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Lie group analysis for the effect of viscosity and thermophoresis particle deposition on free convective heat and mass transfer in the presence of suction/injection

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Abstract

An analysis has been carried out to study heat and mass transfer characteristics of an incompressible and Newtonian fluid having temperature-dependent fluid viscosity and thermophoresis particle deposition over a vertical stretching surface with variable stream condition. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The vertical surface is assumed to be permeable so as to allow for possible wall suction or injection. The governing differential equations are derived and transformed using Lie group analysis. The transformed equations are solved numerically by applying Runge–Kutta Gill scheme with shooting technique. Favorable comparisons with previously published work on various special cases of the problem are obtained. Numerical results for the velocity, temperature and concentration profiles for a prescribed temperature-dependent fluid viscosity and thermophoresis particle deposition parameters are presented graphically to elucidate the influence of the various physical parameters.

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1 Introduction

The study of thermal radiation heat transfer with temperature-dependent fluid viscosity effects from vertical surface is important due to its wide range of applications in industrial, technological and geothermal applications such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids and power generation systems. In this paper, symmetry methods are applied to a natural convection boundary layer problem. The main advantage of such methods is that they can successfully be applied to non-linear differential equations. The symmetries of differential equations are those continuous groups of transformations under which the differential equations remain invariant.

Thermophoresis is the term describing the fact that small micron sized particles suspended in a non-isothermal gas will acquire a velocity in the direction of decreasing temperature. The gas molecules coming from the hot side of the particles have a greater velocity than those coming from the cold side. The faster moving molecules collide with the particles more forcefully. This difference in momentum leads to the particle developing a velocity in the direction of the cooler temperature. The velocity acquired by the particles is called the thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is known as the thermophoretic force. The magnitudes of the thermophoretic force and velocity are proportional to the temperature gradient and depend on many factors like thermal conductivity of aerosol particles and carrier gas. Thermophoresis causes small particles to deposit on cold surfaces. Thermophoresis principle is utilized to manufacture graded index silicon dioxide and germanium dioxide optical fiber preforms used in the field of communications.

In the field of viscous fluids there is a large body of papers dealing with the effect of thermophoresis particle deposition. We limit ourselves to a short description of the status-of-the art in the laminar regime. In the paper [1], by Epstein et al., thermophoretic deposition of particles from a vertical plate in free convection boundary layer flow was analyzed. Further, thermophoresis in the horizontal plate configuration was focused by Goren [2]. The thermophoretic transport of small particles in forced convection flow over inclined plates was studied by Garg and Jayaraj [3]. The paper by Jayaraj et al. [4] dealt with thermophoresis in natural convection with variable properties for a laminar flow of a viscous fluid over a cold vertical flat plate. Selim et al. in document [5] analyzed the effect of surface mass flux on mixed convection flow past a heated vertical flat permeable plate with thermophoresis, by considering a nonuniform surface mass flux through the surface. The combined effects of inertia, diffusion and thermophoresis on particle deposition from a stagnation point flow onto an axisymmetric wavy wafer have been examined by Wang [6]. In another very recent paper, [7], a theoretical model for the coupled transport mechanisms of diffusion, convection and thermophoresis was developed to describe the particle deposition onto a continuous moving wavy surface. Chamka and Pop [8] looked to the effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium; the steady free convection over an isothermal vertical circular cylinder embedded in a fluid-saturated porous medium in the presence of the thermophoresis particle deposition effect was analyzed in [9]. On the other hand, the impetuous research on convective flows in porous media is surveyed in the recents books by Nield and Bejan [10] and Ingham and Pop [11] and [12]. Chieh-Li Chen and Kun-Chieh Chan [13] studied the combined effects of thermophoresis and electrophoresis on particle deposition onto a wavy surface disk. Wang [14] investigated the combined effects of inertia and thermophoresis on particle deposition onto a wafer with wavy surface.

The viscosity is the property of the fluid that resists the flow of the fluids like liquids and gases. Understanding the effect of temperature on the viscosity of the fluid is very important. The temperature-dependent fluid viscosity is the phenomenon by which liquid viscosity tends to decrease (or, alternatively, its fluidity tends to increase) as its temperature increases. Temperature plays an important role in almost all fields of science, including physics, geology, chemistry, and biology. Many physical properties of materials including the phase (solid, liquid, gaseous or plasma), density, solubility, vapor pressure, and electrical conductivity depend on the temperature. The temperature-dependent fluid viscosity with thermophoresis particle deposition becomes more significant when the concentration gradients, temperature gradients are high. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is also affected. Therefore, to predict the flow behavior accurately it is necessary to take into account the viscosity variation for incompressible fluids. Gary et al. [15] and Mehta and Sood [16] showed that, when this effect is included the flow characteristics may changed substantially compared to the constant viscosity assumption. Mukhopadhyay et al. [17] investigated the MHD boundary layer flow with variable fluid viscosity over a heated stretching sheet. Mukhopadhyay and Layek [18] studied the effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface. Vajravelu and Hadjinicolaou [20] studied the flow and heat transfer characteristic in an electrically conducting fluid near an isothermal stretching sheet. Sharma and Mathur [21] investigated steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical infinite plate in the presence of heat source. At present, to the author's best knowledge, two studies dealing with the thermophoresis effect in porous media were published: recently, Partha [22] presented the effects of suction/injection on thermophoresis particle deposition in a non-Darcy porous medium under the influence of Soret, Dufour effects; the effects of thermophoresis particle deposition and chemical reaction on non-Darcy mixed convective heat and mass transfer past a porous wedge with variable viscosity in the presence of suction/injection was analyzed in Kandasamy et al. [23]. On the other hand, the impetuous research on convective flows in porous media is surveyed in the recents books by Nield and Bejan [10] and Ingham and Pop [11,12]. Recently, Kandasamy et al [32,33] analyzed the effects of temperature-dependent fluid viscosity with variable stream condition. Rashad [34] studied the influence of radiation of MHD free convection from a vertical flat plate embedded in porous media with thermophoretic deposition of particles,

