Universal equations of unsteady two-dimensional MHD boundary layer whose temperature varies with time

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Abstract

This paper concerns with unsteady two-dimensional temperature laminar magnetohydrodynamic (MHD) boundary layer of incompressible fluid. It is assumed that induction of outer magnetic field is function of longitudinal coordinate with force lines perpendicular to the body surface on which boundary layer forms. Outer electric filed is neglected and magnetic Reynolds number is significantly lower then one i.e. considered problem is in induction-less approximation. Characteristic properties of fluid are constant because velocity of flow is much lower than speed of light and temperature difference is small enough (under $50^o C$). Introduced assumptions simplify considered problem in sake of mathematical solving, but adopted physical model is interesting from

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practical point of view, because its relation with large number of technically significant MHD flows. Obtained partial differential equations can be solved with modern numerical methods for every particular problem. Conclusions based on these solutions are related only with specific temperature MHD boundary layer problem. In this paper, quite different approach is used. First new variables are introduced and then sets of similarity parameters which transform equations on the form which don't contain inside and in corresponding boundary conditions characteristics of particular problems and in that sense equations are considered as universal. Obtained universal equations in appropriate approximation can be solved numerically once for all. So-called universal solutions of equations can be used to carry out general conclusions about temperature MHD boundary layer and for calculation of arbitrary particular problems. To calculate any particular problem it is necessary also to solve corresponding momentum equation.

Keywords: MHD, magnetic field, electroconductivity, temperature, similarity parameters, universal equations, Prandtl number, Eckert number.

Nomenclature

B	magnetic induction
c_p	specific heat capacity
D	standardization constant
Ec	Eckert number
F	characteristic function, $F = U \partial z / \partial t$
$f_{k,n}$	dynamical parameters
g	time derivate of characteristic function z
$g_{k,n}$	magnetic parameters
h	characteristic linear scale of transversal coordinate
H	characteristic function, $H = \delta^* / \delta^{**}$
H^*	characteristic function, $H = \delta^* / h$
H^{**}	characteristic function, $H = \delta^{**} / h$
$l_{k,n}$	temperature parameters
N	characteristic function, $N = \sigma B^2 / \rho$
Pr	Prandtl number

q	temperature difference between body surface and outer flow
t	time
T	thermodynamic temperature
u, v	longitudinal and transversal velocity in boundary layer respectively
U	velocity on outer border of boundary layer
x, y	longitudinal and transversal coordinate respectively
z	characteristic function, $z = h^2/\nu$

Greek symbols

δ^*	extrusion thickness
δ^{**}	thickness of impulse loss
Φ	dimensionless stream function
η	dimensionless transversal coordinate
λ	thermal conductivity
μ	viscosity
ν	kinematic viscosity
Θ	dimensionless temperature difference
ρ	fluid density
σ	conductivity
τ	shear stress
Ψ	stream function
ξ	characteristic function, $\xi = \tau_w h/(\mu U)$

Subscripts

0	initial time moment
1	known boundary layer cross-section
∞	outer boundary of boundary layer
w	body surface

1 Introduction

Idea of boundary layer control appear when Prandtl form the theory, and this idea came from Prandtl [1] himself. Here control mean, position

of boundary layer separation point control. For a long time following methods was used for boundary layer control: admit the body motion in streamwise direction, increasing the boundary layer velocity, boundary layer suction, second gas injection, profile laminarization, body cooling.

Interest in effect of outer magnetic filed on heat-physical processes appear fifty years ago [2]. Developing of this research was stimulated by two problems: protection of spacecrafts from aerodynamic overheating and destruction during the passage trough dense atmosphere layers; building the operational ability of high temperature MHD generators constructive elements for direct transformation of heat energy in to electric. First problem show that magnetohydrodynamical influence on ionized gases is convenient control method for mass, heat and hydrodynamic processes. Solutions of mentioned problems were followed with rapid increase of analytical papers and experimental procedures about heat transfer in MHD boundary layer [3], [4].

