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On the optimal shape of a compressed column

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Abstract

A new numerical solution to the Lagrange problem is presented. The solution is compared with a priori estimates obtained recently. Also we derive a new relation that shows that the cross-sectional area at the middle of the optimally shaped column is *larger* than the cross-sectional area at the ends. Our numerical experiments confirm that conclusion.

Keywords: Lagrange problem, Pontryagin's principle, Estimates of cross-sectional area.

1 Introduction

The problem of determining the shape of a rod of a given volume that is the strongest against buckling is an important engineering problem. It was formulated by Lagrange (1773) and is now known as the *Lagrange problem*. Correct solution of the problem, with the simply supported boundary conditions, leading to the so called optimally shaped column, was obtained by Clausen in1851. For the historical account of the Lagrange problem see, for example, the articles by [1, 2, 3]. For different approach to Lagrange problem see also [8].

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Our intention in this work is to use Pontryagin's principle, as is done in [4]. For derivation of bimodal optimality conditions of elastic column in which moment of inertia I is proportional to the cross–sectional area A, i.e., $I = kA^{\alpha}$, where k is a constant and $\alpha = 1, 2, 3$. We shall determine the optimal shape and corresponding buckling load. Our numerical results will be compared with recently obtained a priori estimates [5]. Also we shall prove a new property of the optimal solution $A(0) = A(L) < A(L/2)$, that is the cross-sectional area of the optimal rod is larger at the middle point of the rod than the cross-sectional area at the ends. Our numerical scheme will be different than the one used in [6].

2 Formulation of the problem

Consider an elastic rod of length L loaded by an axial force F with the action line coinciding with the x axis of a rectangular coordinate system $x - B - y$ (see Fig. 1.)

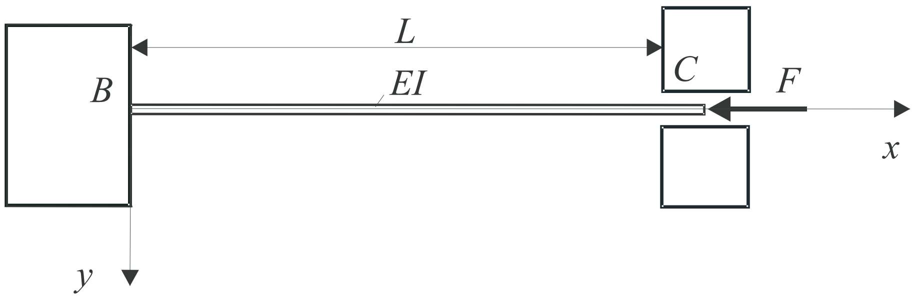


Figure 1: Coordinate system and load configuration

We use the following notation: H and V are components of the contact force (i.e. the resultant force in an arbitrary cross–section) along x and y axes, respectively, M is the bending moment, θ is the angle between the tangent to the column axis and the x axis, S is the arc–length of the column axis measured from the origin of the coordinate system B.

Equilibrium equations for the column are, Atanackovic (1997)

$$\frac{dH}{dS} = 0, \quad \frac{dV}{dS} = 0, \quad \frac{dM}{dS} = -V\cos\theta + H\sin\theta, \tag{1}$$

We adjoin to (10) the geometrical,

$$\frac{d\bar{x}}{dS} = \cos\theta, \quad \frac{d\bar{y}}{dS} = \sin\theta, \tag{2}$$

and the constitutive equation

$$M = EI\frac{d\theta}{dS}. \tag{3}$$

In (2), (3) we used $\bar{x}$ and $\bar{y}$ to denote coordinates of an arbitrary point on the rod axis in the coordinate system $x - B - y$, E is modulus of elasticity and I is the moment of inertia of the cross–section. Equations (2), (3) correspond to the classical Bernoulli–Euler rod theory. The boundary conditions for the column shown in Fig. 1. are

$$\bar{y}(0) = \bar{y}(L) = 0, \quad \theta(0) = \theta(L) = 0, \quad H(L) = -F. \tag{4}$$

