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Thermal diffusion in a binary fluid mixture flows due to a rotating disc of uniform high suction in presence of a weak axial magnetic field

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Abstract

The effect of a weak uniform axial magnetic field on separation of a binary mixture of incompressible viscous thermally and electrically conducting fluids flowing due to a rotating disc of uniform high suction is examined. Neglecting the induced electric field the equations governing the motion, temperature and concentration are solved in cylindrical polar coordinate by expanding the flow parameters as well as the temperature and the concentration in powers of suction parameter. The solution obtained for concentration distribution is plotted against the different axial distances from the disc for various values of non-dimensional parameters. It is found that the temperature gradient, axial magnetic field, Reynolds number, Schmidt number, Prandtl number and suction parameter effect the species separation significantly.

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1 Introduction

Separation processes of components of a binary fluid mixture wherein one of the components is present in extremely small proportion are of much interest due to their applications in science and technology. Separation of isotopes from their naturally occurring mixture is one of such examples. It is well known that only one part of heavy water which is an isotope of water is found in 25000 parts of water in normal occurrence (Arnikar [1], Rastogy et al. [2]) but it is required for use as a (i) moderator in nuclear reactions for slowing down the neutrons, (ii) tracer compound for studying the mechanism of many chemical reaction and (iii) heat transport medium i.e., a coolant in atomic power plant. Because of their small relative mass difference isotopes of heavier molecules offer the greatest practical challenge in attempts to isolate the rarer component. Electromagnetic method of separation (Srivastava [3]) works only at relatively higher values of concentrations.

In a binary fluid mixture the diffusion of individual species takes place by three-mechanisms namely ordinary diffusion, pressure diffusion (or baro-diffusion) and thermal diffusion. The diffusion flux **i** of lighter and rarer component is given by Landau and Lifshitz[4] as:

$$\mathbf{i} = -\,\rho D\,[\,grad\ c_1 + k_p\,grad\ p + k_T\,grad\ T], \tag{1}$$

where ρ is the density of the binary fluid mixture, D is the diffusion coefficient, c_1 is the ratio of mass of the lighter component to the total mass of the fluid, k_pD is the pressure diffusion coefficient, p is pressure, k_TD is the thermal diffusion coefficient and T is temperature. The ordinary diffusion contribution to the mass flux is seen to depend in a complicated way on the concentration gradients of the components present in the mixture. The baro-diffusion indicates that there may be a net movement of the components in a mixture if there is a pressure gradient imposed on the system. An example of baro –diffusion is the process of diffusion in the binary mixture of different kinds of gases present in the atmosphere. By reasons of variation of forces of gravity with height thereby causing a density gradient, different constituents in the atmosphere tend to separate out. The pressure gradient created by the gravity as well as the rotation of the earth separates various components of air. The tendency for a mixture to separate under a pressure gradient is very small but use is made of this effect in centrifuge separations in which tremendous pressure gradient is established. Thermal diffusion describes the tendency for species to diffuse under the influence of a temperature gradient. In many practical problems dealing with flows in porous media one encounters with a multiple component electrically conducting fluids e.g. molten fluid in the earth's crust, crude oil in the petroleum. It is customary to consider one of the components as solvent and the other components as solute. It is shown in ref. Groot and Mazur [5] that if separation due to thermal diffusion occurs then it may even render an unstable system to stable one. This effect is also quite small, but devices can be arranged to produce very steep temperature gradients so that separations of mixtures are effected.

Sarma [6] perhaps was the first to study the problem of baro-diffusion in a binary mixture of incompressible viscous fluids set in motion due to an infinite disk rotation. He obtained results on separation action in this configuration for small baro–diffusion number taking the Schmidt number to be of the order unity and including the effect of separation at the disk. Hurle and Jakeman [7] have discussed the effect of a temperature gradient on diffusion of a binary fluid mixture. Many investigators ([3], [6], [8]-[26]) analyzed the effect of baro-diffusion and thermal diffusion on separation of a binary mixture in different geometry.

In many cases the fluid mixture is found to be electrically conducting and so to study the effect of magnetic field on separation we have considered in this paper a binary mixture of incompressible viscous thermally and electrically conducting fluids flowing due a rotating disc of uniform high suction in presence of a constant uniform axial magnetic field. Velocity distribution under such geometry was investigated by Pande [27]. We have investigated, in this paper, the effect of axial magnetic field on the concentration distribution of the rarer component of a binary fluid mixture.

