

# Radiation effects on free-convection flow past an impulsively started vertical plate in a rotating fluid

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## Abstract

Theoretical study of thermal radiation effects on unsteady free convective flow over a moving vertical plate in a rotating fluid is considered. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of rotation, radiation, free-convection parameters and the skin friction components on the plate are discussed.

**Keywords:** Free-convection; thermal radiation; rotating fluid; heat transfer; vertical plate.

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**List of symbols***Latine symbols*

$a^*$	absorption coefficient
$C_p$	specific heat at constant pressure
$g$	acceleration due to gravity
$Gr$	thermal Grashof number
$k$	thermal conductivity of the fluid
$Pr$	Prandtl number
$q_r$	radiative heat flux in the $y$ -direction
$R$	radiation parameter
$T'_\infty$	temperature of the fluid far away from the plate
$T'_w$	temperature of the plate
$T'$	temperature of the fluid near the plate
$t'$	time
$t$	dimensionless time
$u'$	velocity of the fluid in the $x'$ -direction
$u_0$	velocity of the plate
$u$	dimensionless velocity
$v'$	velocity of the fluid in the $y'$ -direction
$v$	dimensionless velocity
$y'$	coordinate axis normal to $x'$ -axis
$z'$	coordinate axis normal to the plate
$z$	dimensionless coordinate axis normal to the plate

*Greek symbols*

$\beta$	volumetric coefficient of thermal expansion
$\mu$	coefficient of viscosity
$\nu$	kinematic viscosity
$\Omega'$	rotation parameter

$\Omega$	dimensionless rotation parameter
$\rho$	density
$\tau$	dimensionless skin-friction
$\theta$	dimensionless temperature
<i>erfc</i>	complementary error function

## 1 Introduction

The effect of radiation are quite significant at high temperature. Radiative convective flows are encountered in countless industrial and environment processes, particularly in astrophysical studies and space technology. Radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment.

Arpaci [2] studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. England and Emery[3] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. In all above studies, the stationary plate was considered. Singh[4]studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Bestman and Adjepong [5] studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous and optically transparent medium.

Das *et al.*[6] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Raptis and Perdakis [7] considered the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically.

However the effect of thermal radiation on moving infinite vertical

plate in a rotating fluid is not studied in the literature. It is proposed to study thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate with uniform temperature , in a rotating fluid. The dimensionless governing equations are solved by Laplace transform technique.

## 2 Basic equations and analysis

Consider the three dimensional flow of a viscous incompressible fluid past an impulsively started infinite vertical isothermal plate with uniform temperature in a rotating fluid. On this plate , the  $x'$ -axis is taken along the plate in the vertically upward direction and the  $y'$ -axis is taken normal to  $x'$ -axis in the plane of the plate and  $z'$ -axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity  $\Omega'$  about the  $z'$ -axis. Initially, the plate and fluid are at the same temperature . At time  $t' > 0$ , the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity  $u_0$  in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised or lowered to  $T'_w$  which is there after maintained constant. Since the plate occupying the plane  $z' = 0$  is of infinite extent, all the physical quantities depend only on  $z'$  and  $t'$ . The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega'v' = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial z'^2}, \quad (1)$$

$$\frac{\partial v'}{\partial t'} - 2\Omega'u' = \nu \frac{\partial^2 v'}{\partial z'^2}, \quad (2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \quad (3)$$

The term  $\frac{\partial q_r}{\partial z'}$  represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0 : \quad & u' = 0, \quad T' = T'_\infty \quad \text{for all } z' \\ t' > 0 : \quad & u' = u_0, \quad T' = T'_w \quad \text{at } z' = 0 \\ & u' = 0, \quad T' \rightarrow T'_\infty \quad \text{as } z' \rightarrow \infty. \end{aligned} \quad (4)$$