Solving $(1)_{1,2}$ and by using $(4)_3$ we obtain

$$H = -F. \tag{5}$$

Also we assume that the axial moment of inertia I and the cross-sectional area A are connected as $I = kA^{\alpha}$,where k is a constant and $\alpha = 1, 2, 3$. For example, if $\alpha = 2$ than, for a circular cross–section $k = 1/4\pi$. By introducing the dimensionless quantities

$$\begin{aligned} t &= \frac{S}{L}, \quad a = \frac{A}{L^2}, \quad \zeta = \frac{\bar{x}}{L}, \quad \eta = \frac{\bar{y}}{L}, \\ w &= \frac{W}{L^3}, \quad \lambda = \frac{F}{kEL^2}, \quad \nu = \frac{V}{kEL^2}, \quad m = \frac{M}{kEL^3}, \end{aligned} \tag{6}$$

we obtain from (1)–(5)

$$\begin{aligned} \dot{v} &= 0, \quad \dot{m} = -v\cos\theta - \lambda\sin\theta, \\ \dot{\zeta} &= \cos\theta, \quad \dot{\eta} = \sin\theta, \quad \dot{\theta} = \frac{m}{a^{\alpha}}, \end{aligned} \tag{7}$$

subject to boundary conditions

$$\zeta(0) = 0, \quad \eta(0) = 0, \quad \eta(1) = 0, \quad \theta(0) = 0, \quad \theta(1) = 0, \tag{8}$$

where $\dot{(\cdot)} = \frac{d}{dt}(\cdot)$. The system (7), (8) has a trivial solution

$$\theta_0 = \eta_0 = v_0 = 0, \quad \zeta_0 = t, \tag{9}$$

for all values of load parameter λ_1 and stiffness parameter λ_2.To determine $(\lambda_1, \lambda_2) \in \mathbb{R}^2$ for which there is a nontrivial solution to (7), (8) we write $v = v_0 + \Delta v, ..., \theta = \theta_0 + \Delta\theta$ where $\Delta v, ..., \Delta\theta$ are perturbations. By substituting this into (7), (8) and by neglecting the higher order terms in perturbations, we obtain (after omitting Δ in front of Δv etc.)

$$\dot{v} = 0, \quad \dot{m} = -v - \lambda\theta, \quad \dot{\eta} = \theta, \quad \dot{\theta} = \frac{m}{a^{\alpha}}, \tag{10}$$

subject to

$$\eta(0) = 0, \quad \eta(1) = 0, \quad \theta(0) = 0, \quad \theta(1) = 0. \tag{11}$$

The volume of the rod is given as

$$w = \int_0^1 a(t)\,dt. \tag{12}$$

In order to apply Pontryagin's maximum principle, we introduce new dependent variables as

$$x_1 = \eta, \quad x_2 = \theta, \quad x_3 = v, \quad x_4 = m. \tag{13}$$

Then, the system (10), (11) becomes

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{x_4}{a^{\alpha}}, \quad \dot{x}_3 = 0, \quad \dot{x}_4 = -x_3 - \lambda x_2, \tag{14}$$

and

$$x_1(0) = 0, \quad x_1(1) = 0, \quad x_2(0) = 0, \quad x_2(1) = 0. \tag{15}$$

In terms of the optimal control, the problem now becomes: Given λ find the control $a^*(t) \in \mathbf{U}$ such that

$$\min_{a\in\mathbf{U}} I = \min_{a\in\mathbf{U}} \int_0^1 a(t)\,dt = \int_0^1 a^*(t)\,dt. \tag{16}$$

under the state equations (14), (15).