2 Governing equations and boundary conditions

We consider here the case when one of the components of the binary mixture of incompressible thermally and electrically conducting viscous fluids is present in small quantity, hence the density and viscosity of the mixture is independent of the distribution of the components. The concentration c_2 of heavier and more abundant component is given by $c_2 = 1 - c_1$. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is to be understood as the mass average velocity $\mathbf{V}= (\rho_1\mathbf{V}_1 + \rho_2\mathbf{V}_2)/\rho$ and the density $\rho = \rho_1 + \rho_2$, where the subscripts 1&2 denote the rarer and the more abundant components respectively. The equation of continuity and the equation of motion of an incompressible fluid in steady case are respectively,

$$\nabla \cdot \mathbf{V} = 0 \tag{2}$$

and

$$\rho(\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p + \mu\nabla^2\mathbf{V} + \mathbf{J} \times \mathbf{B}, \tag{3}$$

where μ is the coefficient of viscosity of the binary fluid mixture, $\mathbf{J}$ is the current density vector and $\mathbf{B}$ is the magnetic flux density vector. In steady motion the Maxwell equations are given by

$$curl\,\mathbf{H} = 4\pi\mathbf{J}, \tag{4}$$

$$curl\,\mathbf{E} = 0, \tag{5}$$

$$div\,\mathbf{H} = 0, \tag{6}$$

$\mathbf{H}$ is the magnetic field vector, $\mathbf{E}$ is the electric field vector,

It is well known that for most of the fluids used in engineering applications collision frequency exceed the cyclotron frequency for electrons. As the Hall current factor is ratio of the cyclotron frequency to the collision frequency, so, the Hall current is very small and hence we have neglected it in our discussion. Consequently Ohm's law is given by

$$\mathbf{J} = \sigma[\mathbf{E} + \mathbf{V} \times \mathbf{B}], \tag{7}$$

where

$$\mathbf{B} = \mu_e \mathbf{H}, \tag{8}$$

σ is the electric conductivity and μ_e is the magnetic permeability.

The energy equation in steady case is given by

$$\rho c_p \mathbf{V} \cdot \nabla T = k \nabla^2 T + \mu \phi + \mathbf{J}^2/\sigma, \tag{9}$$

where c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid mixture, ϕ represents the heat due to viscous dissipation and the last term $\mathbf{J}^2/\sigma$ represents heat due to Joulean dissipation.

The equation for species conservation of the first component is given by (see Landau and Lifshitz [28])

$$\rho(\mathbf{V} \cdot \nabla) c_1 = -\nabla \cdot \mathbf{i}, \tag{10}$$

where $\mathbf{i}$ is given by (1). The coefficients k_p and k_T may be determined from the thermodynamic properties alone. Landau and Lifshitz [4] have given the explicit expression for the baro-diffusion coefficients k_p as

$$k_p = (m_2 - m_1)[(c_1/m_1) + (c_2/m_2)]c_1 c_2/p_\infty, \tag{11}$$

where p_∞ denotes the working pressure of the medium and m_1, m_2 are masses of two kinds of particles. Neglecting c_1^2 (since concentration of rarer and lighter component c_1 is very small) (11) becomes

$$k_p = (m_2 - m_1)c_1/(m_2 p_\infty) = A c_1, \tag{12}$$

where

$$A = (m_2 - m_1)/(m_2 p_\infty). \tag{13}$$

The expression for k_T has been suggested by Hurl and Jakeman [3] as $k_T = s_T c_1 c_2$, where s_T is the Soret coefficient. For small values of c_1, k_T becomes $(k_T =) s_T c_1$. Substituting the expression for $\mathbf{i}$ from (1), k_p from (12) and $k_T = s_T c_1$ in (10) we get the equation for c_1 as

$$(\mathbf{V}.\nabla)c_1 = D[\nabla^2 c_1 + A\nabla.(c_1 \nabla P) + s_T \nabla.(c_1 \nabla T)]. \tag{14}$$

Boundary conditions for the flow field, temperature field and electromagnetic field are the same as in the usual magneto-hydrodynamic problems. The boundary conditions for the concentration c_1 are different in different cases. At the surface of a body insoluble in the fluid mixture the total mass flux as well as the individual species flux normal to the surface should vanish (see Srivastava [21]) i.e.,

$$\rho c_1 \mathbf{V} \cdot \mathbf{n} + \mathbf{i} \cdot \mathbf{n} = 0, \tag{15}$$

where $\mathbf{n}$ is the unit normal drawn at the solid surface directed outwards. Substituting the expression for $\mathbf{i}$ from (1) into (15), we get,

$$\rho c_1 \mathbf{V} \cdot \mathbf{n} - \rho D[\nabla c_1 \cdot \mathbf{n} + k_p \nabla p \cdot \mathbf{n} + k_T \nabla T \cdot \mathbf{n}] = 0. \tag{16}$$

If, however, there is diffusion from a body that dissolves in the fluid, equilibrium is rapidly established near its surface, and the concentration in the fluid adjoining the body in this case is the saturation concentration c_0 (say); the diffusion out of this layer takes place more slowly than the process of solution. The boundary condition at such surface is, therefore, $c_1 = c_0$.