By Rosseland approximation, radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^* \sigma (T'_\infty{}^4 - T'^4) \quad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding  $T'^4$  in a Taylor series about  $T'_\infty$  and neglecting higher-order terms, thus

$$T'^4 \cong 4T'_\infty{}^3 T' - 3T'_\infty{}^4 \quad (6)$$

By using equations (5) and (6), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma T'_\infty{}^3 (T'_\infty - T') \quad (7)$$

On introducing the following dimensionless quantities:

$$\begin{aligned} (u, v) &= \frac{(u', v')}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad z = \frac{z' u_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ Gr &= \frac{g \beta \nu (T'_w - T'_\infty)}{u_0^3}, \\ Pr &= \frac{\mu C_p}{k}, \quad \Omega = \frac{\Omega' \nu}{u_0^2}, \quad R = \frac{16a^* \nu^2 \sigma T'_\infty{}^3}{k u_0^2}. \end{aligned} \quad (8)$$

and the complex velocity  $q = u + iv, i = \sqrt{-1}$  in equations (1) to (4), the equations relevant to the problem reduces to

$$\frac{\partial q}{\partial t} + 2i\Omega = Gr \theta + \frac{\partial^2 q}{\partial z^2}, \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \frac{R}{Pr} \theta \quad (10)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned} q(z, t) = 0, \quad \theta(z, 0) = 0, \quad \text{for all } z \leq 0 \& t \leq 0 \\ t > 0 : \quad q(z, t) = 1, \quad \theta(0, t) = 1, \quad \text{at } z = 0 \\ q(z, t) = 0, \quad \theta(z, t) \rightarrow 0 \quad \text{as } z \rightarrow \infty. \end{aligned} \quad (11)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for the equations (9) to (10), subject to the boundary conditions (11), by Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2} \left[ \exp(z\sqrt{R}) \operatorname{erfc}\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right) + \exp(-z\sqrt{R}) \operatorname{erfc}\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) \right] \quad (12)$$

$$\begin{aligned} q = \frac{1}{2} \left( 1 + \frac{Gr}{b(1-Pr)} \right) & \left[ \exp(z\sqrt{m}) \operatorname{erfc}(d1) + \exp(-2z\sqrt{m}) \operatorname{erfc}(d2) \right] \\ & - \frac{Gr \exp(bt)}{2b(1-Pr)} \left[ \exp(z\sqrt{b+m}) \operatorname{erfc}(d3) + \exp(-z\sqrt{b+m}) \operatorname{erfc}(d4) \right] \\ & + \exp(z\sqrt{Pr(b+a)}) \operatorname{erfc}(d7) + \exp(-z\sqrt{Pr(b+a)}) \operatorname{erfc}(d8) \\ & - \frac{Gr}{2b(1-Pr)} \left[ \exp(z\sqrt{R}) \operatorname{erfc}(d5) + \exp(-2z\sqrt{R}) \operatorname{erfc}(d6) \right] \end{aligned} \quad (13)$$

where

$$\begin{aligned}
 d1, d2 &= \left[ \frac{z}{2\sqrt{t}} \pm \sqrt{mt} \right], & d3, d4 &= \left[ \frac{z}{2\sqrt{t}} \pm \sqrt{(b+m)t} \right] \\
 d5, d6 &= \left[ \frac{z\sqrt{Pr}}{2\sqrt{t}} \pm \sqrt{at} \right], & d7, d8 &= \left[ \frac{z\sqrt{Pr}}{2\sqrt{t}} \pm \sqrt{(a+b)t} \right], \\
 a &= \frac{R}{Pr}, & b &= \frac{R-m}{1-Pr} \quad \text{and} \quad m = 2i\Omega.
 \end{aligned}$$