MHD research was connected gradually with new applied problems. MHD devices for liquid metals engage metallurgist attention. It was shown that effect of magnetic filed could be very helpful in modernization of technological processes. Developing of nuclear power systems is almost unconceivable without usage of MHD devices. It is determined that magnetic field can have significantly influence on different chemical-technology processes. Controlling of crystallization processes in metallurgy and influence of magnetic field on discrete chemical systems bring magnetohydrodynamics and heat physics in relation with problems that was research subject in other science (physical chemistry, kinetics, biophysics. . .). At the end, analogies, which appear with knowledge of magnetic field influence on mechanics and biological suspensions (especially blood), brought to possibility to transfer heat physics research results into magneto-biological and medical processes [5].

2   Mathematical model

As mentioned in introduction, in this paper, unsteady temperature two-dimensional laminar MHD boundary layer of incompressible neutral fluid is studied.

Outer magnetic field is still in relation to fluid in outer flow. It is assumed that outer magnetic filed induction is function of longitudinal coordinate with force lines perpendicular to the body surface on which boundary layer forms. Further on it is presumed there is no outer electric filed and magnetic Reynolds number is significantly lower then one i.e. considered problem is in induction-less approximation. Velocity of flow is considered much lower then speed of light and usual assumption in temperature boundary layer calculation that temperature difference is small (under 50^oC) is used, accordingly characteristic properties of fluid are constant (viscosity, heat conduction, electro-conductivity, magnetic permeability, mass heat capacity ...). Introduced assumptions simplify considered problem, however obtained physical model is interesting from practical point of view, because its relation with large number of MHD flows significant for technical practice. Of course, all introduced assumptions are related with simplified mathematical model, which can be solved.

Described two-dimensional problem of MHD unsteady temperature boundary layer in inductioneless approximation is mathematically presented with continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \tag{1}$$

moment equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}\left(u - U\right); \tag{2}$$

energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho\, c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2}{\rho\, c_p}\left(u - U\right)^2; \tag{3}$$

and corresponding boundary and initial conditions:

$$u = 0\,,\ v = 0,\ T = T_w\left(x, t\right) \quad for \quad y = 0; \tag{4}$$

$$u \to U\left(x, t\right),\ T \to T_\infty \quad for \quad y \to \infty; \tag{5}$$

$$u = u_0(x, y)\,,\ T = T_0\left(x, y\right) \quad for \quad t = t_0; \tag{6}$$

$$u = u_1(t, y), \ T = T_1(t, y) \quad for \quad x = x_0. \tag{7}$$

In previous equations and initial and boundary conditions the parameter labeling used is common for the theory of MHD boundary layer.

For further consideration stream function, $\Psi(x, y, t)$ is introduced with following relations:

$$\frac{\partial \Psi}{\partial x} = -v, \ \frac{\partial \Psi}{\partial y} = u; \tag{8}$$

which satisfies equation (1) identically and transform moment equation (2) into equation:

$$\frac{\partial^2 \Psi}{\partial t \partial y} + \frac{\partial \Psi}{\partial y}\frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x}\frac{\partial^2 \Psi}{\partial y^2} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \nu\frac{\partial^3 \Psi}{\partial y^3} - \frac{\sigma B^2}{\rho}\left(\frac{\partial \Psi}{\partial y} - U\right); \tag{9}$$

and energy equation into equation:

$$\frac{\partial T}{\partial t} + \frac{\partial \Psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\lambda}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho\, c_p}\left(\frac{\partial^2 \Psi}{\partial y^2}\right)^2 + \frac{\sigma B^2}{\rho\, c_p}\left(\frac{\partial \Psi}{\partial y} - U\right). \tag{10}$$

Boundary and initial conditions are transformed into equations:

$$\Psi = 0, \frac{\partial \Psi}{\partial y} = 0; \ T = T_w(x, t) \quad for \quad y = 0; \tag{11}$$

$$\frac{\partial \Psi}{\partial y} \to U(x, t) \ ; \ T \to T_\infty \quad for \quad y \to \infty; \tag{12}$$

$$\frac{\partial \Psi}{\partial y} = u_0(x, y), \ T = T_0(x, y) \quad for \quad t = t_0; \tag{13}$$

$$\frac{\partial \Psi}{\partial y} = u_1(t, y), \ T = T_1(t, y) \quad for \quad x = x_0. \tag{14}$$

Equation (9) does not depend from equation (10) and it can be solved independently. Solution of equation (9) is used for solving of equation (10).

