

On the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction

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Abstract

This work is concerned with magnetohydrodynamic viscous flow due to a shrinking sheet in the presence of suction. The cases of two dimensional and axisymmetric shrinking are discussed. The governing boundary layer equations are written into a dimensionless form by similarity transformations. The transformed coupled nonlinear ordinary differential equations are numerically solved by using an advanced numeric technique. Favorability comparisons with previously published work are presented. numerical results for the dimensionless velocity, temperature and concentration profiles are obtained and displayed graphically for pertinent parameters to show interesting aspects of the solution.

Keywords: chemical reaction, suction at the surface, porous shrinking sheet, magnetic effect.

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1 Introduction

The flow over a shrinking surface is an important problem in many engineering processes with applications in industries such as the hot rolling, wire drawing and glass wire production. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The present trend in the field of magnetic strength analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with magnetic effect is of considerable importance in chemical and hydrometallurgical industries. Effect of heat and mass transfer on nonlinear MHD boundary layer flow have been discussed by many authors (Bhattacharyya and Gupta, 1985, Brady and Acrivos, 1981, Crane, 1970, Gupta and Gupta, 1977, Jensen, Einset, and Fotiadis, 1991, troy et al., 1987, Usha and Sridharan, 1995, Wang, 1984, Wang, 1988, Wang, 1990, Hakiem, Mohammadeian, Kaheir and Gorla, 1999, Kuo, 2005, Cheng and Lin, 2002, Apelblat, 1982) in various situations.

Magnetohydrodynamic (MHD) mixed convection heat transfer flow in porous and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, high temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids, and (MHD) power generation systems. Chemical reaction can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. This depends on whether they occur at an interface or as a single phase volume reaction. A few representative fields of interest in which combined heat and mass transfer with chemical reaction play important role, are design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, food processing and cooling towers. Cooling towers are the cheapest way to cool large quantities of water. For example, formation of smog is a first

order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke-stacks. This NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope of what is termed as photochemical smog.

One can find abundant number of articles in the literature regarding different problems for newtonian and non-newtonian fluids, with or without heat transfer analysis dealing with the stretching flow problems. However, the investigations regarding the flow problems due to a shrinking sheet are scarcely available in the literature. To the best of our knowledge, only very few such attempts, (e.g., Sajid, Javed, and Hayat, 2008, Hayat, Abbas and Sajid, 2007, Sajid and Hayat, 2007, Wang, 1990a, Miklavcic and Wang, 2006) are yet available in the literature. In Wang, 1990a, Wang presented unsteady shrinking film solution and in Miklavcic and Wang, 2006, Miklavcic and Wang proved the existence and uniqueness of steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter.

The object of this paper is to study the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet in the presence of suction. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2 Mathematical analysis

Let us consider the MHD flow of an incompressible viscous fluid over a shrinking sheet at $y = 0$. The x and y axes are taken along and perpendicular to the sheet respectively, as shown in Fig.1. The fluid is assumed to be Newtonian and electrically conducting and the flow is confined to $y > 0$. A constant magnetic field of strength B_0 acts in the direction of y axis. The induced magnetic field is negligible, which is a valid assumption on a laboratory scale. The assumption is justified when the magnetic Reynolds number is small, Hayat et al. (2007). Since no electric field is applied and the effect of polarization of the ionized fluid is negligible, we can assume that the electric field $E = 0$.