It was shown in [6] that for given λ and $a\,(t)$ the system (14),(15) can have at most two linearly independent solutions $(\overline{x}_1, ...\overline{x}_4)$ and $(\widehat{x}_1, ...\widehat{x}_4)$ corresponding to two buckling modes. Since both solutions correspond to the same λ and $a(t) = a^*(t)$ we have

$$\begin{gathered}\dot{\overline{x}}_1 = \overline{x}_2, \quad \dot{\overline{x}}_2 = \frac{\overline{x}_4}{a^\alpha}, \quad \dot{\overline{x}}_3 = 0, \quad \dot{\overline{x}}_4 = -\overline{x}_3 - \lambda \overline{x}_2 \\ \dot{\widehat{x}}_1 = \widehat{x}_2, \quad \dot{\widehat{x}}_2 = \frac{\widehat{x}_4}{a^\alpha}, \quad \dot{\widehat{x}}_3 = 0, \quad \dot{\widehat{x}}_4 = -\widehat{x}_3 - \lambda \widehat{x}_2 \end{gathered} \tag{17}$$

satisfying

$$\begin{gathered}\overline{x}_1\,(0) = 0, \quad \overline{x}_1\,(1) = 0, \quad \overline{x}_2\,(0) = 0, \quad \overline{x}_2\,(1) = 0, \\ \widehat{x}_1\,(0) = 0, \quad \widehat{x}_1\,(1) = 0, \quad \widehat{x}_2\,(0) = 0, \quad \widehat{x}_2\,(1) = 0. \end{gathered} \tag{18}$$

The Pontryagin's function $\mathcal{H}$, taking into account that differential constraints are given by (17) reads

$$\begin{aligned}\mathcal{H} &= a + \overline{p}_1 \overline{x}_2 + \overline{p}_2 \frac{\overline{x}_4}{a^\alpha} + \overline{p}_4\left(-\overline{x}_3 - \lambda \overline{x}_2\right) \\ &\quad + \widehat{p}_1 \widehat{x}_2 + \widehat{p}_2 \frac{\widehat{x}_4}{a^\alpha} + \widehat{p}_4\left(-\widehat{x}_3 - \lambda \widehat{x}_2\right), \end{aligned} \tag{19}$$

where $\overline{p}_i, \widehat{p}_i, i = 1, ..., 4$ are so-called "co–state" p-variables corresponding to state x-variables. The "co-state" variables $\overline{p}_i, \widehat{p}_i, i = 1, ..., 4$ satisfy

$$\begin{aligned}\dot{\overline{p}}_1 &= -\frac{\partial \mathcal{H}}{\partial \overline{x}_1} = 0, & \overline{p}_2 &= -\frac{\partial \mathcal{H}}{\partial \overline{x}_2} = -\overline{p}_1 + \lambda \overline{p}_4, \\ \dot{\overline{p}}_3 &= -\frac{\partial \mathcal{H}}{\partial \overline{x}_3} = \overline{p}_4, & \dot{\overline{p}}_4 &= -\frac{\partial \mathcal{H}}{\partial \overline{x}_4} = -\frac{\overline{p}_2}{a^\alpha}, \\ \dot{\widehat{p}}_1 &= -\frac{\partial \mathcal{H}}{\partial \widehat{x}_1} = 0, & \widehat{p}_2 &= -\frac{\partial \mathcal{H}}{\partial \widehat{x}_2} = -\widehat{p}_1 + \lambda \widehat{p}_4, \\ \dot{\widehat{p}}_3 &= -\frac{\partial \mathcal{H}}{\partial \widehat{x}_3} = \widehat{p}_4, & \dot{\widehat{p}}_4 &= -\frac{\partial \mathcal{H}}{\partial \widehat{x}_4} = -\frac{\widehat{p}_2}{a^\alpha}, \end{aligned} \tag{20}$$

subject to

$$\begin{aligned}\overline{p}_3\,(0) &= 0, \quad \overline{p}_3\,(1) = 0, \quad \overline{p}_4\,(0) = 0, \quad \overline{p}_4\,(1) = 0, \\ \widehat{p}_3\,(0) &= 0, \quad \widehat{p}_3\,(1) = 0, \quad \widehat{p}_4\,(0) = 0, \quad \widehat{p}_4\,(1) = 0. \end{aligned} \tag{21}$$