Using Lie group analysis, three-dimensional, unsteady, laminar boundary layer equations of non-Newtonian fluids were studied by Yurusoy and Pakdemirli [29] and [30]. Using Lie group analysis, they obtained two different reductions to ordinary differential equations. They studied the effects of a moving surface with vertical suction or injection through the porous surface and also analyzed the exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. Yurusoy et al. [31] investigated the Lie group analysis of creeping flow of a second grade fluid. They constructed an exponential type of exact solution using the translation symmetry and a series type of approximate solution using the scaling symmetry and also discussed some boundary value problems. But so far no attempt has been made to analyze the effect of temperature-dependent fluid viscosity with thermophoresis particle deposition on natural convection flow past a vertical stretching surface for various parameters using Lie group analysis.

2 Mathematical analysis

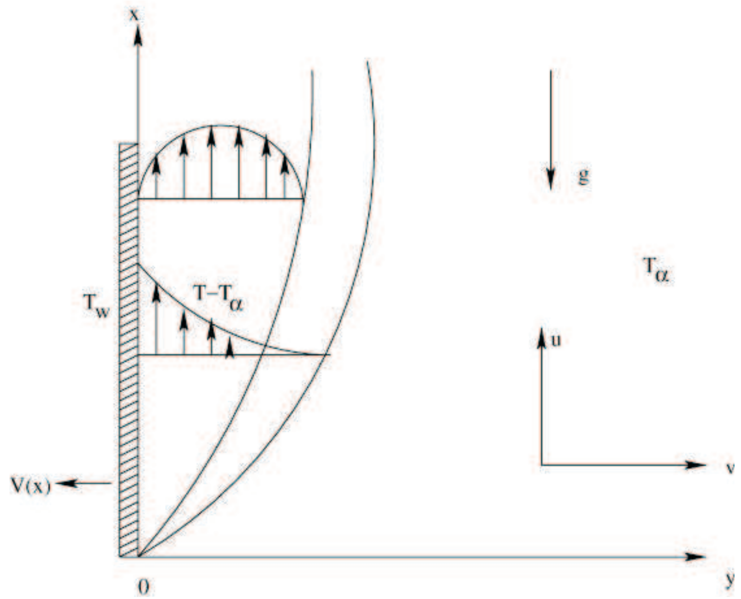

Figure 1: Physical model of boundary layer flow over a vertical stretching surface

We consider a free convective, laminar boundary layer flow and heat and mass transfer of viscous incompressible and Newtonian fluid over a vertical stretching sheet emerging out of a slit at origin (x = 0, y = 0) and moving with non-uniform velocity U(x) in the presence of thermal radiation (Fig.1). A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. The chemical reaction is not taking place in the flow and the effects of thermophoresis are being taken into account to help in the understanding of the mass deposition variation on the surface. The viscous dissipation effect and Joule heat are neglected on account of the fluid is finitely conducting. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall, C_∞, is infinitesimally small and the viscous dissipation term in the energy equation is neglected (as the fluid velocity is very low).

The fundamental equations for steady incompressible flow can be defined as follows:

Continuity equation:

$$div\,\vec{V}\;=0 \tag{1}$$

Momentum equation:

$$(\vec{V}\;.\;grad\,\vec{V})\;=-\frac{1}{\rho}\,grad\,p+\nu\,\nabla^2\,\vec{V}+\vec{g}\,\{\beta(T-T_\infty)+\beta^*(C-C_\infty)\} \tag{2}$$

Energy equation:

$$(\vec{V}.\;grad)T=\;\frac{k_e}{\rho\;c_p}\,\nabla^2T+\vec{Q} \tag{3}$$

Species concentration equation:

$$(\vec{V}.\;grad)C=\;D\,\nabla^2\,C\;-\;(div\;\vec{v}_TC) \tag{4}$$

where $\vec{V}$ the velocity vector, p is the pressure, ν is the kinematic coefficient of viscosity, $\vec{Q}$ is the additional heat force and $\vec{g}$ is the acceleration due to gravity.

Under these conditions, the basic governing boundary layer equation of momentum, energy and diffusion for laminar flow neglecting Joule's viscous dissipation under Boussinesq's approximation including the temperature dependent fluid viscosity can be simplified to the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}+[g\beta(T-T_\infty)+g\beta^*(C-C_\infty)] \quad (6)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho\, c_p}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho\, c_p}(T_\infty-T) \quad (7)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial(V_T C)}{\partial y} \quad (8)$$

$$u=U(x),\;\; v=-V(x),\;\; C=C_w,\;\; T=T_w \;\; at \;\; y=0;$$

$$u=0,\;\; C=C_\infty,\;\; T=T_\infty \;\; as \;\; y\rightarrow\infty \quad (9)$$