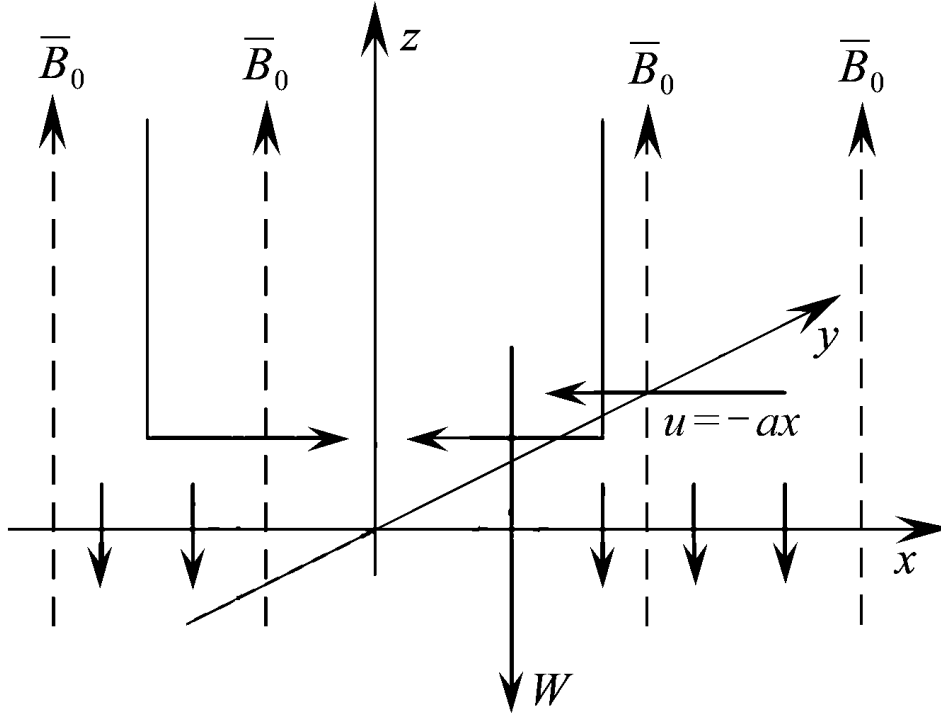


Figure 1: Flow analysis on shrinking surface, Muhaimin et al. (2008)

The chemical reactions are taking place in the flow and a constant suction is imposed at the horizontal surface, see Fig.1. The governing boundary layer equations of momentum, energy and diffusion for the MHD flow in terms of vector notation are defined as follows:

Continuity equation:

$$\text{div } \vec{V} = 0 \quad (1)$$

Momentum equation:

$$(\vec{V} \cdot \text{grad } \vec{V}) = -\frac{1}{\rho} \text{grad } p + \nu \nabla^2 \vec{V} + \frac{1}{\rho} \vec{j} \times \vec{B} \quad (2)$$

d Energy equation:

$$(\vec{V} \cdot \text{grad})T = \frac{k_e}{\rho c_p} \nabla^2 T \quad (3)$$

Species concentration equation:

$$(\vec{V} \cdot \text{grad})C = D \nabla^2 C \pm k_1 C \quad (4)$$

where $\vec{j} = \sigma(\vec{E} + \vec{V} \times \vec{B}) - \frac{1}{en_e} \vec{j} \times \vec{B} - \frac{1}{en_e} \text{grad} p_e \text{div } \vec{B} = 0$, $\text{curl } \vec{H} = 0$ and $\text{curl } \vec{E} = 0$ where \vec{V} is the velocity vector, p is the pressure, ν is the kinematic coefficient of viscosity.

Continuity equation in terms of vector notation (unsteady flow) is $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$. For steady incompressible flow: $\frac{\partial \rho}{\partial t} = 0$ and ρ is a constant. Continuity equation becomes $\nabla \cdot (\rho \vec{V}) = 0$, it implies that $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$. Finally, the continuity equation (steady flow) is reduced to $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

Under these conditions, the basic governing boundary layer equations of momentum, energy and diffusion for mixed convection flow neglecting Joule's viscous dissipation can be simplified to the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (\text{Continuity}) \quad (5)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u \quad (x - \text{Momentum}) \quad (6)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho} v - \frac{\nu}{K} v \quad (y - \text{Momentum}) \quad (7)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial z} +$$

$$\nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (z - \text{Momentum}) \quad (8)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (\text{Energy}) \quad (9)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) - k_1 C \quad (\text{Diffusion}) \quad (10)$$

where u, v, w are the velocity components in the x, y and w directions respectively, ν is the kinematic viscosity, p is the pressure, σ is the electrical conductivity, ρ is the density of the fluid, B_o is the magnetic induction, α is the thermal conductivity of the fluid, μ is the dynamic viscosity, K is the porous medium permeability, $c_p g$ is the specific heat at constant pressure and k_1 is the rate of chemical reaction.

The boundary conditions applicable to the present flow are

$$\begin{aligned} u &= -U = -ax, v = -a(m - 1)y, \\ w &= -W, T = T_w, C = C_w \quad \text{at } y = 0, \\ u &\rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (11)$$

in which $a > 0$ is the shrinking constant, $W > 0$ is the suction velocity, $m = 1$ when sheet shrinks in x -direction only and $m = 2$ when the sheet shrinks axsymmetrically.

