the nonsimilarity aspects of the problem are embodied in the terms containing partial derivatives with respect to ξ. Thus, with ξ derivative terms retained in the system of equations (29) - (31), it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent streamwise locations through the ξ derivatives, a locally autonomous solution, at any given streamwise location, cannot be obtained.

In such a case, an implicit marching numerical solution scheme (the basic marching method is direct, that is, noniterative, but some of the most powerful techniques presented herein utilize it within rapidly converging iterative schemes) is usually applied preceding the solution in the ξ-direction, i.e., calculating unknown profiles at $\xi_{\iota+1}$ when the same profiles at ξ_ι are known. The process starts at $\xi = 0$ and the solution proceeds from ξ_ι to $\xi_{\iota+1}$ but such a procedure is time consuming.

However, when the terms involving $\frac{\partial f}{\partial \xi}, \frac{\partial \theta}{\partial \xi}$ and $\frac{\partial \phi}{\partial \xi}$ and their η derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations, for the functions f, θ and ϕ with ξ as a parameter and the computational task is simplified. Furthermore a locally autonomous solution, for any given ξ, can be obtained because the streamwise coupling is severed.

So, in this work, a modified and improved numerical solution scheme, for local nonsimilarity boundary layer analysis, is used. The scheme is similar to that of Minkowycz and Sparrow (1988) but it deals with the differential equations in lieu of integral equations. In each level of truncation, the governing coupled and nonlinear system of differential equations is solved by applying the common finite difference method, with central differencing, a tridiagonal matrix manipulation, and an iterative procedure. The whole numerical scheme can be programmed and applied easily and has distinct advantages compared to that in Minkowycz and Sparrow (1988) with respect to stability, accuracy, and convergence speed. The details of this scheme are described in Kafoussias and Williams (1993) and Kafoussias and Karabis (1996).

To examine the behavior of the boundary layer over the wedge, numerical calculations were carried out for different values of the dimensionless parameters, entering the problem under consideration for Pr = 0.71, which corresponds to air. The numerical results are shown in Figs. 2 - 5 for the velocity, the temperature and the concentration of the fluid along the wall of wedge.

4 Results and discussion

The computations have been carried out for various values of variable viscosity α_1, chemical reaction (γ), Thermophoresis particle deposition parameter (τ) and porous medium (λ). In order to validate our method, we have compared steady state results of skin friction $f''(\xi, 0)$ and rate of heat transfer $-\theta'(\xi, 0)$ for various values of ξ (Table.1) with those of Minkowycz et al. (1988) and found them in excellent agreement.

The velocity, temperature and concentration profiles obtained in the dimensionless form are presented in the following Figures for Pr = 0.71 which represents air at temperature 20^0C and Sc = 0.62 which corresponds to water vapor that represents a diffusion chemical species of most common interest in air. Grashof number for heat transfer is chosen to be $Gr_x = 9$, since these values corresponds to a cooling problem, and Reynolds number $Re_x = 3.0$. The values of γ are chosen to be 0.1, 2.0 and 5.0. It is important to note that α_1 is negative for liquids and positive