3 Formulation of the problem

We consider here the steady flow of a binary mixture of thermally and electrically conducting viscous incompressible fluids by using cylindrical polar coordinate system (r, θ, z). The binary fluid mixture is flowing in presence of an infinitely rotation heated disc of uniform high suction in its surface at $z = 0$. The disc is maintained at a constant temperature T^* higher than the ambient temperature T_0. The concentration of the rarer and lighter component of the mixture maintained at a constant value c_0 far away from the disc. A weak axial magnetic field of uniform strength B_0 is applied. The induced magnetic field due to the uniform magnetic B_0 is neglected and the physical justification regarding the neglect of induced electric and magnetic fields, we have followed Sparrow and Cess [29]. It is assume that the disc at $z = 0$ rotates with a constant angular velocity Ω in its own plane. Let u, v, w be the velocity components in the directions of r, θ, z respectively.

In axisymmetric case, the governing equations (2), (3) and (9) for the steady flow of a binary mixture of incompressible thermally and electrically conducting viscous fluids due to the rotation of a disc of uniform high suction at its surface in the fluid in presence of a uniform axial magnetic field becomes

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{17}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2}\right) - \frac{\sigma B_0^2 u}{\rho}, \tag{18}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \nu\left(\nabla^2 v - \frac{v}{r^2}\right) - \frac{\sigma B_0^2 v}{\rho}, \tag{19}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w, \tag{20}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\nabla^2 T + \frac{\nu\phi}{c_p} + \frac{\sigma B_0^2}{\rho c_p}(u^2 + v^2), \tag{21}$$

where the viscous dissipation function ϕ is given by

$$\begin{aligned}\phi = 2&\left\{\left(\frac{\partial u}{\partial r}\right)^2 + \frac{u^2}{r^2} + \left(\frac{\partial w}{\partial z}\right)^2\right\} \\ &+ \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}\right)^2,\end{aligned} \tag{22}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{23}$$

and $\nu\,(= \mu/\rho)$ is the kinematic coefficient of viscosity.

In the flow of binary mixture if we ignore the pressure gradient, the diffusion of the individual species takes place by two mechanisms, namely, the concentration gradient and the temperature gradient. Under this condition the equation (14) of the species conservation for solving the mass transfer problem can be written as

$$u\frac{\partial c_1}{\partial r} + w\frac{\partial c_1}{\partial z} = D\left[\nabla^2 c_1 + s_T\left\{c_1\nabla^2 T + \frac{\partial c_1}{\partial r}\frac{\partial T}{\partial r} + \frac{\partial c_1}{\partial z}\frac{\partial T}{\partial z}\right\}\right]. \tag{24}$$

The boundary conditions of the problem are given by

$$u = 0,\ v = r\Omega,\ w = -w_0,\ T = T^* \quad at \quad z = 0 \tag{25}$$

and

$$u = v = 0,\ T = T_0,\ c_1 = c_0,\ \frac{\partial c_1}{\partial z} = 0 \quad at \quad z \to 0. \tag{26}$$

Following Evans [30] we define a suction parameter by means of $a \equiv= w_0/\sqrt{\nu\Omega}$.

In view of the boundary conditions (25)-(26) we assume the following form for the velocity components, the temperature, the pressure and the concentration within the boundary layer region as

$$u = r\Omega F(\zeta), \quad v = r\Omega G(\zeta), \quad w = \sqrt{\nu\Omega}\left[-a + H(\zeta)\right], \quad p = \mu\Omega\, p_1(\zeta),$$

$$T = T_0 + (\nu\Omega/c_p)\left[f_1(\zeta) + R^2 f_2(\zeta)\right], \quad c_1 = c_0\left[g_1(\zeta) + R^2 g_2(\zeta)\right]$$

where

$$\zeta = \sqrt{\Omega/\nu}z \quad \text{and} \quad R = \sqrt{\Omega/\nu}r. \tag{27}$$

On physical grounds, H may be assumed to be nearly constant for large values of suction parameter a, and consequently $H' = F = 0$. Following Stuart [31] we get as a first approximation $G = e^{-a\zeta}$. This suggest that a solution may be obtained in descending powers of suction parameter a, and that $a\varsigma$ is more suitable than ς as an independent variable. We therefore define a new parameter $\eta = a\zeta$.

Using (27) and $\eta =$ a ζ in (17)-(21), (24) and equating the coefficient of like powers of suction parameter a from both sides, we get

$$aH' + 2F = 0, \tag{28}$$

$$a^2\left(F'' + F'\right) = F^2 - G^2 + aF'H + M^2 F, \tag{29}$$

$$a^2\left(G'' + G'\right) = 2FG + aG'H + M^2 G, \tag{30}$$

$$a^2\left(f''_1 + P_r f'_1\right) = -4f_2 + P_r\left(aHf'_1 - 12F^2\right), \tag{31}$$

$$a^2\left[f''_2+P_rf'_2+\left(G'^2+F'^2\right)P_r\right] \\ =P_r\left[2Ff_2+af'_2H-M^2\left(F^2+G^2\right)\right], \tag{32}$$

$$a^2\left[g''_1+\alpha\left(g_1f'_1\right)'+S_mg'_1\right]=-4g_2+S_maHg'_1-4\alpha g_1f_2, \tag{33}$$