Here u and v are the components of velocity respectively in the x and y directions, μ is the coefficient of fluid viscosity, ρ is the fluid density, T is the temperature, κ is the thermal conductivity of the fluid, D is diffusion coefficient, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of concentration expansion, g is the gravity field, T_∞ is the temperature at infinity, $U(x)$ is the streamwise velocity, $V_T(=-k\frac{\nu}{T}\frac{\partial T}{\partial y})$ is the thermophoretic velocity, where k is the thermophoretic coefficient and $V(x)$ is the velocity of suction / injection of the fluid, T_w is the wall temperature. Using Rosseland approximation for radiation (Brewster [24]) we can write $q_r=-\frac{4\sigma_1}{3k^*}\frac{\partial T^4}{\partial y}$ where σ_1 is the Stefan–Boltzman constant, k^* is the absorption coefficient and the term $Q(T_\infty-T)$ is assumed to be the amount of heat generated / absorbed per unit volume. Q is a constant, which may take on either positive or negative values. When the wall temperature T_w exceeds the free stream temperature T_∞, the source term represents the heat source when $Q>0$ and heat sink when $Q<0$ whereas $T_w<T_\infty$, the opposite relationship is true.

Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞ and neglecting higher orders we get $T^4\cong 4T_\infty^3 T-3T_\infty^4$.

Therefore, the Eq. (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2}-\frac{16\sigma_1 T_\infty^3}{3\rho\, c_p k^*}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho\, c_p}(T_\infty-T) \quad (10)$$

We now introduce the following relations for u, v, θ *and* ϕ as

$$u = \frac{\partial\psi}{\partial y},\ \ v = -\frac{\partial\psi}{\partial x},\ \ \theta = \frac{T - T\infty}{T_w - T_\infty},\ \ and\ \ \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{11}$$

The streamwise velocity and the suction/injection velocity are taken as

$$U(x) = c\,x^m,\ V(x) = V_0\,x^{\frac{m-1}{2}} \tag{12}$$

Here $c > 0$ is constant, T_w is the wall temperature, the power-law exponent m is also constant. In this study we take $c = 1$.

The temperature-dependent fluid viscosity is given by (Batchelor [25]),

$$\mu = \mu^*[a + b(T_w - T)]$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, b are constants and $b(> 0)$. For a viscous fluid, Ling and Dybbs [26] suggested a viscosity dependence on temperature T of the form $\mu = \dfrac{\mu_\infty}{[1 + \gamma(T - T_\infty)]}$ where c is a thermal property of the fluid and μ_∞ is the viscosity away from the hot sheet. This relation does not differ at all with our formulation. The range of temperature, i.e. $(T_w - T_\infty)^0C$ studied here is $(0 - 23^0)C$.

Using the relations (11) into equations (6)-(8), we obtain,

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = -\zeta\ v^*\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2} + v^*[a + \zeta(1-\theta)]\frac{\partial^3\psi}{\partial y^3} + g\frac{\zeta}{b}(\beta\theta + \beta^*\phi) \tag{13}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = (\frac{\kappa}{\rho\ c_p} + \frac{16\sigma_1 T_\infty^3}{3\rho\ c_p k^*})\frac{\partial^2\theta}{\partial y^2} + \frac{Q}{\rho\ c_p}\theta \tag{14}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y} = D\frac{\partial^2\phi}{\partial y^2} - \tau(\frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y} + \frac{\partial^2\theta}{\partial y^2}\phi) \tag{15}$$

where $\zeta = b(T_w - T_\infty)$, $\quad v^* = \dfrac{\mu^*}{\rho}$, $\tau = -\dfrac{k(T_w - T_\infty)}{T_r}$

The boundary conditions becomes

$$\frac{\partial\psi}{\partial y} = x^m,\ \frac{\partial\psi}{\partial x} = V_0 x^{\frac{m-1}{2}},\ \theta = \phi = 1\ at\ y = 0;$$

$$\frac{\partial \psi}{\partial y} \to 0, \quad \theta \to 0, \quad \phi \to 0 \;\; as \;\; y \to \infty \tag{16}$$

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (Mukhopadhyay et al. [17]),

$$\Gamma: \; x^* = x\, e^{\varepsilon\, \alpha_1}, \; y^* = y\, e^{\varepsilon\, \alpha_2}, \; \psi^* = \psi\, e^{\varepsilon\, \alpha_3}, u^* = u\, e^{\varepsilon\, \alpha_4},$$
$$v^* = v\, e^{\varepsilon\, \alpha_5}, \; \theta^* = \theta\, e^{\varepsilon\, \alpha_6}, \phi^* = \phi\, e^{\varepsilon\, \alpha_7} \tag{17}$$

Equation (17) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, v, \theta, \phi)$ to the coordinates $(x^*, y^*, \psi^*, u^*, v^*, \theta^*, \phi^*)$.