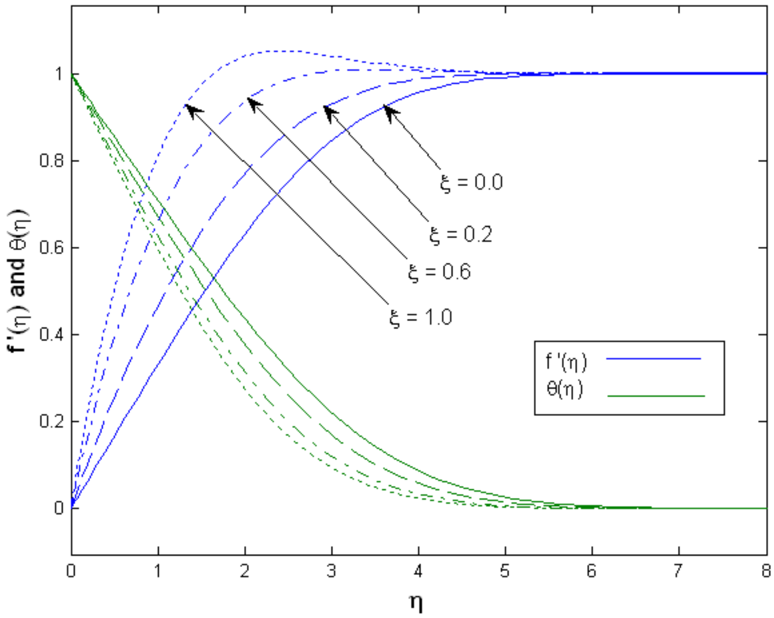
	Minkowycz et al. (1988)		Present works	
ξ	$f''(\xi,0)$	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\theta'(\xi,0)$
0	0.33206	0.29268	0.33206	0.29268
0.2	0.55713	0.33213	0.55707	0.33225
0.4	0.75041	0.35879	0.75007	0.35910
0.6	0.92525	0.37937	0.92449	0.37986
0.8	1.08792	0.39640	1.08700	0.39685
1.0	1.24170	0.41106	0.24062	0.41149
2.0	1.92815	0.46524	1.92689	0.46551
10.0	5.93727	0.64956	5.93502	0.64968

Table 1: Comparison with previous published work N = 3, $\gamma_1 = 1.0, Sc = 0.62, \lambda = 0.1$, S = 3.0, α_1= 0 and $\Omega = 30^0$

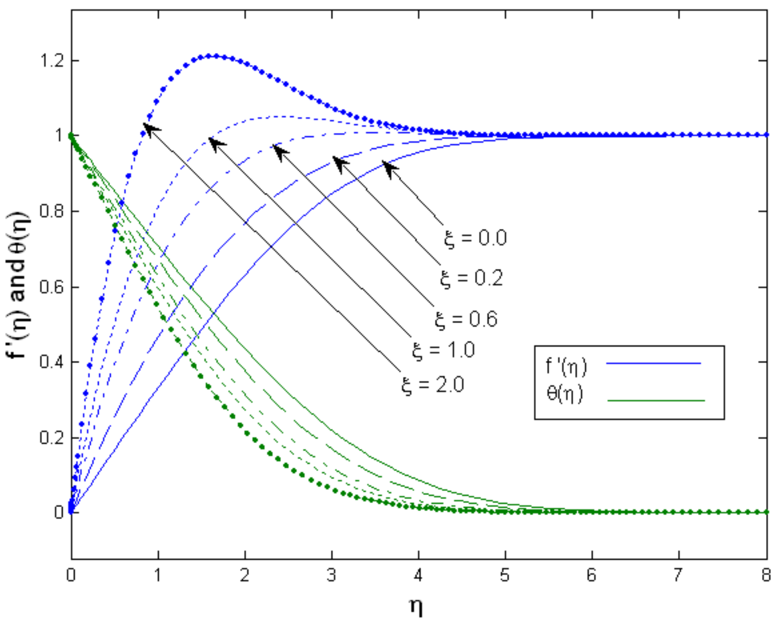
for gases when $T_w - T_\infty$ is positive. The values of α_1 (for air $\alpha_1 > 0$) are chosen to be 0.1, 0.3 and 0.5 and the value of suction, S is chosen to be 3.0.

In the absence of species concentration equation, in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for ξ are compared with the available exact solution of Minkowycz et al. (1988), is shown in Fig.2. It is observed that the agreements with the theoretical solution of velocity and temperature profiles are excellent.

Effect of chemical reaction with thermophoresis particle deposition plays a very important role on the concentration field. The effect of thermophoretic parameter, τ and chemical reaction, γ on velocity, temperature and concentration field are shown in Figs.3 and 4. It is observed that the velocity, temperature and concentration of the fluid decrease with increase of thermophoresis and chemical reaction parameters. In particular, the effect of increasing the thermophoretic parameter τ is limited to be increasing slightly the wall slope of the concentration profiles but decreasing the concentration. This is true only for small values of Schmidt number for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of Schmidt number (Sc > 100) the diffusion effect is minimal compared to the convection effect and, therefore, the thermophoretic and chemical reaction parameters are