3 Universal equations

Obtained partial differential equations (1), (2), (3) can be solved for every particular case using modern numerical methods and computer. In this paper, quite different approach is used based on ideas in papers [6,7,8] which is extended in papers [9,10,11]. Essence of this approach is in introducing adequate transformations and sets of parameters in starting equations of laminar two-dimensional unsteady temperature MHD boundary layer of incompressible fluid, which transform the equations system and corresponding boundary conditions into form unique for all particular problems and this form is considered as universal. Solution of universal equations obtained using modern numerical methods, can be on convenient wave saved and used for general conclusions derivation about developing of described temperature MHD boundary layer and for boundary layer calculation of observed problem special cases. Integration of obtained universal equations is performed once for all. In order to solve particular problems it is necessary to determine impulse equation using obtained universal solutions. In this paper, universal equations of described problem are given and numerical solving is subject of future research.

In order to realize described procedure following new variables are introduced:

$$x = x, \ t = t, \ \eta = \frac{Dy}{h(x,t)}, \ \Phi(x,t,\eta) = \frac{D\Psi(x,y,t)}{U(x,t)\,h(x,t)},$$

$$\Theta(x,t,\eta) = \frac{T_w - T}{T_w - T_\infty} \tag{15}$$

where D is normalizing constant, and $h(x,t)$ is characteristic linear scale of transversal coordinate in boundary layer. According to introduced variables, equation (9) is transformed in new form:

$$D^2 \frac{\partial^3 \Phi}{\partial \eta^3} + f_{1,0}\left(\Phi \frac{\partial^2 \Phi}{\partial \eta^2} - \left(\frac{\partial \Phi}{\partial \eta}\right)^2 + 1\right) + (f_{0,1} + g_{1,0})\left(1 - \frac{\partial \Phi}{\partial \eta}\right) +$$

$$\frac{1}{2}(F\Phi + \eta g)\frac{\partial^2 \Phi}{\partial \eta^2} = z\frac{\partial^2 \Phi}{\partial t \partial \eta} + UzX(\eta; x) \tag{16}$$

where for the sake of shorter expression, the notations are introduced:

$$z = \frac{h^2}{\nu}; \quad g = \frac{\partial z}{\partial t}; \quad N = \frac{\sigma B^2}{\rho}; \quad g_{1,0} = Nz; \quad F = U\frac{\partial z}{\partial x};$$

$$f_{1,0} = z\frac{\partial U}{\partial x}; \quad f_{0,1} = \frac{z}{U}\frac{\partial U}{\partial t}; \tag{17}$$

$$X(x_1; x_2) = \frac{\partial \Phi}{\partial x_1}\frac{\partial^2 \Phi}{\partial \eta \partial x_2} - \frac{\partial \Phi}{\partial x_2}\frac{\partial^2 \Phi}{\partial x_1 \partial \eta};$$

and equation (10) now have the form:

$$\frac{D^2}{P_r}\frac{\partial^2 \Theta}{\partial \eta^2} - D^2 E_c\left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2 - E_c g_{1,0}\left(1 - \frac{\partial \Phi}{\partial \eta}\right)^2 +$$

$$(1 - \Theta)\left(l_{0,1} + l_{1,0}\frac{\partial \Phi}{\partial \eta}\right) + \frac{1}{2}\eta g\frac{\partial \Theta}{\partial \eta} + \tag{18}$$

$$\frac{1}{2}(F + 2f_{1,0})\Phi\frac{\partial \Theta}{\partial \eta} = z\frac{\partial \Theta}{\partial t} - UzY(x; \eta)$$

where the following notations have been used for the sake of shorter statement:

$$\mathrm{Pr} = \frac{\nu \rho c_p}{\lambda} \quad - Prandtl\ number$$

$$E_c = \frac{U^2}{c_p(T_w - T_\infty)} \quad - Eckert\ number$$

$$l_{0,1} = \frac{z}{T_w - T_\infty}\frac{\partial T_w}{\partial t} \quad ; \quad l_{1,0} = \frac{Uz}{T_w - T_\infty}\frac{\partial T_w}{\partial x},$$

$$Y(x_1; x_2) = \frac{\partial \Phi}{\partial x_1}\frac{\partial \Theta}{\partial x_2} - \frac{\partial \Phi}{\partial x_2}\frac{\partial \Theta}{\partial x_1}. \tag{19}$$