$$a^2\left[g''_2+\alpha\left\{\left(g_2f'_1\right)'+\left(g_1f'_2\right)'\right\}+S_mg'_2\right] \\ =S_m\left(2Fg_2+aHg'_2\right)-8\alpha g_2f_2, \tag{34}$$

where $\alpha = t_dE_c$, $M = B_0\sqrt{\sigma/\rho\Omega}$ is the Hartmann number, $P_r = \rho c_p\nu/\kappa$ is the Prandtl number, $S_m = \nu/D$ is the Schmidt number, $t_d = S_T\left(T^*-T_0\right)$ is the thermal diffusion number and $E_c = \nu\Omega/c_p(T^*-T_0)$ is the Eckert number.

The boundary conditions (25)- (26) under the forms in (27) become,

$$F=0,\ G=1,\ H=0,\ f_1=N,\ f_2=0 \quad at \quad \eta=0. \tag{35}$$

Here

$$N=c_p\left(T^*-T_0\right)/\nu\Omega$$

and

$$F=0,\ G=n,\ f_1\to 0,\ f_2\to 0,\ g_1\to 1,\ g_2\to 0, \\ g'_1\to 0,\ g'_2\to 0 \quad at \quad \eta\to\infty. \tag{36}$$

For large values of suction parameter a, a regular perturbation scheme can be developed by expanding F, G, H, f_1, f_2, g_1 and g_2 in descending powers of the suction parameter a. We assume,

$$F\left(\eta\right)=\sum_{i=0}^{\infty}a^{-i}F_i\left(\eta\right),\quad G\left(\eta\right)=\sum_{i=0}^{\infty}a^{-i}G_i\left(\eta\right),\quad H\left(\eta\right)=\sum_{i=0}^{\infty}a^{-i}H_i\left(\eta\right),$$

$$f_1\left(\eta\right)=\sum_{i=0}^{\infty}a^{-i}f_{1i}\left(\eta\right),\qquad f_2\left(\eta\right)=\sum_{i=0}^{\infty}a^{-i}f_{2i}\left(\eta\right),$$

$$g_1(\eta) = 1 + \alpha \sum_{i=0}^{\infty} a^{-i} g_{1\alpha_i}(\eta) \quad \text{and} \quad g_2(\eta) = \alpha \sum_{i=0}^{\infty} a^{-i} g_{2\alpha_i}(\eta). \tag{37}$$

Substituting the expansions (37) in the equations (28)-(34) and equating the coefficients of various powers of suction parameter a on both sides, we get

$$H_0' = 0, \quad H_1' = -2F_0, \quad H_2' = -2F_1, \quad H_3' = -2F_2, \tag{38}$$

$$\begin{aligned} F''_0 + F'_0 &= 0, \qquad F''_1 + F'_1 = F'_0 H_0, \\ F''_2 + F'_2 &= -G_0^2, \quad F''_3 + F'_3 = 0, \end{aligned} \tag{39}$$

$$\begin{aligned} G''_0 + G'_0 &= 0, \quad G_1'' + G_1' = 0, \quad G_2'' + G_2' = M^2 G_0, \\ G_3'' + G_3' &= 0, \quad G_4'' + G_4' = 2F_2 G_0 + H_3 G_0' + M^2 G_2, \end{aligned} \tag{40}$$

$$\begin{aligned} f_{10}'' + P_r f_{10}' &= 0, \quad f_{11}'' + P_r f_{11}' = 0, \quad f_{12}'' + P_r f_{12}' = -4 f_{20}, \\ f_{13}'' + P_r f_{13}' &= 0, \qquad f_{14}'' + P_r f_{14}' = -4 f_{22}, \end{aligned} \tag{41}$$

$$\left.\begin{aligned} & f_{20}'' + P_r f_{20}' + P_r {G_2'}^2 = 0, \qquad f_{21}'' + P_r f_{21}' = 0, \\ & f_{22}'' + P_r f_{22}' + 2P_r G_0' G_2' = -P_r M^2 G_0^2, \qquad f_{23}'' + P_r f_{23}' = 0, \\ & f_{24}'' + P_r f_{24}' + P_r \left({F_2'}^2 + 2G_0' G_4' + {G_2'}^2 \right) = \\ & \qquad P_r \left[2F_2 f_{20} + f_{20}' H_3 - 2M^2 G_0 G_2 \right], \end{aligned}\right\} \tag{42}$$