Substituting (17) in (13), (14) and (15) we get,

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial y^{*2}} \right) =$$
$$-\zeta\, v^* e^{\varepsilon(3\alpha_2-\alpha_3-\alpha_6)} \left(\frac{\partial \theta^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial y^{*2}} \right) + v^*[a+\zeta] e^{\varepsilon(3\alpha_2-\alpha_3)} \frac{\partial^3 \psi^*}{\partial y^{*3}} \tag{18}$$
$$-\zeta\, v^* e^{\varepsilon(3\alpha_2-\alpha_3-\alpha_6)} \theta^* \frac{\partial^3 \psi^*}{\partial y^* 3} + g \frac{\zeta}{b} (\beta\, e^{-\varepsilon \alpha_6} \theta^* + \beta^* e^{-\varepsilon \alpha_7} \phi^*)$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \theta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \theta^*}{\partial y^*} \right) =$$
$$\left(\frac{\kappa}{\rho\, c_p} + \frac{16 \sigma_1 T_\infty^3}{3\rho\, c_p k^*} \right) e^{\varepsilon(2\alpha_2-\alpha_6)} \frac{\partial^2 \theta^*}{\partial y^* 2} + \frac{Q}{\rho\, c_p} e^{-\varepsilon \alpha_6} \theta^* \tag{19}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \phi^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \phi^*}{\partial y^*} \right) = D e^{\varepsilon(2\alpha_2-\alpha_7)} \frac{\partial^2 \phi^*}{\partial y^* 2}$$
$$-\tau\, e^{\varepsilon(2\alpha_2-\alpha_6-\alpha_7)} \frac{\partial \theta^*}{\partial y^*} \frac{\partial \phi^*}{\partial y^*} - \tau\, e^{\varepsilon(2\alpha_2-\alpha_6-\alpha_7)} \frac{\partial^2 \theta^*}{\partial y^* 2} \phi^* \tag{20}$$

The system will remain invariant under the group of transformations Γ, we would have the following relations among the parameters, namely

$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 - \alpha_6 = 3\alpha_2 - \alpha_3 =$
$\alpha_2 - \alpha_3 - \alpha_6 = -\alpha_6 = -\alpha_7$;
$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = \; 2\alpha_2 - \alpha_6 = \; -\alpha_6$ *and* $\alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 =$
$2\alpha_2 - \alpha_7 = 2\alpha_2 - \alpha_6 - \alpha_7$

These relations give $\alpha_6 = \alpha_7 = 0,\ \alpha_2 = \frac{1}{4}\alpha_1 = \ \frac{1}{3}\alpha_3$.

The boundary conditions yield $\alpha_4 = m\alpha_1 = d\frac{1}{2}\alpha_1, \alpha_5 = d\frac{m-1}{2}\alpha_1 = -\frac{1}{4}\alpha_1\ (as\ m = \frac{1}{2})$

In view of these, the boundary conditions become

$$\frac{\partial \psi^*}{\partial y^*} = x*^{\frac{1}{2}},\ \frac{\partial \psi^*}{\partial x} = V_0 x*^{(-\frac{1}{4})},\ \theta^* = \phi^* = 1\ at\ y^* = 0$$

$$and\ \frac{\partial \psi^*}{\partial y^*} \to 0, \theta^* \to 0, \phi^* \to 0\ as\ y^* \to \infty \tag{21}$$

The set of transformations Γ reduces to $x^* = x\ e^{\varepsilon\ \alpha_1},\ y^* = y\ e^{\varepsilon \frac{\alpha_1}{4}}\ ,\ \psi^* = \psi\ e^{\varepsilon \frac{3\alpha_1}{4}}\ , u^* = u\ e^{\varepsilon \frac{\alpha_1}{2}}\ , v^* = v\ e^{-\varepsilon \frac{\alpha_1}{4}}\ ,\ \theta^* = \theta, , \phi^* = \phi$.

Expanding by Taylor's method in powers of ε and keeping terms up to the order ε we get

$$x^* - x\ = x\varepsilon\ \alpha_1,\ \ y^* - y\ = y\varepsilon\frac{\alpha_1}{4},\ \ \psi^* - \psi\ = \psi\varepsilon\frac{3\alpha_1}{4}\ ,$$

$$u^* - u\ = u\varepsilon\frac{\alpha_1}{2},\ \ v^* - v\ = -v\varepsilon\frac{\alpha_1}{4},\ \ \theta^* - \theta = \phi^* - \phi = 0$$

The characteristic equations are

$$\frac{dx}{x\ \alpha_1} = \frac{dy}{y\frac{\alpha_1}{4}} = \frac{d\psi}{\psi\frac{3\alpha_1}{4}} = \frac{du}{u\frac{\alpha_1}{2}} = \frac{dv}{-v\frac{\alpha_1}{4}} = \frac{d\theta}{0} = \frac{d\phi}{0}$$

Solving the above equations we get,

$$y^* x^{*\frac{-1}{4}} = \eta\ \ \psi^* = x^{*\frac{3}{4}} F(\eta),\ \theta^* = \theta(\eta),\ \phi^* = \phi(\eta) \tag{22}$$

with the help of these relations, the (18), (19) and (20) become

$$2F'^2 - 3FF'' = -4\zeta v^* \vartheta' F'' + 4(a+\zeta)v^* F''' - 4\zeta v^* \theta F''' + 4g\frac{\zeta}{b}(\beta\theta + \beta^*\phi) \tag{23}$$

$$4\left(\frac{\kappa}{\rho\ c_p} + \frac{16\sigma_1 T_\infty^3}{3\rho\ c_p k^*}\right)\theta'' + 4\frac{Q}{\rho\ c_p}\theta + 3F\theta' = 0 \tag{24}$$

$$4D\phi'' + 3(F - \frac{4}{3}\tau\ \theta')\ \phi' - 4\tau\ \theta''\ \phi\ = 0 \quad (25)$$

The boundary conditions take the following form

$$F' = 1,\ \ F = \frac{4V_0}{3},\ \ \theta = \phi = 1\ \ at\ \ \eta = 0\ \ and\ \ F' \to 0,\ \ \theta \to 0,\ \ \phi \to 0\ \ as\ \ \eta \to \infty \quad (26)$$