Now we introduce sets of parameters: dynamical

$$f_{k,n} = U^{k-1}\frac{\partial^{k+n}U}{\partial x^k \partial t^n}z^{k+n}(k, n = 0, 1, 2, ...; k \vee n \neq 0); \tag{20}$$

magnetic

$$g_{k,n} = U^{k-1}\frac{\partial^{k-1+n}N}{\partial x^{k-1}\partial t^n}z^{k+n}(k, n = 0, 1, 2, ...,; k \neq 0); \tag{21}$$

temperature

$$l_{k,n} = \frac{U^k}{q} \frac{\partial^{k+n} q}{\partial x^k \partial t^n} z^{k+n} \ , (k, n = 0, 1, 2, ...; \ k \vee n \neq 0), \qquad (22)$$

where

$$q = T_w - T_\infty; \qquad (23)$$

and constant parameter:

$$g = \frac{\partial z}{\partial t} = const. \qquad (24)$$

which can have different values. It can be noticed that first parameters are given in the Eqs. (17) and (19). Introduced sets of parameters reflect the nature of velocity change on outer edge of boundary layer, alteration characteristic of variable N and temperature change on body surface, and a part from that, in the integral form (by means of z and $\partial z / \partial t$) pre-history of flow in boundary layer. Introduced sets of parameters enable transformations of differential equations (16) and (18) into universal form in sense that neither equations nor boundary conditions explicitly depends from values that characterized particular problems.

Procedure for obtaining "universal" equations has following steps. First, we find the derivates in Eqs. (16) and (18) using the operators given in equations (25)-(26), and then we transform this Eqs. with introduced independent variables $\eta; f_{k,n}; g_{k,n}; l_{k,n}$.

For derivate along longitudinal coordinate x we used operator:

$$\frac{\partial}{\partial x} = \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left(\frac{\partial f_{k,n}}{\partial x} \frac{\partial}{\partial f_{k,n}} + \begin{cases} 0, \ for \ Eq.\,(16) \\ \frac{\partial l_{k,n}}{\partial x} \frac{\partial}{\partial l_{k,n}}, \ for \ Eq.\,(18) \end{cases} \right) +$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial x} \frac{\partial}{\partial g_{k,n}}; \qquad (25)$$

and for time derivate operator:

$$\frac{\partial}{\partial t} = \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left(\frac{\partial f_{k,n}}{\partial t} \frac{\partial}{\partial f_{k,n}} + \begin{cases} 0, \ for \ Eq.\,(16) \\ \frac{\partial l_{k,n}}{\partial t} \frac{\partial}{\partial l_{k,n}}, \ for \ Eq.\,(18) \end{cases} \right) +$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial t} \frac{\partial}{\partial g_{k,n}}; \tag{26}$$

Parameter derivates along coordinate x and time t are obtained by differentiation of Eqs. (20)-(22):

$$\frac{\partial f_{k,n}}{\partial x} = \frac{1}{Uz} \left\{ [(k-1) f_{1,0} + (k+n) F] f_{k,n} + f_{k+1,n} \right\} = \frac{1}{Uz} Q_{k,n}; \tag{27}$$

$$\frac{\partial f_{k,n}}{\partial t} = \frac{1}{z} \left\{ [(k-1) f_{0,1} + (k+n) g] f_{k,n} + f_{k,n+1} \right\} = \frac{1}{z} E_{k,n}; \tag{28}$$

$$\frac{\partial g_{k,n}}{\partial x} = \frac{1}{Uz} \left\{ [(k-1) f_{1,0} + (k+n) F] g_{k,n} + g_{k+1,n} \right\} = \frac{1}{Uz} K_{k,n}; \tag{29}$$