Introducing the following similarity transformations

$$\begin{aligned} u &= axf'(\eta), v = a(m - 1)yf'(\eta), w = -\sqrt{avm} f(\eta), \eta = \sqrt{\frac{a}{\nu}} z, \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \quad (12)$$

Equation (1) is identically satisfied and Equation (8) can be integrated to give

$$\frac{p}{\rho} - \nu\frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{constan}t \quad (13)$$

Equations (6)-(11) reduces to the following boundary value problem

$$f''' - (M^2 + \text{Pr } \lambda)f' - f'^2 + m f f'' = 0 \quad (14)$$

$$\theta'' + m \text{Pr } f \theta' - \text{Pr } \theta f' = 0 \quad (15)$$

$$\phi'' - Sc f' \phi + m Sc f \phi' - Sc \gamma \phi = 0 \quad (16)$$

The boundary conditions can be written as

$$\eta = 0 : \quad f(0) = S, f'(0) = -1, \theta(0) = 1, \phi(0) = 1$$

$$\eta \rightarrow \infty, \quad f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \quad (17)$$

where the Prandtl number, p_r , Schmidt number, Sc , Magnetic parameter, M^2 , Chemical reaction parameter, γ , porosity parameter, λ and suction parameter S can be defined as follows:

$$p_r = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \quad M^2 = \frac{\sigma B_0^2}{\rho a}, \quad \gamma = \frac{k_1}{a},$$

$$\lambda = \frac{\alpha}{aK} \quad \text{and} \quad S = \frac{W}{m\sqrt{a\nu}} \quad (18)$$

when $m = 1$ (sheet shrinks in x-direction) and $m = 2$ (sheet shrinks in axisymmetrically)

The mass diffusion equation (16) can be adjusted to meet these circumstances if one takes $\gamma > 0$ for destructive reaction, $\gamma = 0$ for no reaction and $\gamma < 0$ for generative reaction.

3 Numerical solution

The set of nonlinear ordinary differential equations (14) to (16) with boundary conditions (17) were solved numerically using Runge Kutta Gill algorithm, Gill (1951) with a systematic guessing of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ by the shooting technique until the boundary conditions at infinity $f'(0)$, $\theta(0)$ and $\phi(0)$ decay exponentially to zero. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution with η_{\max} , and an accuracy to the fifth decimal place is sufficient for convergence.

The computations were done by a program which uses a symbolic and computational computer language Matlab. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of η_∞ was found to each iteration loop by assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ , to each group of parameters α, γ, M^2 and m determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7} . Effects of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow over a porous shrinking sheet is studied in the presence of suction. In the following section, the results are discussed in detail.

4 Results and Discussion

Numerical computations are carried out for $1 \leq M^2 \leq 3, 1.0 \leq \gamma \leq 3.0, 0.1 \leq \lambda \leq 4.0$, and $1 \leq m \leq 3.0$. Typical velocity, temperature and concentration profiles are shown in following Figures for $Pr = 0.71$ and some values for the governing parameters γ, M^2, Sc, m and λ .

In the absence of energy and diffusion equations, in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for $M^2 = 2.0, m = 1.0$ are compared with the available exact solution of Sajid and Hayat 2007, is shown in Fig.2. It is observed that the agreement with the theoretical solution of velocity profile is excellent.

Effect of chemical reaction in the presence of magnetic field plays a very important role on concentration field. The dimensionless velocity, temperature and concentration distribution for different destructive reaction of chemical reaction is shown in Fig.3. It is seen from the figure that the concentration of the fluid decreases with increase of destructive reaction ($\gamma > 0$) whereas the velocity and temperature are significant with increase of destructive reaction. Also, it is observed that the concentration of the fluid decreases uniformly near the wall of the wedge. Figure 4 presents typical profiles for velocity, temperature and concentration for different values of magnetic parameter. In