Again, we introduce the following transformations for η, F, θ *and* ϕ in Eq. (23), (24) and (25):

$$\eta = (\frac{g\beta_1}{b})^{\alpha_1} v^{*b_1}\eta^*, F = (\frac{g\beta_1}{b})^{\alpha'_1} v^{*b'_1}F^*,\ \theta = (\frac{g\beta}{b})^{\alpha''_1} v^{*b''_1}\theta^*, \phi = (\frac{g\beta^*}{b})^{\alpha''_1} v^{*b''_1}\phi^*$$

$$were\ \beta_1 = \frac{\beta+\beta^*}{2} \quad (27)$$

Taking $F^* = f,\ \bar{\theta} = \theta\ and\ \bar{\phi} = \phi$ the Equations (23),(24) and (25) finally take the following form

$$4(a+\zeta)F''' - 4\zeta\ \theta\ F''' - 4\zeta\ \vartheta' F'' - 2F'^2\ + 3FF'' + 4\zeta(\theta+\phi) = 0 \quad (28)$$

$$\frac{4}{\Pr}\left(1 + \frac{4}{3N}\right)\theta'' + 4\delta\theta + 3F\theta' = 0 \quad (29)$$

$$\frac{4}{Sc}\phi'' + 3(F - \frac{4}{3}\tau\ \theta')\ \phi' - 4\tau\ \theta''\ \phi = 0 \quad (30)$$

where $\Pr = \frac{v^*\rho\ c_p}{\kappa} = \frac{\mu^* c_p}{\kappa}$ is the Prandtl number, $N = \frac{\kappa\ k^*}{4\sigma_1 T_\infty^3}$ is the Radiation parameter, $\tau = -\frac{k(T_w - T_\infty)}{T_r}$ is the thermophoresis parameter, $\delta = \frac{Q}{c\rho\ c_p}$ is the heat source ($\delta > 0$) and sink ($\delta < 0$) parameter and $Sc = \frac{v^*}{D}$ is the Schmidt number.

The boundary conditions take the following forms.

$$f' = 1,\ \ f = S,\ \ \theta = \phi = 1,\ \ at\ \ \eta^* = 0\ \ and\ \ f' \to 0,\ \ \theta \to 0,\ \ \phi \to 0\ \ as\ \ \eta^* \to \infty \quad (31)$$

where $S = \frac{4}{3}V_0(\frac{g\beta_1}{b})^{\frac{-1}{4}} v^{\frac{-1}{2}}$, $S > 0$ corresponds to suction and $S < 0$ corresponds to injection.

3 Numerical solution

The set of non-linear ordinary differential equations (28) to (30) with boundary conditions (31) have been solved by using the Runge-Kutta Gill scheme, (Gill [27]) along with shooting technique with ζ, τ, δ *and* N as prescribed parameters. The numerical solution was done using Matlab computational software. A step size of $\Delta\eta$= 0.001 was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of η_∞ was found to each iteration loop by assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters ζ, τ, δ *and* N, determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7}. Effects of heat and mass transfer are studied for different values of thermophoresis particle deposition, strength of thermal radiation and the temperature-dependent fluid viscosity in the presence of suction / injection. In the following section, the results are discussed in detail.

4 Results and Discussion

To analyze the results, numerical computation has been carried out using the method described in the previous section for various values of the temperature-dependent fluid viscosity parameter ζ, suction / injection parameter S, Prandtl number Pr, thermophoresis parameter τ, heat source and sink parameter δ, Schmidt number Sc and thermal radiation parameter N. For illustrations of the results, numerical values are plotted in the Figs. 2-7. In all cases we take $a = 1.0$.

In the absence of diffusion equations, in order to validate our method, we have compared steady state results of skin friction $f''(0)$ and rate of heat transfer $\theta'(0)$ for various values of N (Table.1) with those of Anwar Hossain et al. [28] and found them in excellent agreement.

Table 1: Comparison with previous published work

	Anwar Hossain et al.[28]		*Present work*		*Remark*
N	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$\xi = 0.8$ *and* $\mathrm{Pr} = 1.0$
0.0	0.5050	0.5469	0.504874	0.546651	
0.5	0.5994	0.3107	0.599663	0.310587	
1.0	0.6423	0.2466	0.642271	0.246579	
2.0	0.6886	0.1900	0.688591	0.189882	
3.0	0.7150	0.1615	0.714878	0.161489	

In the absence of diffusion equations, in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The temperature profiles for Prandtl number Pr are compared with the available exact solution of Mukhopadhyay and Layek [18] is shown in Fig.2a & 2b. It is observed that the agreements with the theoretical solution of temperature profiles are excellent. For a given N, it is clear that there is a fall in temperature with increasing the Prandtl number. This is due to the fact that there would be a decrease of thermal boundary layer thickness with the increase of Prandtl number as one can see from Fig. 2a & 2b by comparing the curves with $\mathrm{Pr} = 0.3$ and $\mathrm{Pr} = 1.0$. This behavior implies that fluids having a smaller Prandtl number are much more responsive to thermal radiation than fluids having a larger Prandtl number.