Using equation (13) we get the following expression for skin- friction components  $\tau_x$  and  $\tau_y$ :

$$\begin{aligned}
 \tau_x + i\tau_y &= - \left( \frac{\partial q}{\partial z} \right)_{z=0} \\
 &- \frac{1}{\sqrt{\pi t}} \left\{ \left( 1 + \frac{Gr}{b(1-Pr)} \right) \left( 1 + \sqrt{m\pi t} \operatorname{erf}(\sqrt{mt}) \right) \right. \\
 &- \frac{Gr \exp(bt)}{b(1-Pr)} \left( 1 + \sqrt{(m+b)\pi t} \operatorname{erf}(\sqrt{(m+b)t}) \right) \\
 &+ \frac{Gr \exp(bt)}{b(1-Pr)} \left( \sqrt{Pr(a+b)\pi t} \operatorname{erf}(\sqrt{(a+b)t} + \sqrt{Pr}) \right) \\
 &\left. - \frac{Gr}{b(1-Pr)} \left( \sqrt{R\pi t} \operatorname{erf}(\sqrt{a t}) + \sqrt{Pr} \right) \right\} \quad (14)
 \end{aligned}$$

In equations (13) and(14),the argument of the complementary error function and error function is complex. Hence in order to obtain the  $u$  and  $v$  components of the velocity and skin -friction, we have used the following formula due to Abramowitz and Stegun [1]:

$$\begin{aligned}
 \operatorname{erf}(a+ib) &= \operatorname{erf}(a) + \frac{\exp(-a^2)}{2a\pi} [1 - \cos(2ab) + i\sin(2ab)] \\
 &+ \frac{2\exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-n^2/4)}{n^2 + 4a^2} [f_n(a,b) + ig_n(a,b)] \\
 &+ \epsilon(a,b)
 \end{aligned}$$

where

$$\begin{aligned} f_n &= 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab) \\ g_n &= 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab) \end{aligned}$$

and  $|\epsilon(a, b)| \approx 10^{-16} |\operatorname{erf}(a + ib)|$ .

### 3 Discussion of results

Using the above formula, expressions for  $u, v$  and  $\tau_x$  and  $\tau_y$  are obtained but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of  $u, v, \tau_x$  and  $\tau_y$  for different values of the various parameter like rotation, radiation and thermal Grashoff number.

The primary velocity profiles of air for different values of the radiation parameter are shown in Fig.1. It is observed that the primary velocity increases with decreasing radiation parameter  $R$  in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate.

The primary velocity profiles of air for different values of the rotation parameter are shown in Fig.2. It depicts that the primary velocity increases with decreasing rotation parameter  $\Omega$ . It is also observed that greater cooling of the plate, due to free- convection currents, increases the primary velocity of the plate in this case too.

The secondary velocity profiles of air for different values of the radiation

parameter are shown in Fig.3, the effect of radiation increases the secondary

velocity  $v$ . Fig.4. shows the effect of rotation on  $v$  which is just reverse to that of radiation parameter. Further, greater cooling of the plate,



due to free- convection currents, decreases the secondary velocity of the plate.

The temperature profiles for air ( $Pr=0.71$ ) are calculated for different values of thermal radiation parameter from Equation (12) and these are shown in Fig.5. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. The skin-friction components for air ( $Pr = 0.71$ ) at the wall for different values of  $Gr$ ,  $t$ ,  $R$  and  $\Omega$  are shown in Table 1.

**Table 1 Skin-friction  $\tau$**

Gr	$\Omega$	t	R	$\tau_x$	$\tau_y$
5.0	0.5	0.2	0.2	0.1502	1.2278
			2.0	-0.7553	0.4117
	2.0	0.2	0.6808	2.3058	
		2.0	0.9978	2.1903	
10.0	0.5	0.4	0.2	-0.7419	1.6623
	0.5	0.2	0.2	-1.1477	2.1411
			2.0	-2.9587	0.5089

The effect of radiation decreases the value of both the components of skin-friction, but the effect of rotation on skin-friction increases the component  $\tau_x$  and decreases  $\tau_y$  as  $\Omega$  increases. As time advances the component  $\tau_x$  decreases, but reverse phenomenon occurs for the component  $\tau_y$ . Greater cooling of the plate, due to free- convection currents, lowers  $\tau_x$  and raises  $\tau_y$ .