The optimality condition $\min\limits_{a\in\mathbf{U}}\mathcal{H}$ leads to

$$\frac{\partial\mathcal{H}}{\partial a}=1-\alpha\overline{p}_2\frac{\overline{x}_4}{a^{\alpha+1}}-\alpha\widehat{p}_2\frac{\widehat{x}_4}{a^{\alpha+1}}=0. \tag{22}$$

By solving (22) for a we obtain

$$a=a^{*}=\left[\alpha\left(\overline{p}_2\overline{x}_4+\widehat{p}_2\widehat{x}_4\right)\right]^{1/(\alpha+1)}. \tag{23}$$

We now identify the state $\overline{x}_i,\widehat{x}_i, i=1,...4$ and co–state $\overline{p}_i,\widehat{p}_i, i=1,...4$ variables. It could be easily seen that $\overline{p}_1=\overline{x}_3$, $\overline{p}_2=\overline{x}_4$, $\overline{p}_3=-\overline{x}_1$, $\overline{p}_4=-\overline{x}_2$, $\widehat{p}_1=\widehat{x}_3$, $\widehat{p}_2=\widehat{x}_4$, $\widehat{p}_3=-\widehat{x}_1$, $\widehat{p}_4=-\widehat{x}_2$. With these values, we obtain, in original variables

$$a\left(t\right)=\left[\alpha\left(\overline{m}^2+\widehat{m}^2\right)\right]^{1/(\alpha+1)}. \tag{24}$$

In original variables the bimodal solutions are

$$\begin{aligned}
&\dot{\overline{\eta}}=\overline{\theta}, \quad \dot{\overline{\theta}}=\frac{\overline{m}}{a^{\alpha}}, \quad \dot{\overline{v}}=0, \quad \dot{\overline{m}}=-\overline{v}-\lambda\overline{\theta}\\
&\dot{\widehat{\eta}}=\widehat{\theta}, \quad \dot{\widehat{\theta}}=\frac{\widehat{m}}{a^{\alpha}}, \quad \dot{\widehat{v}}=0, \quad \dot{\widehat{m}}=-\widehat{v}-\lambda\widehat{\theta},
\end{aligned} \tag{25}$$

subject to

$$\begin{aligned}
&\overline{\eta}_1\left(0\right)=0, \quad \overline{\eta}_1\left(1\right)=0, \quad \overline{\theta}_2\left(0\right)=0, \quad \overline{\theta}_2\left(1\right)=0,\\
&\widehat{\eta}_1\left(0\right)=0, \quad \widehat{\eta}_1\left(1\right)=0, \quad \widehat{\theta}_2\left(0\right)=0, \quad \widehat{\theta}_2\left(1\right)=0.
\end{aligned} \tag{26}$$

We showed in [6] that $\mathcal{H}$ given by (19) is a first integral. Therefore

$$\begin{aligned}
\mathcal{H}=\;&a\left(t\right)+\overline{\theta}\overline{v}+\frac{\left(\overline{m}\right)^2}{\left[\alpha\left(\left(\overline{m}\right)^2+\left(\widehat{m}\right)^2\right)\right]^{\alpha/(\alpha+1)}}\\
&+\overline{\theta}\left(\overline{v}+\lambda\overline{\theta}\right)+\widehat{\theta}\widehat{v}+\frac{\left(\widehat{m}\right)^2}{\left[\alpha\left(\left(\overline{m}\right)^2+\left(\widehat{m}\right)^2\right)\right]^{\alpha/(\alpha+1)}}\\
&+\widehat{\theta}\left(\widehat{v}+\lambda\widehat{\theta}\right)=const.
\end{aligned} \tag{27}$$

From [3] p. 69 we cite the other two first integrals of the system (here in the generalized version with arbitrary α)

$$
\begin{aligned}
\mathcal{K} &= \lambda \frac{\alpha+1}{\alpha} a(t) + \left(\overline{v} + \lambda \overline{\theta}\right)^2 + \left(\widehat{v} + \lambda \widehat{\theta}\right)^2 = const., \\
\mathcal{D} &= -\left(\overline{v} + \lambda \overline{\theta}\right) \widehat{m} + \left(\widehat{v} + \lambda \widehat{\theta}\right) \overline{m} = const.
\end{aligned}
\tag{28}
$$