$$\frac{\partial g_{k,n}}{\partial t} = \frac{1}{z} \left\{ [(k-1) f_{0,1} + (k+n) g] g_{k,n} + g_{k,n+1} \right\} = \frac{1}{z} L_{k,n}; \tag{30}$$

$$\frac{\partial l_{k,n}}{\partial x} = \frac{1}{Uz} \left\{ [(k-1) l_{1,0} + (k+n) F] l_{k,n} + l_{k+1,n} \right\} = \frac{1}{Uz} M_{k,n}; \tag{31}$$

$$\frac{\partial l_{k,n}}{\partial t} = \frac{1}{z} \left\{ [(k-1) l_{0,1} + (k+n) g] l_{k,n} + l_{k,n+1} \right\} = \frac{1}{z} N_{k,n}; \tag{32}$$

where $Q_{k,n}; E_{k,n}; K_{k,n}; L_{k,n}; M_{k,n}; N_{k,n}$ are terms in curly brackets in obtained equations. It is important to notice $Q_{k,n}; K_{k,n}; M_{k,n}$ beside the parameters depend on value $U \partial z/\partial x = F$. Using Eqs. (20)-(22) and (24), operators (25) and (26), terms (27)-(32) Eqs. (16) and (18) are transformed into equations:

$$\Im_1 = \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left[Q_{k,n} X(\eta; f_{k,n}) + E_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial f_{k,n}} \right] +$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \left[L_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial g_{k,n}} + K_{k,n} X(\eta; g_{k,n}) \right]; \tag{33}$$

$$\Im_2 = \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left[Q_{k,n} Y(\eta; f_{k,n}) + E_k \frac{\partial \Theta}{\partial f_{k,n}} + N_{k,n} \frac{\partial \Theta}{\partial l_{k,n}} + M_{k,n} Y(\eta; l_{k,n}) \right] +$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \left[L_{k,n} \frac{\partial \Theta}{\partial g_{k,n}} + K_{k,n} Y\left(\eta; g_{k,n}\right) \right], \tag{34}$$

where the following markings have been used for shorter statement:$\Im_1$- left side of first equation of system (16), $\Im_2$- left side of equation (18). In order to make Eqs. (33) and (34) universal it is necessary to show that value F which appear in terms for $Q_{k,n}; K_{k,n}; M_{k,n}$ can be expressed by means of introduced parameters. In order to prove mentioned we start from impulse equation of described problem:

$$\frac{\partial}{\partial t}\left(U\delta^*\right) + \frac{\partial}{\partial x}\left(U^2\delta^{**}\right) + U\left(\frac{\partial U}{\partial x} + N\right)\delta^* - \frac{\tau_w}{\rho} = 0, \tag{35}$$

in which:

$$\delta^*\left(x,t\right) = \int_0^{\infty} \left(1 - \frac{u}{U}\right) dy \quad extrusion\ thickness \tag{36}$$

$$\delta^{**}\left(x,t\right) = \int_0^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \quad thicknes\ of\ impulse\ loss \tag{37}$$

and

$$\tau_w\left(x,t\right) = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad friction\ stress\ on\ the\ body. \tag{38}$$

Introducing dimensionless characteristic functions:

$$H^*\left(x,t\right) = \frac{\delta^*}{h} \quad ; \quad H^{**}\left(x,t\right) = \frac{\delta^{**}}{h} \quad ; \quad \xi\left(x,t\right) = \frac{\tau_w h}{\mu U} \tag{39}$$

which, according to Eqs. (14) and (36)-(38), can be expressed in the following form:

$$H^*\left(x,t\right) = \frac{1}{D}\int_0^{\infty}\left(1 - \frac{\partial \Phi}{\partial \eta}\right)d\eta, \quad H^{**}\left(x,t\right) = \frac{1}{D}\int_0^{\infty}\frac{\partial \Phi}{\partial \eta}\left(1 - \frac{\partial \Phi}{\partial \eta}\right)d\eta,$$

$$\xi\left(x,t\right) = D\left.\frac{\partial^2 \Phi}{\partial \eta^2}\right|_{\eta=0}; \tag{40}$$