The effect of thermophoretic parameter τ on concentration field is shown in Fig.3. In the presence of uniform temperature-dependent fluid viscosity, it is observed that the concentration of the fluid decrease whereas the velocity and temperature of the fluid are not significant with increase of thermophoretic parameter. In particular, the effect of increasing the thermophoretic parameter τ is limited to increasing slightly the wall slope of the concentration profiles but decreasing the concentration. This is true only for small values of Schmidt number for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of Schmidt number, the diffusion effect is minimal compared to the convection effect and, therefore, the thermophoretic parameter τ is

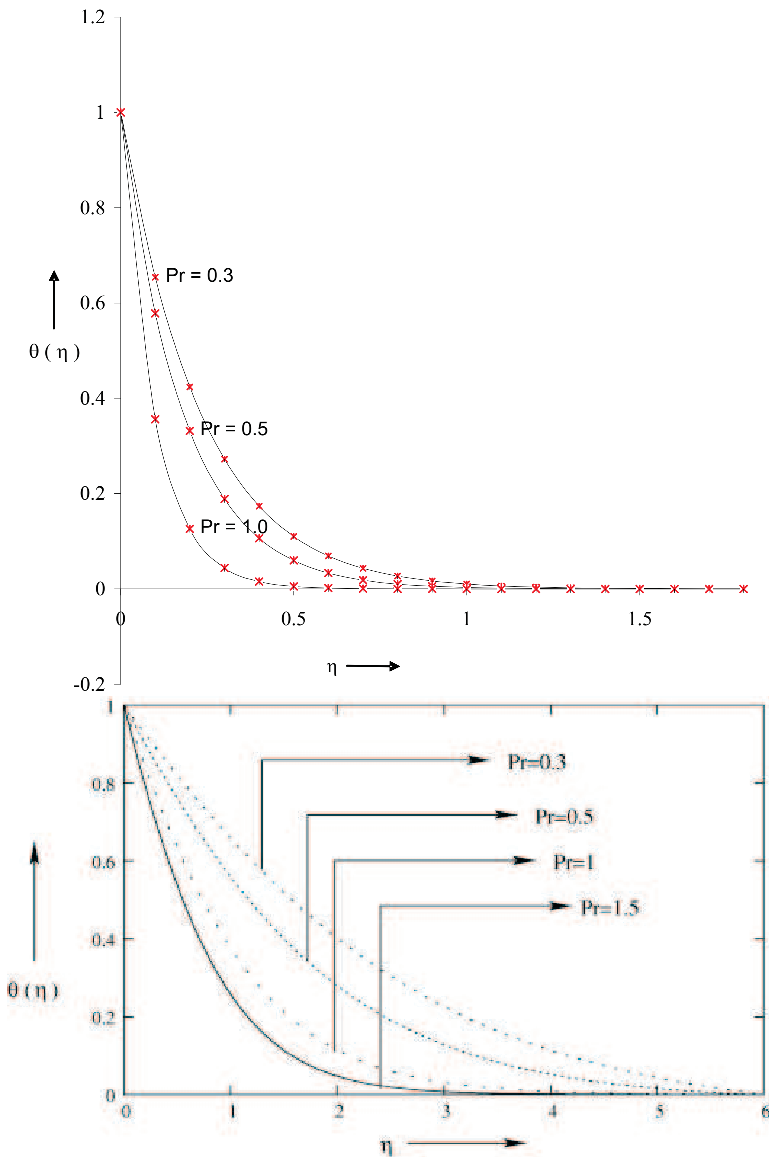

Figure 2: a&b: Comparison of the temperature profiles(present result) with Mukhopadhyay and Layek [18]; Symbol: Result for Mukhopadhyay and Layek Solid line: Current result

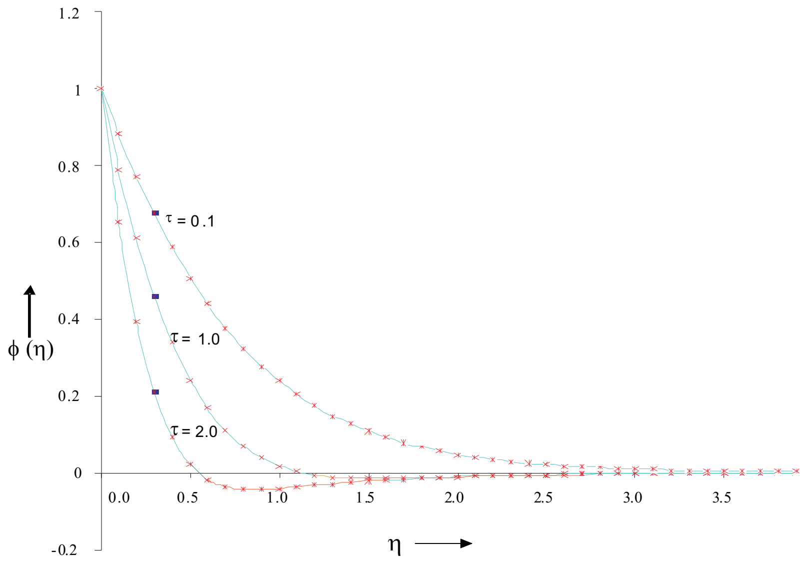

Figure 3: $Sc = 0.67, N = 0.1,\ a = 1.0,\ S = 0.5,\ \Pr = 0.71, \delta = 0.5, \xi = 0.5$

expected to alter the concentration boundary layer significantly. In particular, the concentration of the fluid gradually changes from higher value to the lower value only when the strength of the thermophoresis particle deposition is higher than the temperature-dependent fluid viscosity strength. For large particle deposition mechanism, interesting result is the large distortion of the concentration field caused for $\tau \geq 1.0$. Negative value of the concentration profile is seen in the outer boundary region for $\tau = 1.0$ *and* $\tau = 2.0;\ \xi = 0.5$. All these physical behavior are due to the combined effects of the strength of thermophoresis particle deposition and the temperature-dependent fluid viscosity at wall surface.