Note that

$$
\begin{aligned}
&\frac{(\overline{m})^2}{\left[\alpha\left((\overline{m})^2 + (\widehat{m})^2\right)\right]^{\alpha/(\alpha+1)}} + \frac{(\widehat{m})^2}{\left[\alpha\left((\overline{m})^2 + (\widehat{m})^2\right)\right]^{\alpha/(\alpha+1)}} \\
&\qquad = \frac{(\overline{m})^2 + (\widehat{m})^2}{\left[\alpha\left((\overline{m})^2 + (\widehat{m})^2\right)\right]^{\alpha/(\alpha+1)}} = \frac{a}{\alpha},
\end{aligned}
$$

where we used (24). Also $\dot{\overline{m}} = -\left(\overline{v} + \lambda \overline{\theta}\right), \dot{\widehat{m}} = -\left(\widehat{v} + \lambda \widehat{\theta}\right)$, so that

$$
\mathcal{H} = \frac{\alpha+1}{\alpha} a(t) + \overline{\theta}\overline{v} + \widehat{\theta}\widehat{v} - \overline{\theta}\dot{\overline{m}} - \widehat{\theta}\dot{\widehat{m}}. \tag{29}
$$

Since

$$
\begin{aligned}
\overline{\theta}\dot{\overline{m}} &= \left(\overline{\theta}\overline{m}\right)^{\cdot} - \dot{\overline{\theta}}\overline{m} = \left(\overline{\theta}\overline{m}\right)^{\cdot} - \frac{\overline{m}}{a^\alpha}\overline{m}, \\
\widehat{\theta}\dot{\widehat{m}} &= \left(\widehat{\theta}\widehat{m}\right)^{\cdot} - \dot{\widehat{\theta}}\widehat{m} = \left(\widehat{\theta}\widehat{m}\right)^{\cdot} - \frac{\widehat{m}}{a^\alpha}
\end{aligned}
$$

we obtain after the integration of (29) and use of the boundary conditions

$$
\begin{aligned}
\mathcal{H} &= \frac{1+\alpha}{\alpha} w + \int_0^1 \frac{(\overline{m})^2 + (\widehat{m})^2}{a^\alpha} \\
&= \frac{1+\alpha}{\alpha} w + \int_0^1 \frac{a(t)}{\alpha} dt = \frac{2+\alpha}{\alpha} w.
\end{aligned}
\tag{30}
$$

Also by substituting $t = 0$ (or $t = 1$) and using the boundary conditions $\overline{\theta}(0) = 0, \widehat{\theta}(0) = 0$ in (29) we obtain

$$
\mathcal{H} = \frac{\alpha+1}{\alpha} a(0) = \frac{\alpha+1}{\alpha} a(1). \tag{31}
$$

Finally by combining (30) and (31) we obtain $a(0) = a(1) = \frac{2+\alpha}{1+\alpha}w$. Thus, for $w = 1$ we obtain:$\alpha = 1, a(0) = \frac{3}{2}, \alpha = 2, a(0) = \frac{4}{3}, \alpha = 3, a(0) = \frac{5}{4}$. This estimates are presented in [5]. We call $\left(\overline{\eta}, \overline{\theta}, \overline{v}, \overline{m}\right)$ and $\left(\widehat{\eta}, \widehat{\theta}, \widehat{v}, \widehat{m}\right)$ the first and second mode solution, respectively. Since, $\overline{\eta}(t) = \overline{\eta}(1-t)$ we have $\overline{\theta}(0.5) = 0$. This relation with global equilibrium equation for the rod leads to $\overline{v} = 0$. Let $a_0 = a(0), \overline{m}_0 = \overline{m}(0), \widehat{m}_0 = \widehat{m}(0), a_s = a(0.5), \widehat{\theta}_s = \widehat{\theta}(0.5), \overline{m}_s = \overline{m}(0.5), \widehat{m}_s = \widehat{m}(0.5)$. Then, from $(28)_1$, evaluated at $t = 0$ and $t = 0.5$ we have

$$\mathcal{K} = \lambda \frac{\alpha+1}{\alpha} a_0 + \widehat{v}^2 = \lambda \frac{\alpha+1}{\alpha} a_s + \left(\widehat{v} + \lambda \widehat{\theta}_s\right)^2. \tag{32}$$