and after transition to new independent variables (introduced parameters) in terms (40) values H^*, H^{**}, ξ become functions only from parameters $f_{k,n}$, $g_{k,n}$, $l_{k,n}$, g. Now, using parameters (20), (21) and (22) as new independent variables and derivative operators (25), (26) from impulse Eq. (35) after simple transformation next equation is obtained:

$$F = \frac{P}{Q} \tag{41}$$

where, for the sake of shorter expression following marks are used:

$$P = \xi - f_{1,0}\left(2H^{**} + H^*\right) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H^* -$$

$$\sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty} \left\{E_{k,n}\frac{\partial H^*}{\partial f_{k,n}} + \left[(k-1)f_{1,0}f_{k,n} + f_{k+1,n}\right]\frac{\partial H^{**}}{\partial f_{k,n}}\right\} -$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \left\{L_{k,n}\frac{\partial H^*}{\partial g_{k,n}} + \left[(k-1)f_{1,0}g_{k,n} + g_{k+1,n}\right]\frac{\partial H^{**}}{\partial g_{k,n}}\right\}; \tag{42}$$

$$Q = \frac{1}{2}H^{**} + \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty}(k+n)f_{k,n}\frac{\partial H^{**}}{\partial f_{k,n}} + \sum_{\substack{k=1 \\ n=0}}^{\infty}(k+n)g_{k,n}\frac{\partial H^{**}}{\partial g_{k,n}}. \tag{43}$$

Last two equations define function F in terms of values, which depends only from introduced parameters. Eqs. (33) and (34) are now universal equations of described problem. Boundary conditions, also universal, are given with terms:

$$\Phi = 0, \ \frac{\partial \Phi}{\partial \eta} = 0, \ \Theta = 0 \ for \ \eta = 0 \tag{44}$$

$$\Phi \to 1, \ \Theta \to 1 \ for \ \eta \to \infty \tag{45}$$

$$\Phi = \Phi_0\left(\eta\right), \ \Theta = \Theta_0\left(\eta\right) \quad for$$

$$\left\{\begin{array}{ll} f_{k,n} = 0, \ l_{k,n} = 0 & (k, n = 0, 1, 2, ... k \vee n \neq 0) \\ g_{k,n} = 0 & (k, n = 0, 1, 2, ..., k \neq 0) \\ g = 0 & \end{array}\right\} \tag{46}$$

where $\Phi_0(\eta)$-Blasius solution for stationary boundary layer on the plate, $\Theta_0(\eta)$ is solution of following equation:

$$\frac{D^2}{\text{Pr}}\frac{d^2\Theta_0}{d\eta^2} - D^2 Ec\left(\frac{d^2\Phi_0}{d\eta^2}\right)^2 + \frac{\xi_0}{H^{**}}\Phi_0\frac{d\Theta_0}{d\eta} = 0. \tag{47}$$

Universal Eqs. (33) and (34) with boundary conditions (44)-(46) are strictly for wide class of problems in which $z = At + C(x)$, where A is arbitrary constant and $C(x)$ some function of longitudinal coordinate. For other problems this equations are approximated "universal" equations.

Eqs.(33) and (34) are integrated in m-parametric approximation once for good and all. Obtained characteristic function can be used to yield general conclusions about development of described boundary layer and to solve any particular problem.

Before integration for scale of transversal coordinate in boundary layer $h(x,t)$ some characteristic value is chosen. In this case $h = \delta^{**}$ and accordingly to Eq. (39) $H^{**} = 1$, $H^* = \delta^*/\delta^{**} = H$, and equality (40) now have form:

$$F = 2\left[\xi - f_{1,0}(2 + H) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H - \right.$$

$$\left. \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty} E_{k,n}\frac{\partial H}{\partial f_{k,n}} - \sum_{\substack{k=1 \\ n=0}}^{\infty} L_{k,n}\frac{\partial H}{\partial g_{k,n}}\right] \tag{48}$$