## 4 Conclusions

Theoretical analysis is performed to study flow past an impulsively started infinite vertical plate with uniform temperature, in the pres-

ence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

- (i) The influence of the radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.
- (ii) The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.
- (iii) The skin-friction components increases with decreasing radiation parameter. But the effect of rotation on  $\tau_x$  increases and  $\tau_y$  decreases as rotation parameter increases.

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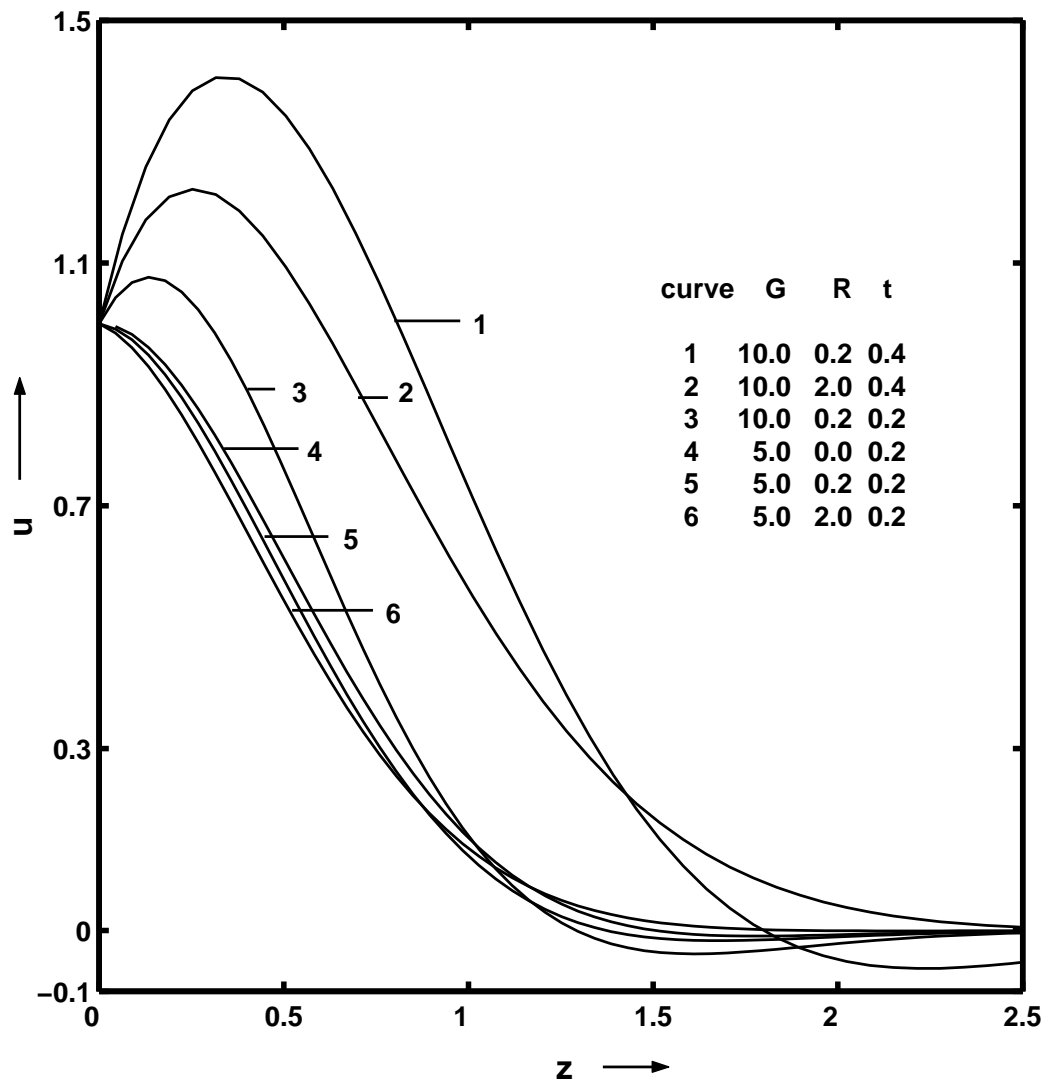