$$\left.\begin{aligned} & g_{1\alpha_0}'' + S_m g_{1\alpha_0}' + f_{10}'' = 0, \qquad g_{1\alpha_1}'' + S_m g_{1\alpha_1}' = 0, \\ & g_{1\alpha_2}'' + S_m g_{1\alpha_2}' + f_{12}'' = -4 \left(g_{2\alpha_0} + f_{20} \right), \\ & g_{1\alpha_3}'' + S_m g_{1\alpha_3}' + f_{13}'' = 0, \qquad g_{1\alpha_4}'' + S_m g_{1\alpha_4}' + f_{14}'' = \\ & \qquad S_m H_3 g_{1\alpha_0}' - 4 f_{22}, \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}
&g''_{2\alpha_0} + S_m g'_{2\alpha_0} + f''_{20} = 0, \qquad g''_{2\alpha_1} + S_m g'_{2\alpha_1} = 0,\\
&g''_{2\alpha_2} + S_m g'_{2\alpha_2} + f''_{22} = 0, \qquad g''_{2\alpha_3} + S_m g'_{2\alpha_3} + f''_{23} = 0,\\
&g''_{2\alpha 4} + S_m g'_{2\alpha_4} + f''_{24} = S_m \left(2F_2 g_{2\alpha_0} + H_3 g'_{2\alpha_0}\right).
\end{aligned}\right\} \tag{44}$$

Also, the boundary conditions (35)-(36) under the assumptions (37) become

$$\begin{aligned}
&F_i = 0, \quad G_0 = 1, \quad G_{i+1} = 0, \quad H_i = 0,\\
&f_{10} = N, \quad f_{1(1+i)} = 0, \quad f_{2i} = 0 \quad at \quad \eta = 0
\end{aligned} \tag{45}$$

and

$$\begin{aligned}
&F_i = 0, \quad G_0 = n, \quad G_{i+1} = 0, \quad f_{1i} = 0, \quad f_{1i} = 0, \quad g_{1\alpha_i} = 0,\\
&g_{2\alpha_i} = 0, \quad g'_{1\alpha_i} = 0, \quad g'_{2\alpha_i} = 0 \quad at \quad \eta \to \infty,
\end{aligned} \tag{46}$$

$\forall i \in W$, where W represents the set of whole numbers.

4 Solutions of the problem

The solutions of equations (38) to (44) under boundary conditions (45)-(46) are obtained as

$$\left.\begin{aligned}
&F_0 = 0, \quad H_0 = 0, \quad G_0 = e^{-\eta}, \quad f_{10} = Ne^{-P_r\eta},\\
&f_{20} = \{P_r/2\left(P_r - 2\right)\}\left(e^{-2\eta} - e^{-P_r\eta}\right),\\
&g_{1\alpha_0} = \{NP_r/\left(S_m - P_r\right)\} e^{-P_r\eta},
\end{aligned}\right\} \tag{47}$$

$$\begin{aligned}
&F_1 = 0, \quad G_1 = 0, \quad H_1 = 0,\\
&f_{11} = 0, \quad f_{21} = 0, g_{1\alpha_1} = 0, \quad g_{2\alpha_1} = 0,
\end{aligned} \tag{48}$$

$$\left.\begin{aligned}
F_2 = {} & \frac{1}{2}\left(e^{-\eta} - e^{-2\eta}\right), \quad H_2 = 0, \quad G_2 = -M^2\eta e^{-\eta}, \\
f_{12} = {} & -\frac{1}{\left(P_r-2\right)^2}\left[-P_r e^{-2\eta} + e^{-P_r\eta}\left\{2\eta\left(P_r-2\right)+P_r\right\}\right], \\
f_{22} = {} & -\frac{P_r M^2}{\left(P_r-2\right)^2}\left[e^{-2\eta}\left\{\left(P_r-2\right)\left(1-\eta\right)+1\right\}\right. \\
& \left. -\left(P_r-1\right)e^{-P_r\eta}\right], \\
g_{1\alpha_2} = {} & -\frac{P_r^2}{\left(P_r-2\right)^2\left(S_m-P_r\right)}e^{-P_r\eta} \\
& +\frac{P_r\left(P_r S_m-4\right)}{\left(P_r-2\right)^2\left(S_m-2\right)^2}e^{-2\eta} \\
& -\frac{2P_r}{\left(P_r-2\right)\left(S_m-P_r\right)}\eta e^{-P_r\eta}, \\
g_{2\alpha_2} = {} & \frac{P_r M^2}{(P_r-2)^2}\left[\left\{\frac{2\left(P_r-2\right)\left(1-\eta\right)+P_r}{\left(S_m-2\right)}\right.\right. \\
& \left.\left. +\frac{2(P_r-2)}{(S_m-2)^2}\right\}e^{-2\eta} - \frac{P_r(P_r-1)}{(S_m-P_r)}e^{-P_r\eta}\right],
\end{aligned}\right\} \tag{49}$$

$$\begin{aligned}
& F_3 = 0, \quad G_3 = 0, \quad H_3 = e^{-\eta} - \frac{1}{2}e^{-2\eta} - \frac{1}{2}, \\
& f_{13} = 0, \quad f_{23} = 0, \quad g_{1\alpha_3} = 0, \quad g_{2\alpha_3} = 0,
\end{aligned} \tag{50}$$

$$G_4 = \left(-\frac{1}{2}\eta + \frac{1}{12}\right)e^{-\eta} - \frac{e^{-3\eta}}{12} + M^4\left(\eta + \frac{\eta^2}{2}\right)e^{-\eta}, \tag{51}$$