The effects of the thermal radiation parameter N on the velocity and temperature profiles in the boundary layer are illustrated in Figs. 4 and 5, respectively. In the presence of uniform thermophoresis particle deposition, it is observed that the velocity and temperature of the fluid decrease whereas the concentration of the fluid is not significant with increase of thermal radiation parameter N. This result can be explained by the fact that a decrease in the values of $N(\dfrac{\kappa\, k^*}{4\sigma_1 T_\infty^3})$ for given $k*$ *and* T_∞ means a

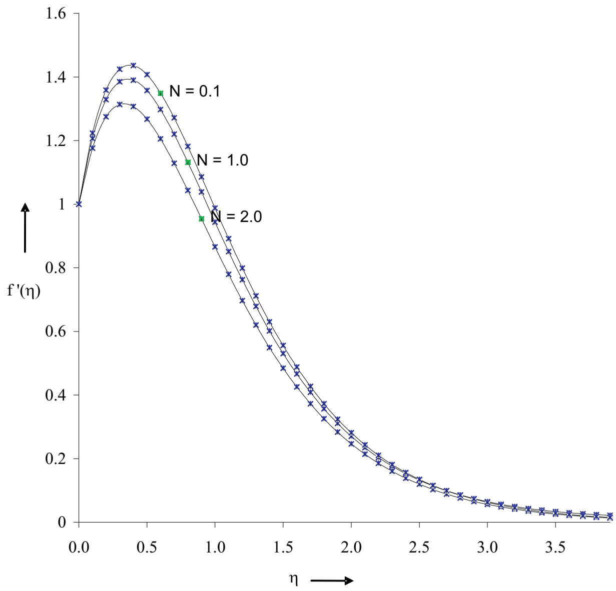

Figure 4: $Sc = 0.67,\ a = 1.0,\ S = 0.5,\ \Pr = 0.71, \delta = 0.5, \xi = 0.5, \tau = 1.0$

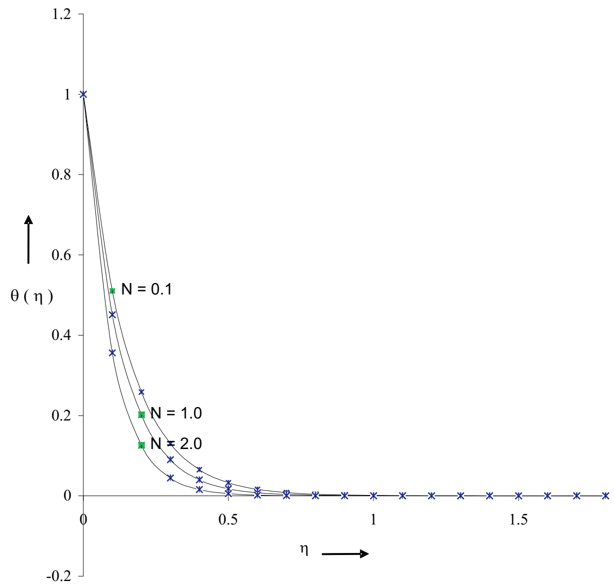

Figure 5: $Sc = 0.67,\ a = 1.0,\ S = 0.5,\ \Pr = 0.71, \delta = 0.5, \xi = 0.5, \tau = 1.0$