Figure 1: Primary velocity profiles for different R

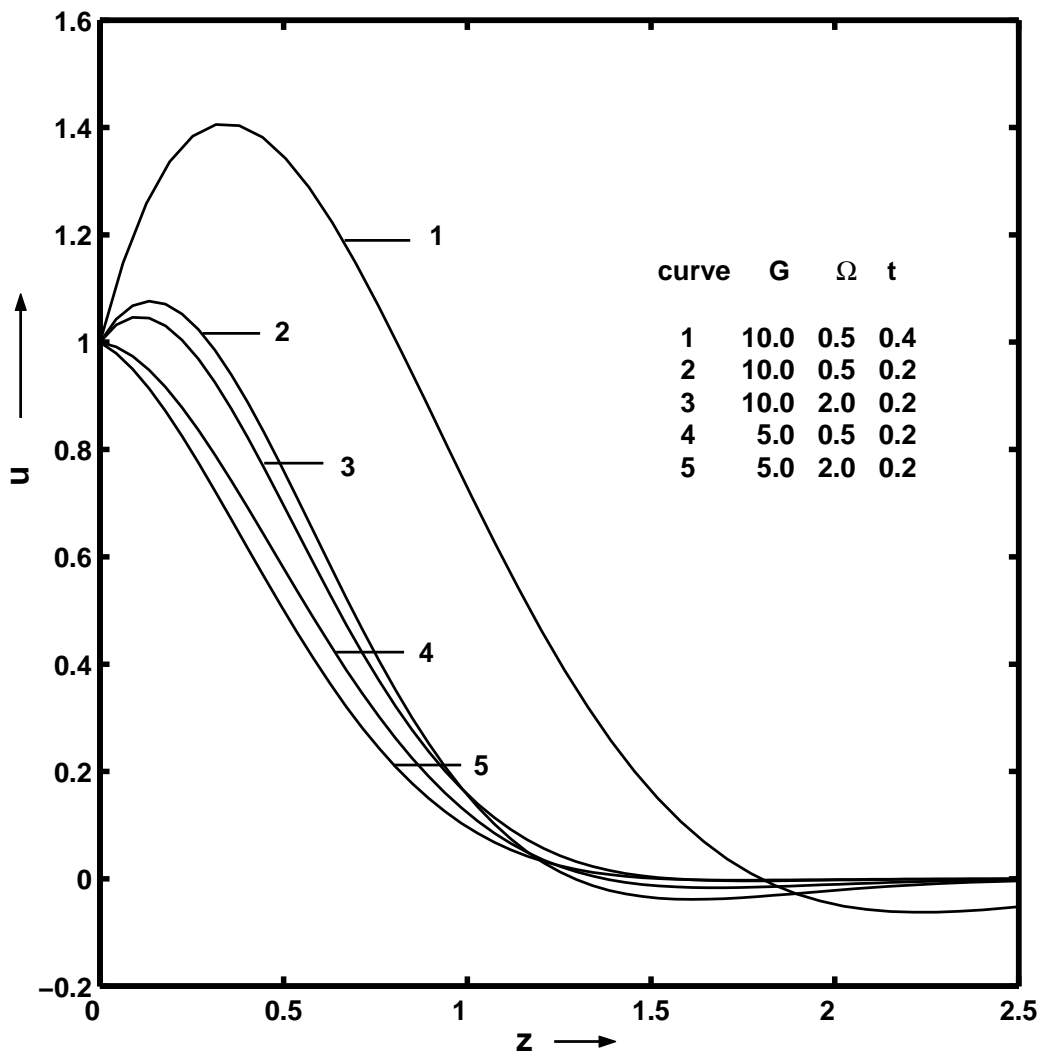


Figure 2: Primary velocity profiles for different  $\Omega$

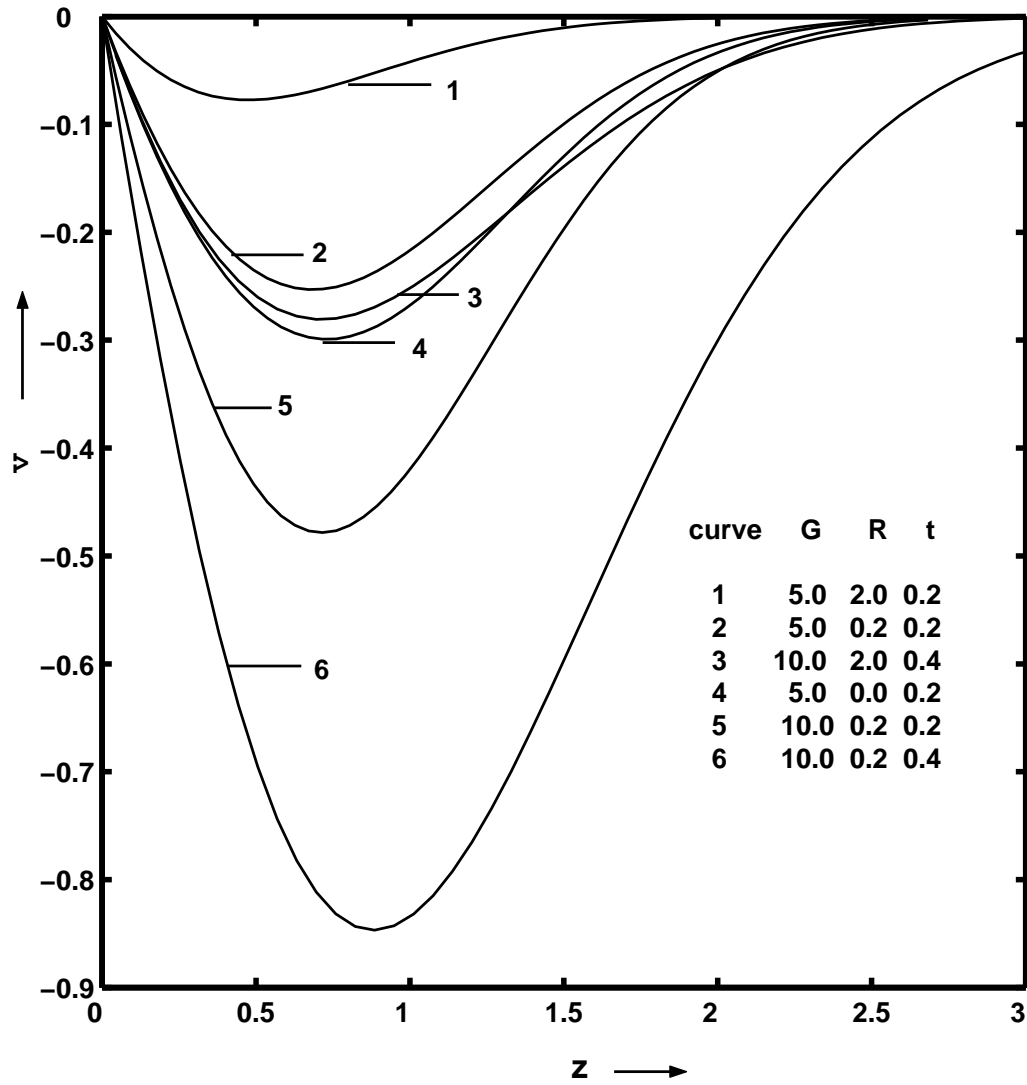


Figure 3: Secondary velocity profiles for different R