Taking parameters $f_{k,n} = 0$, $g_{k,n} = 0$, $g = 0$ Eq. (33) is simplified into form:

$$\frac{d^3\Phi_0}{d\eta^3} + \frac{\xi_0}{D^2}\Phi_0\frac{d^2\Phi_0}{d\eta^2} = 0 \tag{49}$$

and if $D^2 = \xi_0$ then previous Eq. became well-known Blasius equation. According to previous statement for normalizing constant D value 0,47 must be chosen. For selected value h Eq. (47) for determining variable Θ_0 became:

$$\frac{1}{\text{Pr}}\frac{d^2\Theta_0}{d\eta^2} + \Phi_0\frac{d\Theta_0}{d\eta} - Ec\left(\frac{d^2\Phi_0}{d\eta^2}\right)^2 = 0. \tag{50}$$

In this paper adequate approximations of Eqs. (33) and (34) are given in which influence of parameters $f_{1,0}$, $f_{0,1}$, $g_{1,0}$, $l_{1,0}$, $l_{0,1}$ and g are detained and influence of parameters $f_{0,1}$, $l_{1,0}$, $l_{0,1}$ derivatives are disregarded. In this way Eq. (33) is simplified into following form:

$$\Im_1 = Ff_{1,0}X\left(\eta; f_{1,0}\right) + gf_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial f_{1,0}} + Fg_{1,0}X\left(\eta; g_{1,0}\right) + gg_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial g_{1,0}} \quad (51)$$

and function F is obtained from Eq. (48) in same approximation:

$$F = 2\left[\xi - f_{1,0}\left(2 + H\right) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H - gf_{1,0}\frac{\partial H}{\partial f_{1,0}} - gg_{1,0}\frac{\partial H}{\partial g_{1,0}}\right].$$
$$(52)$$

Eq.(50) is four-parametric once localized approximation of Eq. (33).

Eq.(34) is simplified into form:

$$\Im_2 = Ff_{1,0}Y\left(\eta; f_{1,0}\right) + gf_{1,0}\frac{\partial\Theta}{\partial f_{1,0}} + Fg_{1,0}Y\left(\eta; g_{1,0}\right) + gg_{1,0}\frac{\partial\Theta}{\partial g_{1,0}} \quad (53)$$

where function F is given with Eq. (52). Last equation is six-parametric three times localized approximation of Eq. (34).

Boundary conditions which coincide to Eqs. (51) and (52) are conditions (44), (45) and condition:

$$\Phi = \Phi_0\left(\eta\right), \ \Theta = \Theta_0\left(\eta\right) \ for \ f_{1,0} = 0, \ f_{0,1} = 0,$$

$$g_{1,0} = 0, \ l_{1,0} = 0, \ l_{0,1} = 0 \ and \ g = 0 \quad (54)$$

which is obtained from condition (46), using same simplifications like as equations.

Universal equations (51) and (53) need to be solved with corresponding boundary conditions (44), (45) and (54) using three-diagonal method, known in Russian literature as the "progonka" method. Obtained universal solutions can be saved and then used for general conclusions about boundary layer development and also for calculations for every particular problem.

As we early mentioned numerical solving of universal equations will be subject of future research of authors.

4   Conclusion

In this paper unsteady two-dimensional MHD boundary layer whose temperature varies with time is considered. This problem can be analyzed for every particular case i.e. for given outer flow characteristics. Here is used quite different approach in order to use benefits of multi-parametric method and universal equations of observed problem are derived. These equations in some approximation are solved once for all. Approximation is in relation with taking into account definite number of parameters, which is in direct connection with available computer memory. Some approximated equations are given in paper. Solving of these equations is subject of future research.

References

[1] H. Schlichting, Grenzschicht-Theorie, Braun-Verlag, Karlsruhe, 1982.

[2] E.A.Blum, Yu.A.Mihailov and R.Ya.Ozolc, Heat and mass transfer in a magnetic field, (in Russian) Institute of Physics of the Lituanian Academy of Sciences, Riga, 1980.

[3] E.A.Blum and Yu.A.Mihailov, *Heat transfer in conductinga liquids with presence of a transverse magnetic field*, (in Russian), Fifth MHD meeting of Riga: Turbulent MHD-flows, Riga, (1966) 2-18.