$$H_4 = 0, \tag{52}$$

$$f_{14} = -\frac{P_r M^2}{(P_r-2)^3}\Big[\{2(P_r-2)\eta-(P_r+2)\}e^{-2\eta} + \frac{4(P_r-2)(P_r-1)}{P_r}\eta e^{-P_r\eta} + (P_r+2)e^{-P_r\eta}\Big] - P_r^2 N\left[\frac{e^{-(P_r+1)\eta}}{(P_r+1)} - \frac{e^{-(P_r+2)\eta}}{4(P_r+2)} + \frac{1}{2P_r}\eta e^{-P_r\eta} - \frac{3P_r+7}{4(P_r+1)(P_r+2)}e^{-P_r\eta}\right], \tag{53}$$

$$f_{24} = P_r\Big[\frac{5}{24(P_r-2)}\,e^{-2\eta} + \frac{B}{6(P_r-1)(P_r-3)}e^{-3\eta} + \frac{1}{8(P_r-4)}e^{-4\eta} - \frac{BP_r}{2(P_r+1)(P_r-4)}e^{-(P_r+1)\eta} + Ae^{-P_r\eta} - \frac{P_r}{8(P_r+2)}e^{-(P_r+2)\eta} + \frac{P_r}{4(P_r-2)}\eta e^{-P_r\eta} - \frac{1}{2(P_r-2)}\eta e^{-2\eta} + M^4\left\{\frac{1}{P_r-2}\eta^2 e^{-2\eta} - \frac{P_r}{(P_r-2)^2}\eta e^{-2+\eta} + \frac{B}{(P_r-2)^2}e^{-2\eta} - Be^{-P_r\eta}\right\}\Big], \tag{54}$$

$$
\begin{aligned}
g_{1\alpha_4} &= \frac{P_r M^2}{(P_r-2)^3}\left[De^{-2\eta} - \frac{P_r(P_r+2)}{(S_m-P_r)}e^{-P_r\eta}\right. \\
&-\frac{2(P_r-2)(P_rS_m+2S_m-6)+P_r}{(S_m-2)^2}\eta e^{-2\eta}+ \\
&\left.-\frac{4(P_r-1)(P_r-2)}{(S_m-P_r)}\eta e^{-P_r\eta}\right] \\
&-P_rN\left[Ee^{-(P_r+1)\eta} - Fe^{-(P_r+2)\eta} - Ge^{-P_r\eta}\right. \\
&\left.+\frac{P_r}{2(S_m-P_r)}\eta e^{-P_r\eta}\right],
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
g_{2\alpha_4} &= P_r\left[\frac{5e^{-2\eta}}{12(P_r-2)(S_m-2)} + \frac{e^{-4\eta}}{2(P_r-4)(S_m-4)}\right. \\
&+\frac{S_m(-P_r+6)+6(P_r-4)}{6(P_r-2)(S_m-2)(P_r-3)(S_m-3)}e^{-3\eta} \\
&+\frac{P_r}{(S_m-P_r)}\left\{A-\frac{S_m}{4(P_r-2)(S_m-P_r)}\right\}e^{-P_r\eta} \\
&-\frac{P_r^2B}{2(P_r-4)(S_m-P_r)(P_r+1)}e^{-(P_r+1)\eta} \\
&-\frac{P_r^2}{8(S_m-P_r)(P_r+2)}e^{-(P_r+2)\eta} \\
&\left.+\frac{P_r^2}{4(S_m-P_r)(P_r-2)}\eta e^{-P_r\eta} - \frac{\eta e^{-2\eta}}{(S_m-2)(P_r-2)}\right] \\
&+M^4\left[\frac{2\eta^2e^{-2\eta}}{(S_m-2)(P_r-2)} + \frac{4(S_mP_r-S_m-P_r)}{(S_m-2)^2(P_r-2)^2}\eta e^{-2\eta}\right.
\end{aligned} \tag{56}
$$

$$-\frac{4\left(S_mP_r-S_m-P_r\right)+\left(2B+P_r\right)\left(S_m-2\right)^2}{\left(S_m-2\right)^3\left(P_r-2\right)^2}e^{-2\eta}$$
$$\left.+\frac{BP_r}{\left(S_m-P_r\right)}e^{-P_r\eta}\right],$$