Evaluating $\mathcal{D}$ given by $(28)_2$ at $t = 0$ and $t = 0.5$ we get $\mathcal{D} = \widehat{v}\overline{m}_0 = \left(\widehat{v} + \lambda\widehat{\theta}_s\right)\overline{m}_s$. Note also that $a(t) = a(1-t)$ (see [7], p. 108) so that $\dot{a}(0.5) = 0$. This together with $\dot{\mathcal{K}} = 0$ leads to $\widehat{m}_s = 0$. Then from (24) we have $\overline{m}_s = \sqrt{\frac{a_s^{\alpha+1}}{\alpha}}$. Therefore

$$\mathcal{D} = \widehat{v}\overline{m}_0 = \left(\widehat{v} + \lambda\widehat{\theta}_s\right)\sqrt{\frac{a_s^{\alpha+1}}{\alpha}} \tag{33}$$

Also, from (24) evaluated at $t = 0$ we get $a_0^{\alpha+1} = \alpha\left[\overline{m}_0^2 + \widehat{m}_0^2\right]$. Since $\widehat{m}(0) = -\widehat{m}(1)$ the global equilibrium conditions imply $\widehat{m}_0 = \widehat{v}/2$. Therefore $a_0^{\alpha+1} = \alpha\left[\overline{m}_0^2 + \frac{\widehat{v}^2}{4}\right]$, so that $\overline{m}_0 = \sqrt{\frac{a_0^{\alpha+1}}{\alpha} - \frac{\widehat{v}^2}{4}}$. Using this in (33) we get

$$\left(\widehat{v} + \lambda\widehat{\theta}_s\right)^2 = \frac{\widehat{v}^2\left[a_0^{\alpha+1} - \alpha\frac{\widehat{v}^2}{4}\right]}{a_s^{\alpha+1}} \tag{34}$$

By substituting (34) into (33) we finally obtain

$$\frac{\alpha}{4}\frac{\widehat{v}^4}{a_s^{\alpha+1}} + \widehat{v}^2\left[1 - \frac{a_0^{\alpha+1}}{a_s^{\alpha+1}}\right] + \lambda\frac{\alpha+1}{\alpha}\left[a_0 - a_s\right] = 0. \tag{35}$$

Note that $\widehat{v} \neq 0$ (otherwise, $\mathcal{D} = 0$ and $\overline{m}(t)$ and $\widehat{m}(t)$ are linearly dependent, see [3] p. 69). Then, from (35) we have $a_0 \neq a_s$! Our numerical results show that $a_0 < a_s$. Also (35) leads to a *unique* real solution $\widehat{v}$ only if $a_0 < a_s$.

3 Numerical results

We solved (25-26) numerically for $\alpha = 1, 2, 3$ and with the condition $\overline{v} = 0$ and $w = 1$. The results are shown in the next Table

Table 1

α	λ	a_0	a_s
1	47.993050138	1.4999999587	1.500003492
2	52.356254741	1.3333328969	1.3339386287
3	54.825435481	1.2499977373	1.251667967

In Fig. 2 we show buckling modes and cross-sectional area for $\alpha = 3$.

Our values for λ agree with the values obtained earlier. For example in [7] for $\alpha = 2$ the value $\lambda = 52.3564$ and for $\alpha = 3$ the value $\lambda = 54.8248$ was obtained. In [8], the value $\lambda = 52.3562$ and $\lambda = 52.356254$ for $\alpha = 2$ is obtained. Both values are obtained with different method. Also in [6] the values $\lambda = 47.99305032, \lambda = 52.3562542669$ and $\lambda = 54.82543305$ for $\alpha = 1, 2, 3$ is obtained, respectively.