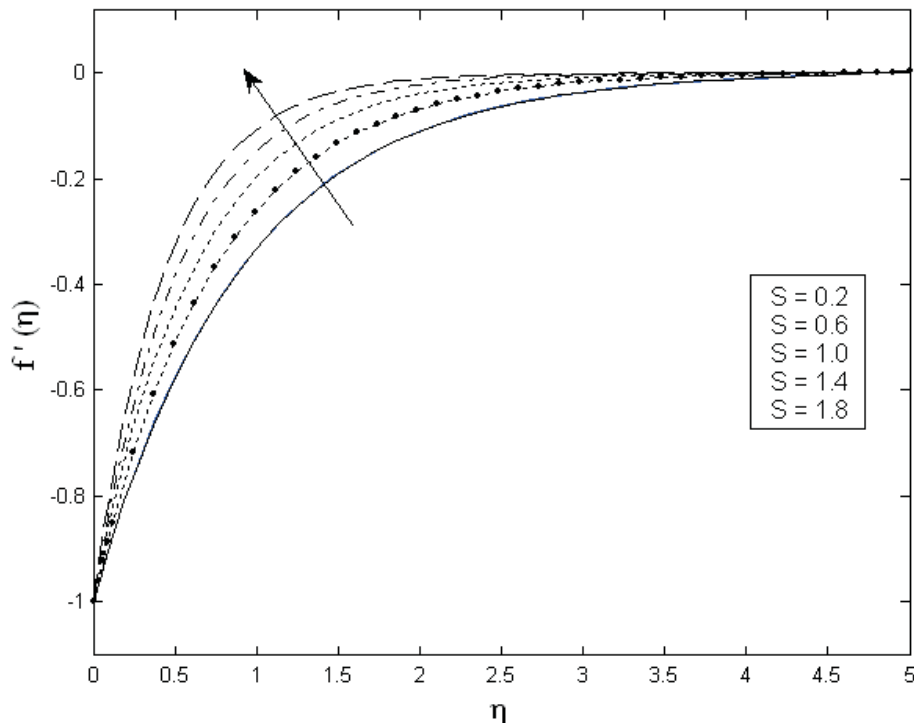


Figure 2: Suction effects on velocity profile. $Pr = 0.71, \lambda = 0, m = 1, M^2 = 2.0$

the presence of uniform chemical reaction, it is clearly shown that the velocity of the fluid increases and the temperature and concentration of the fluid slightly decrease, but not significant with increase of the strength of magnetic field. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the mixed convection flow. Application of a magnetic field moving with the free stream has the tendency to induce a motive force which decreases the

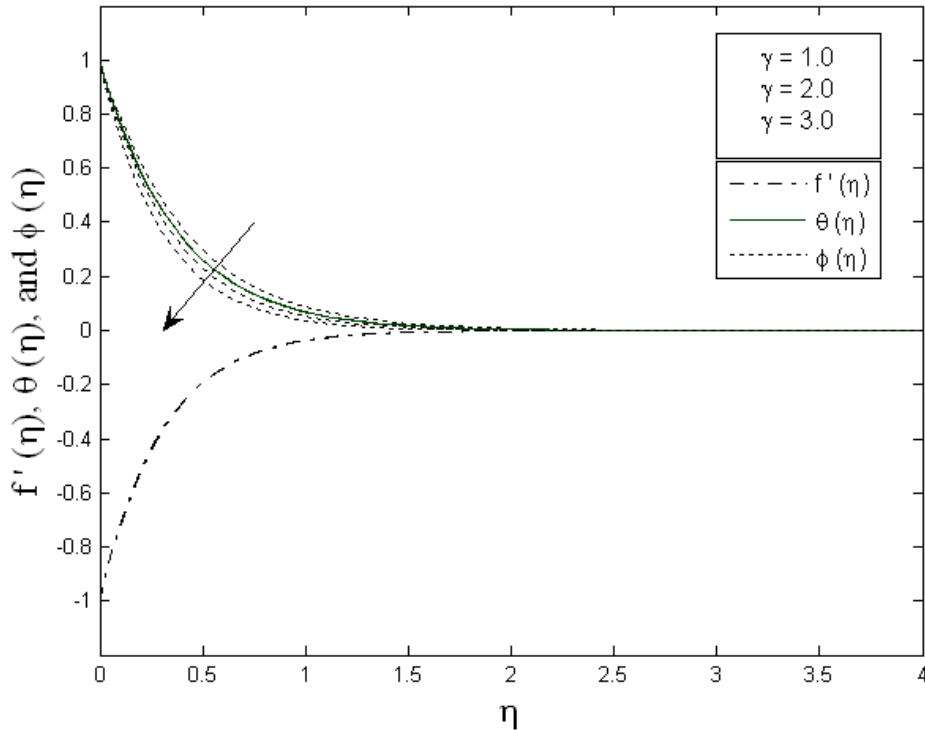


Figure 3: Chemical reaction effect on velocity, temperature and concentration profiles. $Pr = 1, Re_x = 3, \lambda = 1, Sc = 0.62, m = 1, M^2 = 1, S = 3$

motion of the fluid and increases its boundary layer. The effect of shrinking sheet parameter m on velocity, temperature and concentration field are shown in Fig.5. It is clearly predicted that the velocity of the fluid increases and the temperature and concentration of the fluid decrease with increase of the strength of shrinking sheet parameter. Due to the shrinking of the sheet, it is also seen that the changes of velocity, temperature and concentration of fluid are very fast and all these physical behavior are due to the combined effect of the strength of shrinking of sheet and porosity at the wall of the surface. From the