[4] T.Ašković and R.Ašković, *Sur un probléme de la couche limite magnétohydrodynamique laminare non stationnare avec une conductivité electrique variable*, ZAMM 57 (1977) 170-173.

[5] E.E.Tzirtzilakis, *A mathematical model for blood flow in magnetic field*, Physics of Fluids, Vol. 17, Issue 7 (2005) p. 15.

[6] V.Shkadov, *Boundary layer with pressure gradient in a stream of a compressible liquid* (in Russian), Izv. AN SSR - OTN, Mechanics and mashine design (Mekhanika i mashinostroenie), 2 (1963) 28-32.

[7] L.G.Lojcjanskij, *Universal equations and parametric approximations in the theory of laminar boundary layer* (in Russian), PMM (Applied mathematics and mechanics) Vol. 29, (1965) 70-87

[8] Saljnikov, V. N., 1978, A Contribution to Universal Solutions of the Boundary Layer Theory, Theoretical and Applied Mechanics 4, pp. 139 - 163.

[9] O.N.Bushmarin and Yu.V.Sarajev, Parametrical method in the theory of nonstationary boundary layer, Journal of engineering and phyasics (Inz-fiz zurnal), Vol. XXVII, No. 1, (1974) 110-118.

[10] Boričić, Z., Nikodijević, D., Milenković, D., Stamenković, Ž., 2005, A form of MHD universal equations of unsteady incompressible fluid flow with variable electroconductivity on heated moving plate, Theoret. Appl. Mech., Vol.32, No.1, pp. 65–77.

[11] Obrović, B., Nikodijević, D., Savić S., 2005, Boundary layer of the dissociated gas flow over a porous wall under the conditions of equilibrium dissociation Theoret. Appl. Mech., Vol.32, No.2, pp. 165–190.

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Univerzalne jednačine nestacionarnog dvodimenzijskog MHD graničnog sloja na telu duž koga se temperature menja tokom vremena

U ovom radu se razmatra nestacionarni temperaturski dvodimenzionalni magnetnohidrodinamički (MHD) laminarni granični sloj nestišljivog neutralnog fluida. Pretpostavlja se da je indukcija spoljašnjeg magnetnog polja funkcija samo uzdužne koordiante sa linijama sila upravnim na površ tela na kome se formira granični sloj. Dalje se pretpostavlja da spoljašnje električno polje ne postoji i da je vrednost magnetnog Rejnoldsovog broja značajno manja od jedinice tj. Razmatra se problem u tzv. Bezindukcionoj aproksimaciji. Osim toga, smatra se da su brzine strujanja mnogo manje od brzine svetlosti, i da su razlike temperature dovoljno male (ispod $50^o C$), odnosno da su karakteristične veličine fizičkih svojstava fluida konstantne. Iako učinjene pretpostavke značajno uprošćavaju razmatrani problem, tako da se može matematički rešavati, usvojeni fizički model strujanja je interesantan i sa praktične tačke gledišta, jer se odnosi na veliki broj magnetnohidrodinamičkih strujanja od tehničkog značaja. U radu dobijene parcijalne diferencijalne jednačine mogu se u svakom konkretnom slučaju rešiti korišćenjem savremenih numeričkih metoda i računara. Zaključci donešeni na osnovu ovakvih rešenja odnose se samo na konkretni problem temperaturskog MHD graničnog sloja. U ovom radu se koristi sasvim drugačiji pristup. Uvode se nove promenljive, a zatim i skupovi parametara sličnosti i tako jednačine svode na oblike koji u sebi i odgovarajućim graničnim uslovima ne sadrže karakteristike konkretnih problema i u tom smislu ih smatramo univerzalnim. Dobijene univerzalne jednačine, u odgovarajućoj aproksimaciji, mogu se numerički rešiti jedared za svagda. Na ovaj način dobijena tzv. univerzalna rešenja mogu se koristiti za donošenje generalnih zaključaka o temperaturskom MHD graničnom sloju, a i za proračune konkretnih problema temperaturskog graničnog sloja. Za proračune konkretnih problema potrebno je još rešiti samo odgovarajuću impulsnu jednačinu.