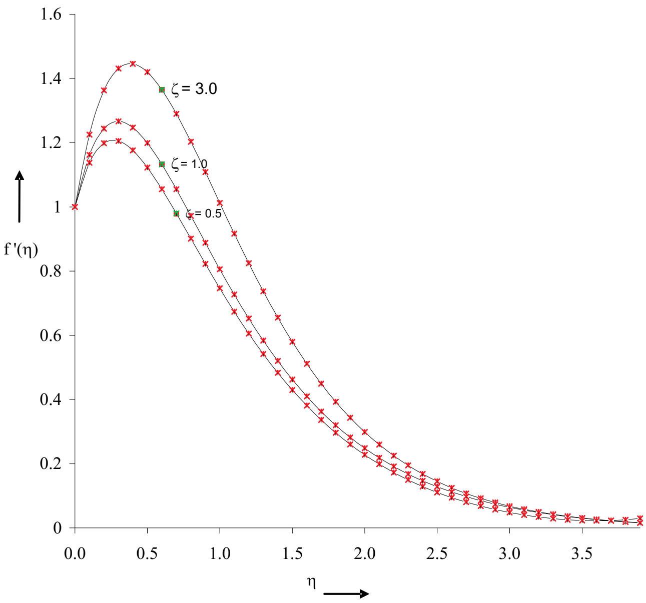

Figure 6: $\delta = 0.5,\ a = 1.0,\ Sc = 0.67,\ \Pr = 0.71, N = 0.1, \tau = 1.0$

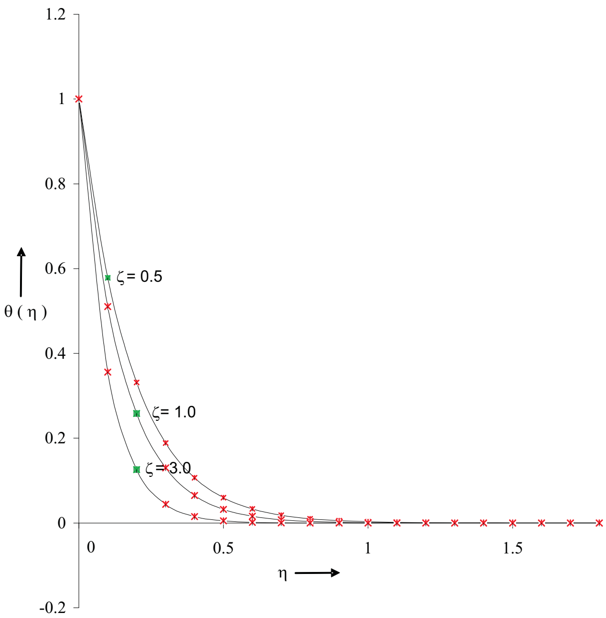

Figure 7: $S = \delta = 0.5,\ a = 1.0,\ Sc = 0.67,\ \Pr = 0.71, N = 0.1, \tau = 1.0$

decrease in the Rosseland radiation absorptive κ. According to equations (2) and (3), the divergence of the radiative heat flux $\frac{\partial q_r}{\partial y}$ increases as κ decreases which in turn increase the rate of radiative heat transferred to the fluid and hence the fluid motion and temperature decrease. In view of this explanation, the effect of radiation becomes more significant as $N \to 0 (N \neq 0)$ and can be neglected when $N \to \infty$. The effect of radiation parameter N is to reduce the velocity and temperature of the fluid significantly in the flow region. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid motion and temperature decrease as the momentum and thermal boundary layer thickness become thinner.

Fig.6 exhibits the velocity profiles for several values of temperature-dependent fluid viscosity ζ. In the presence of uniform suction, the velocity of the fluid is found to increase with the increase of the temperature-dependent fluid viscosity parameter ζ at a particular value of η except very near the wall as well as far away of the wall (at $\eta = 5$). This means that the velocity decreases (with the increasing value of η) at a slower rate with the increase of the parameter ζ at very near the wall as well as far away of the wall. This can be explained physically as the parameter ζ increases, the fluid viscosity decreases the increment of the boundary layer thickness.

In Fig. 7, variations of temperature field $\theta(\eta)$ with η for several values of ζ (with Pr = 0.71 and N = 0.1) in the presence of suction (S = 0.5) are shown. It is very clear from the figure that the temperature decreases whereas the concentration of the fluid is not significant with increase of ζ. The increase of temperature-dependent fluid viscosity parameter ζ makes decrease of thermal boundary layer thickness, which results in decrease of temperature profile $\theta(\eta)$. Decrease in $\theta(\eta)$ means a decrease in the velocity of the fluid particles. So in this case the fluid particles undergo two opposite forces: one increases the fluid velocity due to decrease in the fluid viscosity (with increasing ζ) and other decreases the fluid velocity due to decrease in temperature $\theta(\eta)$(since θ decreases with increasing ζ). Near the surface, as the temperature θ is high so the first force dominates and far away from the surface θ is low and so the second force dominates here.

5 Conclusions

By using Lie group analysis, we first find the symmetries of the partial differential equations and then reduce the equations to ordinary differential equations by using scaling and translational symmetries. Exact solutions for translation symmetry and numerical solution for scaling symmetry are obtained. From the numerical results, it is predict that the effect of increasing temperature-dependent fluid viscosity with uniform thermal radiation on a viscous incompressible fluid is to increase the flow velocity which in turn, causes the temperature to decrease. It is interesting to note that the temperature of the fluid decreases at a very fast rate in the case of water in comparison with air. So, the impact of the temperature-dependent fluid viscosity with thermophoresis particle deposition have a substantial effect on the flow field and, thus, on the heat and mass transfer rate from the sheet to the fluid. It is found that the concentration boundary layer is significantly suppressed by the thermophoretical force, and the effect of thermophoresis plays a dominant role than that of diffusion thus almost uniform deposition efficiency is achieved for clusters of different sizes. Particularly, the effect of thermophoresis particle deposition with temperature-dependent fluid viscosity plays an important role on the rapid growth of World's economy has led to severe air pollution characterized by acid rain, severe pollution in cities, and regional air pollution. The results of the problem are also of great interest in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls.

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Analiza pomoću Liove grupe uticaja viskoznosti i termoforeze na prenos toplote i mase u prisustvu usisavanja/ubrizgavanja

Proučavaju se karakteristike prenosa toplote i mase nestišljivog Njutnovskog fluida koji ima viskoznost zavisnu od temperature i taloženje čestica termoforezom preko vertikalne zategnute površi sa promenljivim uslovom struje. Roslendova aproksimacija se koristi za dobijanje toplotnog fluksa radijacije u energijskoj jednačini. Za vertikalnu površ se pretpostavlja da je propustljiva dozvoljavajući usisavanje ili ubrizgavanje. Diferencijalne jednačine problema su izvedene i transformisane analizom Liove grupe. Transformisane jednačine su rešene primenom Runge-Kuta-Gilove šeme sa snimajućom tehnikom. Uporedjenja sa objavljenim radovima su povoljna. Numerički rezultati za brzinu, temperaturu i koncentracione profile za propisane zadate parametre viskoznosti i taloženja su prikazani grafički da bi objasnili uticaj raznih fizičkih parametara.

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