As is seen from Table, in all cases $a_0 < a_s$. By substituting values from the Table in (35) we obtain

$$R\left(1\right) = -3.5942483042 \times 10^{-6},$$
$$R\left(2\right) = -3.4275646673 \times 10^{-5},$$
$$R\left(3\right) = -1.7020189244 \times 10^{-4}$$

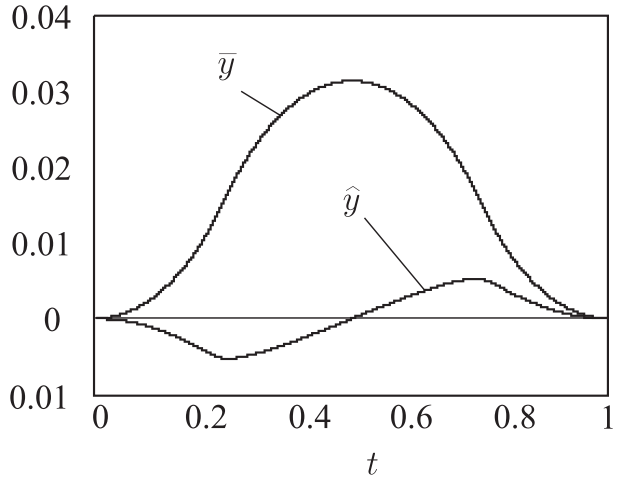
with

$$R\left(\alpha\right) = \frac{\alpha}{4}\frac{\widehat{v}^4}{a_s^{\alpha+1}} + \widehat{v}^2\left[1 - \frac{a_0^{\alpha+1}}{a_s^{\alpha+1}}\right] + \lambda\frac{\alpha+1}{\alpha}\left[a_0 - a_s\right].$$

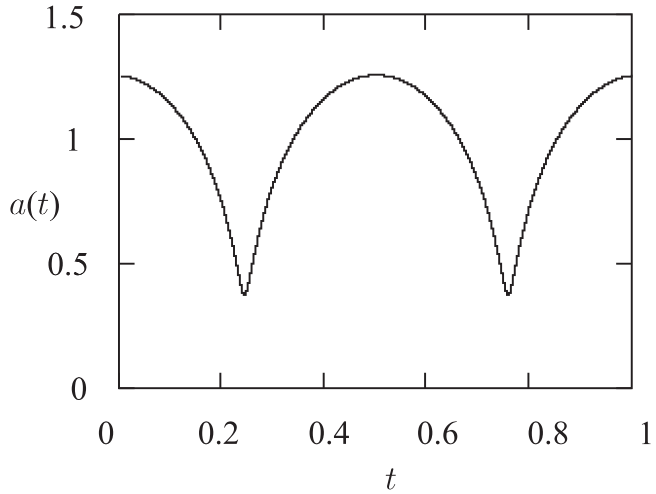
We note that if we assume $a_0 = a_s$ then

$$R_{a_0=a_s}\left(1\right) = 3.3537843901 \times 10^{-4},$$
$$R_{a_0=a_s}\left(2\right) = 0.0468940396,$$
$$R_{a_0=a_s}\left(3\right) = 0.1186057782.$$

Thus, residual in equation (35) is much smaller when we use $a_0 < a_s$ which follows from our numerical solution.



(a) Modes



(b) Area

Figure 2: Buckling modes and optimal cross-sectional area for $\alpha = 3$

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O optimalnom obliku pritisnutog stuba

Prikazano je novo numeričko rešenje Lagrange-ovog problema. Ovo rešenje je upoređeno sa apriornim ocenama nedavno dobijenim. Takođe je izvedena nova relacija koja pokazuje da je površina poprečnog preseka na sredini optimalno oblikovanog štapa *veća* od površina poprečnih preseka na krajevima. Naši numerički eksperimenti potvrđuju ovaj zaključak.

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