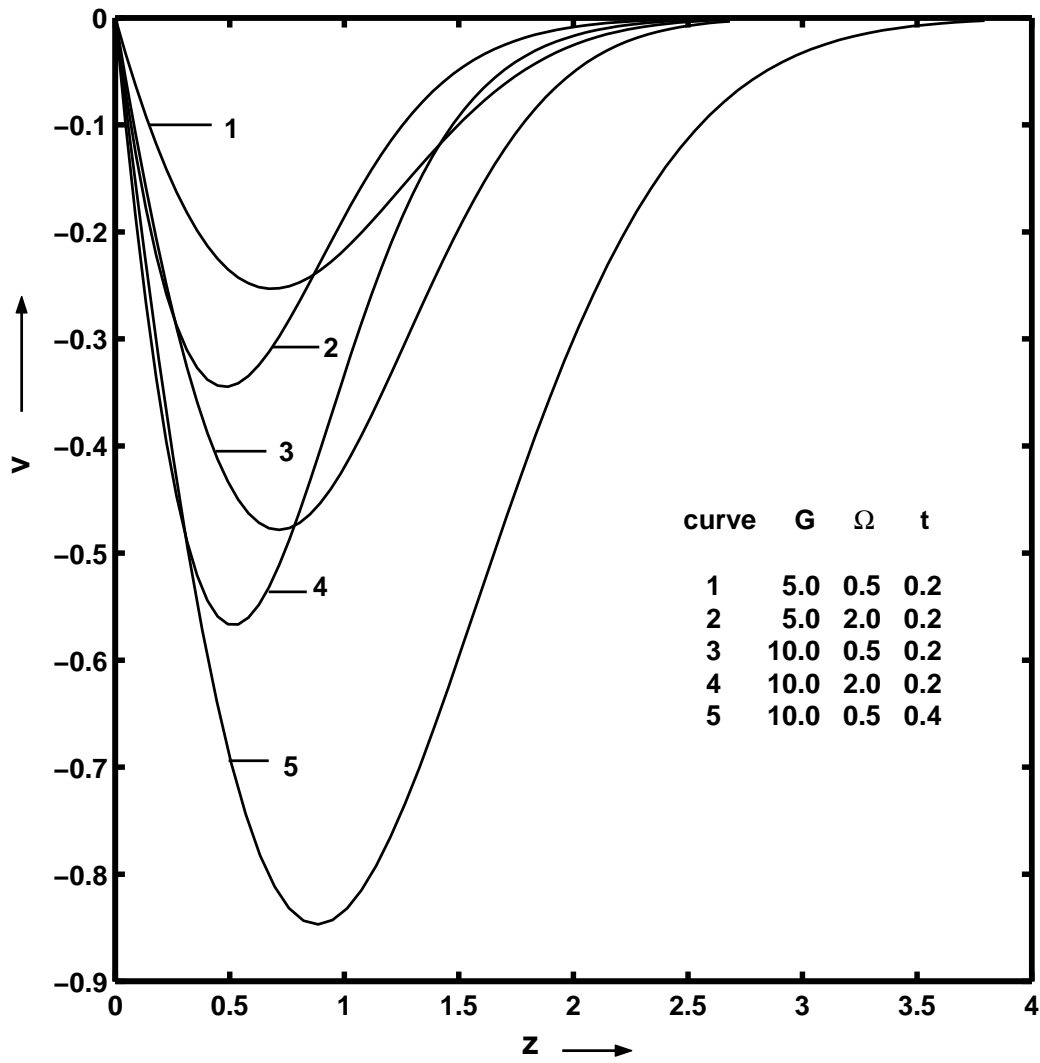


Figure 4: Secondary velocity profiles for different  $\Omega$

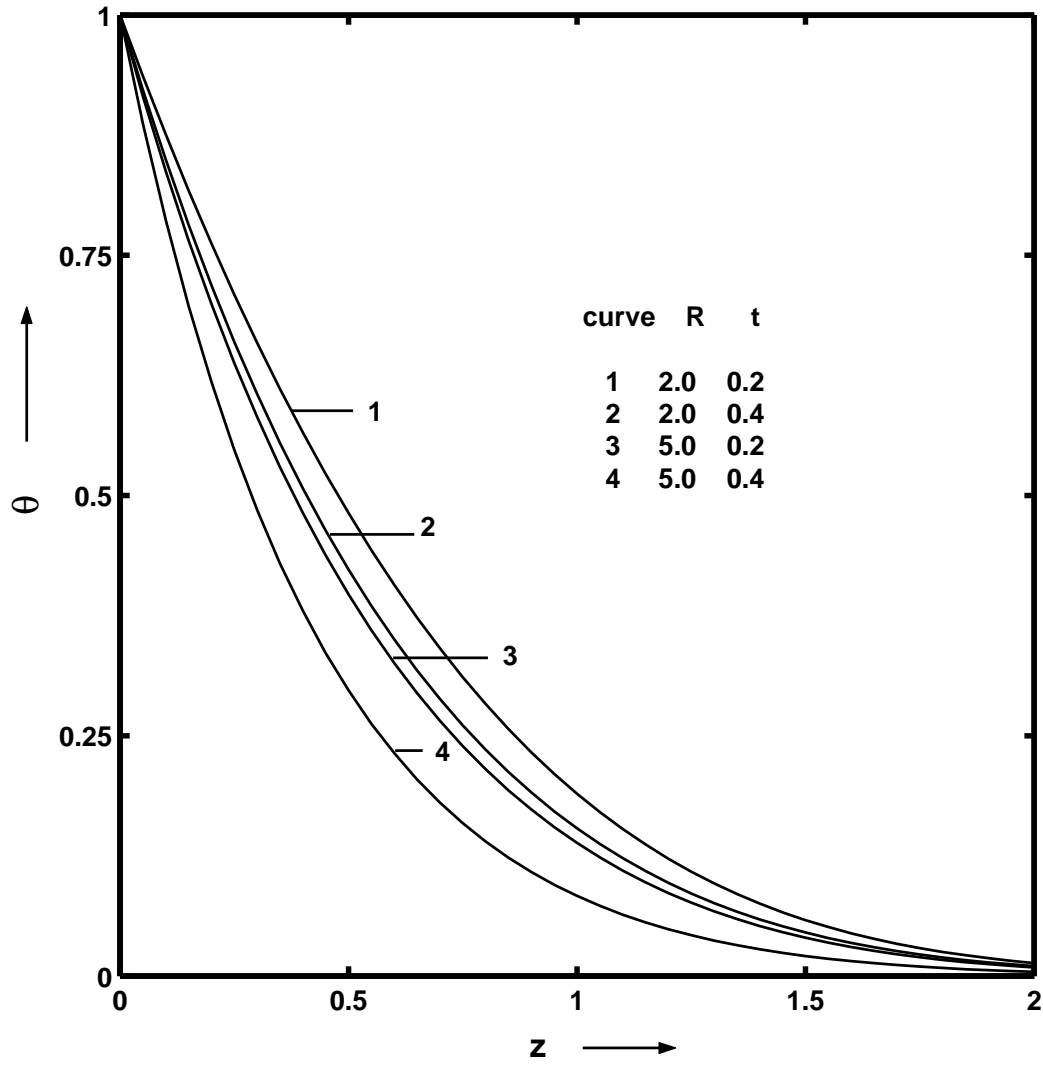


Figure 5: Temperature profiles for different R



### **Uticaji radijacije na slobodno konvekciono tečenje preko naglo pokrenute uspravne ploče u obrtnom fluidu**

Izvršeno je teorijsko proučavanje termičkih radijacionih uticaja na slobodno konvekciono tečenje preko naglo pokrenute uspravne ploče u obrtnom fluidu. Pomoću definicije kompleksne brzine dobijeno je tačno rešenje njenih uzdužnih i poprečnih komponenata. Obrazloženi su uticaji obrtanja, zračenja, parametara slobodne konvekcije kao i komponenata trenja na zidu ploče.