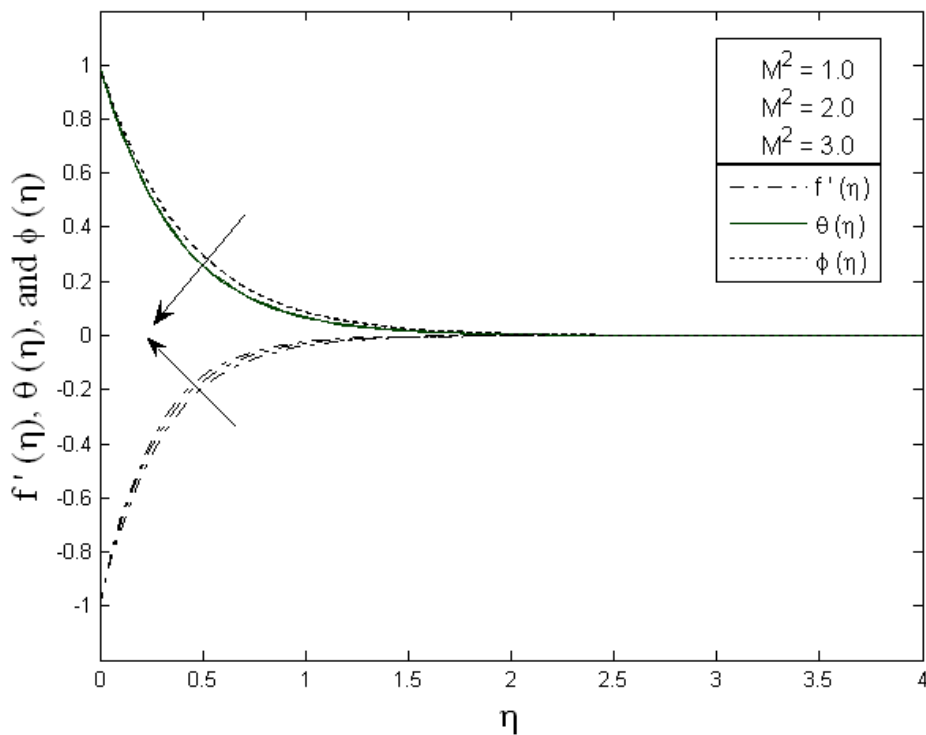


Figure 4: Magnetic effect on velocity, temperature and concentration profile. $Pr = 1, Re_x = 3, \lambda = 1, Sc = 0.62, m = 1, S = 3, \gamma = 1$

table 1, it is observed that the skin friction increases and the rate heat and mass transfer decrease with increase of shrinking, suction, porosity and magnetic parameters. But it is interesting to note that the skin friction and the rate of heat transfer are uniform whereas the rate of mass transfer decreases with increase of chemical reaction parameter.

5 Conclusions

In the present work, the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow past a porous shrinking