where

$$\begin{aligned}
A &= -\frac{5}{24\left(P_r-2\right)^2}+\frac{P_r-2}{6\left(P_r-2\right)\left(P_r-3\right)}-\frac{1}{8\left(P_r-4\right)} \\
&\quad -\frac{P_r\left(P_r-1\right)}{2\left(P_r-2\right)\left(P_r+1\right)}+\frac{P_r}{8\left(P_r+2\right)}, \\
B &= -\frac{\left(P_r-4\right)\left(P_r-1\right)}{\left(P_r-2\right)}, \\
D &= \frac{2S_mP_r^2+2S_m^2P_r-20S_mP_r+4P_r+S_m^2P_r^2+24}{\left(S_m-2\right)^2}, \\
E &= \frac{1}{\left(S_m-P_r-1\right)}\left\{P_r-\frac{S_m}{\left(S_m-P_r\right)\left(P_r+1\right)}\right\}, \\
F &= \frac{1}{\left(S_m-P_r-2\right)}\left\{\frac{P_r}{4}-\frac{S_m}{2\left(S_m-P_r\right)\left(P_r+2\right)}\right\}, \\
G &= \frac{S_m\left(P_r-1\right)}{2\left(S_m-P_r\right)^2P_r}+\frac{P_r^2\left(3P_r+7\right)}{4\left(P_r+1\right)\left(P_r+2\right)\left(S_m-P_r\right)}.
\end{aligned} \tag{57}$$

To get an estimate of mass concentration of the lighter and rarer component of the mixture the average value of concentration $\{\bar{c}_1(\eta)\}_{average}$ is calculated from

$$\{\bar{c}_1(\eta)\}_{average}=\frac{1}{\pi\tau^2}\int_0^{\tau}2\pi r c_1(\eta)\,dr \quad , \tag{58}$$

where 'τ' is the radius of the finite disc.

Substituting $c_1(\eta)$ from (27) in (58) by making the use of (47)-(56), we get,

$$
\begin{aligned}
&\left\{\frac{\bar{c}_1(\eta)}{c_0}\right\}_{average} = 1 + \frac{\alpha}{a} \\
&+ \alpha\left[\frac{NP_r}{S_m - P_r}e^{-P_r\eta} + \frac{R_eP_r}{4(P_r-2)}\left(\frac{2}{S_m-2}e^{-2\eta} - \frac{P_r}{S_m-P_r}e^{-P_r\eta}\right)\right] \\
&+ \frac{\alpha}{a^2}\left[-\frac{P_r^2}{(S_m-P_r)(P_r-2)^2}e^{-P_r\eta} + \frac{P_r(S_mP_r-4)}{(S_m-P_r)^2(P_r-2)^2}e^{-2\eta}\right. \\
&-\frac{2P_r}{(S_m-P_r)(P_r-2)}\eta e^{-P_r\eta} - \frac{R_eP_rM^2}{2(P_r-2)^2} \\
&\left.\left\{\frac{2(P_r-2)(1-\eta)+P_r}{S_m-2}e^{-2\eta} + \frac{2(P_r-2)}{(S_m-2)^2}e^{-2\eta} - \frac{P_r(P_r-1)}{S_m-P_r}e^{-P_r\eta}\right\}\right] \\
&+ \frac{\alpha}{a^3} + \frac{\alpha}{a^4}\left[P_r\left\{\frac{5R_e}{24(S_m-2)(P_r-2)^2}e^{-2\eta} + \right.\right. \\
&\frac{R_eS_m(-P_r+6)+6R_e(P_r-4)}{12(P_r-2)(P_r-3)(S_m-2)(S_m-3)}e^{-3\eta} \\
&+ \frac{R_e}{4(S_m-4)(P_r-4)^2}e^{-4\eta} + \frac{R_eP_rA}{2(S_m-P_r)}e^{-P_r\eta} \\
&- \frac{R_eS_mP_r}{8(S_m-P_r)^2(P_r-2)}e^{-P_r\eta} + NGe^{-P_r\eta} \\
&- \frac{R_eP_r^2B}{4(S_m-P_r)(P_r-4)(P_r+1)}e^{-(P_r+1)\eta} - NEe^{-(P_r+1)\eta} \\
&- \frac{R_eP_r^2}{16(S_m-P_r)(P_r+2)}e^{-(P_r+2)\eta} + NFe^{-(P_r+2)\eta} \\
&+ \frac{R_eP_r^2}{8(S_m-P_r)(P_r-2)}\eta e^{-P_r\eta} - \frac{NP_r}{2(S_m-P_r)}\eta e^{-P_r\eta} \\
&\left. - \frac{1}{(S_m-2)(P_r-2)^2}\eta e^{-2\eta}\right\} + \frac{P_rM^2}{(P_r-2)^3} \\
&\left\{-\frac{2(P_r-2)(S_mP_r+2S_m-6)}{(S_m-2)^2}\eta e^{-2\eta}\right.
\end{aligned}
\tag{59}
$$

$$+ De^{-2\eta} - \frac{4\left(P_r-1\right)\left(P_r-2\right)}{S_m-P_r}\eta e^{-P_r\eta} - \frac{P_r\left(P_r+2\right)}{S_m-P_r}e^{-P_r\eta}\Bigg\}$$

$$+ \frac{R_eM^4}{2}\Bigg\{-\frac{2}{\left(S_m-2\right)\left(P_r-2\right)}\eta^2 e^{-2\eta} + \frac{4\left(S_mP_r-S_m-P_r\right)}{\left(P_r-2\right)^2\left(S_m-2\right)^2}\eta e^{-2\eta}$$

$$- \frac{4\left(S_mP_r-S_m-P_r\right)+\left(2B+P_r\right)\left(S_m-2\right)^2}{\left(P_r-2\right)^2\left(S_m-2\right)^3}e^{-2\eta} + \frac{P_rB}{S_m-P_r}e^{-P_r\eta}\Bigg\}\Bigg],$$

where $R_e = \Omega a^2/\nu$ is the Reynolds number.