(a) Minkowycz et al. (1988)



(b) Present work

Figure 2: Comparison of the velocity and temperature profiles with Minkowycz et al. (1988)

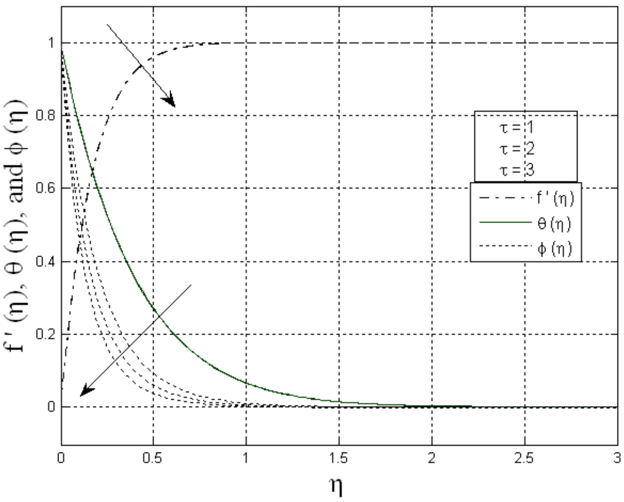


Figure 3: Thermophoretic effect on velocity, temperature and concentration profiles. N = 3, $\gamma_1 = 1.0, Sc\ =\ 0.62, \lambda = 0.1$, S = 3.0, α_1= 0.5, ξ= 0.01 and $\Omega = 30^0$

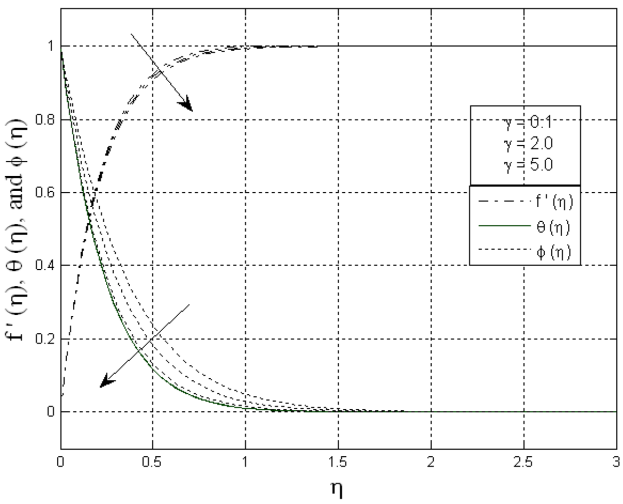


Figure 4: Chemical reaction over velocity, temperature and concentration profiles. $\gamma_1 = 1.0$, N = 3, $Sc\ =\ 0.62, \lambda = 0.1$, $\tau = 0.5$, S = 3.0, α_1= 0.5, ξ = 0.01 and $\Omega = 30^0$

expected to alter the concentration boundary layer significantly. This is consistent with the work of Goren (1977) on thermophoresis of aerosol particles in flat plate boundary layer. It is evident to note that the increase of chemical reaction and thermophoretic particle deposition are significantly altered the concentration boundary layer thickness but not momentum and thermal boundary layers.

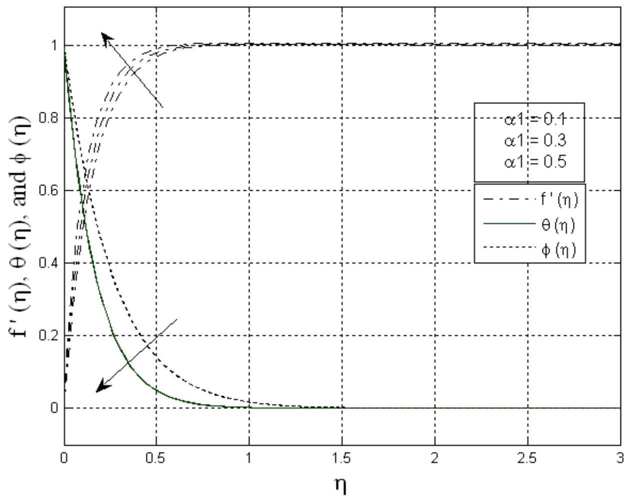


Figure 5: Viscosity effect on velocity, temperature and concentration profiles. γ_1 = 1.0, m = 0.0909, N = γ = 1.0, τ = 0.5, λ = 0.1, S = 3.0, ξ= 0.01 and $\Omega = 30^0$