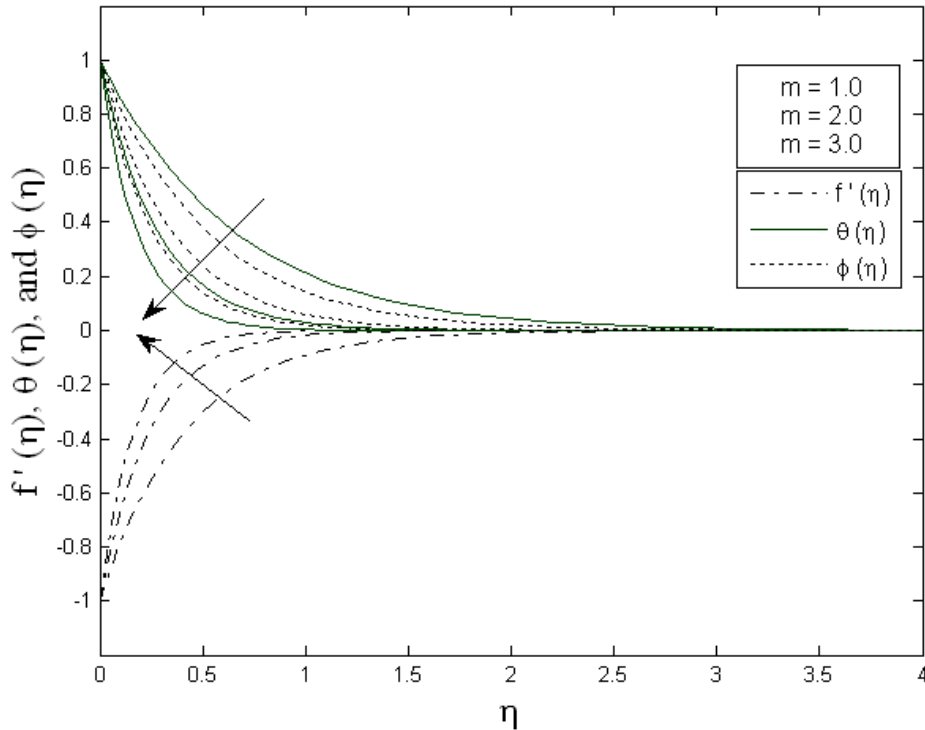


Figure 5: Shrinking parameter effect on velocity, temperature and concentration profile $Pr = 1$, $M^2 = 1$, $Re_x = 3$, $\lambda = 1$, $Sc = 0.62$, $S = 3$, $\gamma = 1$

sheet in the presence of suction is investigated. It is noticed that the increase of the strength of shrinking parameter is expected to alter the momentum, thermal and concentration boundary layers significantly. Particularly, it is seen that the porosity of the shrinking sheet reduces with increase of rate of heat and mass transfer. It is observed that the shrinking of the sheet with chemical reaction have a substantial effect on the flow field and, thus, on the heat and mass transfer rate from the sheet to the fluid. It is expected that this research may prove to be useful for the study of movement of oil or gas and water through the

$f''(0)$	$\theta'(0)$	$\phi'(0)$		Parameter
3.302776	-2.665537	-2.410283	$\lambda = 1.0$	Porosity
3.561553	-2.680315	-2.417000	$\lambda = 2.0$	
4.000000	-2.702455	-2.427225	$\lambda = 4.0$	
3.302776	-2.665537	-2.410283	$\gamma = 1.0$	Chemical reaction parameter
3.302776	-2.665537	-2.929028	$\gamma = 2.0$	
3.302776	-2.665537	-3.342858	$\gamma = 3.0$	
3.302776	-2.665537	-2.410283	$M^2 = 1.0$	Magnetic strength
3.561553	-2.680315	-2.417000	$M^2 = 2.0$	
3.791288	-2.692318	-2.422522	$M^2 = 3.0$	
2.414214	-1.493292	-1.917597	$m = 1.0$	Shrinking
4.124816	-3.608226	-2.865386	$m = 2.0$	
6.001955	-5.653797	-3.944462	$m = 3.0$	

Table 1: Analysis for skin friction and rate of heat and mass transfer.

reservoir of an oil or gas field, in the migration of under ground water and in the filtration and water purification processes. It is interesting to note that the results of the investigation are playing a very important role on packaging unit. In comparison to the stretching sheet problem it is found that the results in the case of hydrodynamic flow are not stable for the shrinking surface and only flows are meaningful in the case of shrinking surface. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

Acknowledgement: The authors wish to express their cordial thanks to our beloved The Vice-Chancellor and The Director of Centre for Science Studies, UTHM Malaysia for their encouragements and acknowledge the financial support received from SFRG 02-01-13- SF0060.

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Submitted on June 2008, revised on February 2009.

O uticaju hemijske reakcije, prenosa toplote i mase na nelinearni MHD granični sloj preko skupljajuće ploče sa usisavanjem

Ovaj rad se bavi uticajem hemijske reakcije, prenosa toplote i mase na nelinearni MHD granični sloj preko skupljajuće ploče sa usisavanjem. Diskutovani su slučajevi dvodimenzionog i osno simteričnog skupljanja. Jednačine graničnog sloja posmatranog problema su transformacijama sličnosti bezdimenzione. Transformisane spregnute nelinearne diferencijalne jednačine su numerički rešene. Izvršeno je uporedjenje sa prethodno objavljenim radovima. Numerički rezultati za bezdimenzionu brzinu, temperaturu i profile koncentracije su dobijeni i grafički prikazani.