Increase of viscosity accelerates the fluid motion and reduces the temperature of the fluid along the wall and the concentration of the fluid is almost not affected with increase of the viscosity and these are shown in Fig.5. The results presented demonstrate quite clearly that α_1, which is an indicator of the variation of viscosity with temperature, has a substantial effect on fluid motion within the boundary layer over a heat surface as well as the drag and heat transfer characteristics.

From the Table 2, it is observed that the skin friction increases and the rate of heat and mass transfer decrease with increase of and viscosity parameter, whereas the skin friction and the rate of mass transfer decrease and the rate of heat transfer increases with increase of chemical reaction and thermophoretic parameters.

$f''(0)$	$\theta'(0)$	$\phi'(0)$		Parameter
4.946190	-4.058611	-2.689660	$\gamma = 0.1$	Chemical reaction parameter
4.855660	-4.054048	-3.482368	$\gamma = 3.0$	
4.818363	-4.052252	-3.910915	$\gamma = 5.0$	
6.034338	-2.580389	-5.260410	$\tau = 1.0$	Thermophoretic parameter
6.034184	-2.580390	-6.804620	$\tau = 2.0$	
6.034067	-2.580391	-8.366696	$\tau = 3.0$	

Table 2: Analysis for skin friction and rate of heat and mass transfer. $\gamma_1 = 1.0$, m = 0.0909, N = 1.0, $\lambda = 0.1$, S = 3.0, ξ= 0.01 and $\Omega = 30^0$

5 Conclusions

In the present paper, the effect of variable viscosity on Darcy mixed convection boundary layer flow over a porous wedge with thermophoresis particle deposition in the presence of chemical reaction has been studied numerically. There are many parameters involved in the final form of the mathematical model. The problem can be extended on many directions, but the first one seems to be to consider the effects of chemical reaction with thermophoresis particle deposition. In mixed convection regime, the concentration boundary layer thickness decreases with increase of the thermophoretic and chemical reaction parameters. So, the thermophoretic with chemical reaction effects in the presence of viscosity of the fluid have a substantial effect on the flow field and, thus, on the heat and mass transfer rate from the sheet to the fluid. Thermophoresis is an important mechanism of micro-particle transport due to a temperature gradient in the surrounding medium and has found numerous applications, especially in the field of aerosol technology. The numerical results are influenced by mixed convection parameter and the variable viscosity parameter which defines the effect of variable viscosity of the fluid α_1. It is observed that with variable viscosity, the separation of boundary layer is delayed for $\alpha_1 > 0$ than $\alpha_1 < 0$. When the effect of variable viscosity is considered for the assisting flow case, the heat transfer for liquids is higher and for gases is lower compared to the constant viscosity case. It is expected that this research may prove to be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of under ground water and in the filtration and water purification processes. Such kind of numerical solution for the effect of thermophoresis particle deposition on nonlinear boundary layer flow over a porous wedge with variable viscosity in the presence of chemical reaction is presented first time in the literature.

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Uticaji promenljive viskoznosti i termoforeze na Darsijev prenos toplote i mase preko poroznog klina u prisustvu hemijske reakcije

Proučava se uticaj taloženja čestica termoforezom kao i promenljive viskoznosti na Darsijev prenos toplote i mase viskoznog nestišljivog fluida preko poroznog klina u prisustvu hemijske reakcije. Zid klina je potopljen u uniformnu Darsijevu poroznu sredinu da bi se dozvolilo usisavanje ili ubrizgavanje. Za viskoznost fluida se pretpostavlja da je inverzno linearna funkcija temperature. Analizirani su uticaji promenljive viskoznosti, magnetskih i parametara termoforeze kao i hemijske reakcije na karakteristike tečenja, prenos toplote i mase.

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