5 Results

If we put $\alpha = 0$ in expressions (59) for average concentration of the first component of the binary fluid mixture we get $\bar{c}_1(\eta) = c_0$ for all values of η. From this we can conclude that the separation of species of rarer and lighter component present in the binary fluid mixture ceases to take place if we neglect the effect of temperature gradient. Our results are found to be in good agreement with the results of the researchers ([3], [6], [8]-[26]).

Fig.1, fig.2 and fig.3 reveal that the separation of species of rarer and lighter component of the binary fluid mixture increases with decrease of the magnetic parameter M, Prandtl number P_r and the Reynolds number R_e. Fig.4 and fig.6 reveal that the separation of species of the binary mixture increases with the increase of the Schmidt number S_m and the product of Eckert number and the thermal diffusion number i.e. E_ct_d. From Fig.5 it clear that the concentration of the rarer and lighter component of the binary fluid mixture is remains constant on the disc and at the infinite from the disc for all values of the suction parameter. The separation of the species in between the disc and the infinity from the disc increases with the increase of the suction parameter.

6 Conclusions

The problem of mass transfer due to the flow of an electrically and thermally conducting, viscous incompressible binary fluid mixture in pres-

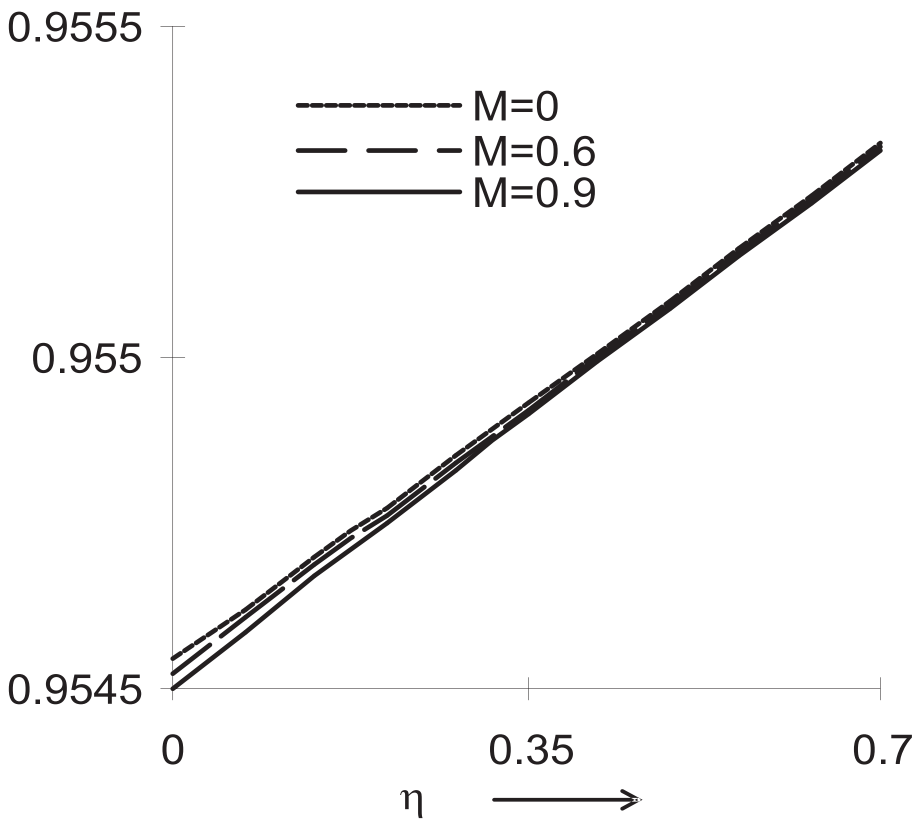

Figure 1: Graph of $\overline{c_1}(\eta)/c_0$ for various values of Hartmann Number against the normal distance η from the disc taking $N = 10, P_r = 0.07$, $S_m = 0.224$, $R_e = 1, a = 1.5, \alpha = -0.01$.

ence of an infinitely rotating heated disc of uniform high suction on its surface, in presence of a uniform axial magnetic field, has been investigated under the assumption that one of the components, which is rarer and lighter, is present in the mixture in a very small quantity. Analytical solutions of the governing equations have been obtained by expanding the flow parameters as well as the temperature and the concentration in powers of suction parameter. Different analytic expressions are obtained for non-dimensional velocity, temperature and concentration profile in presence of the magnetic field. The specific conclusions derived from

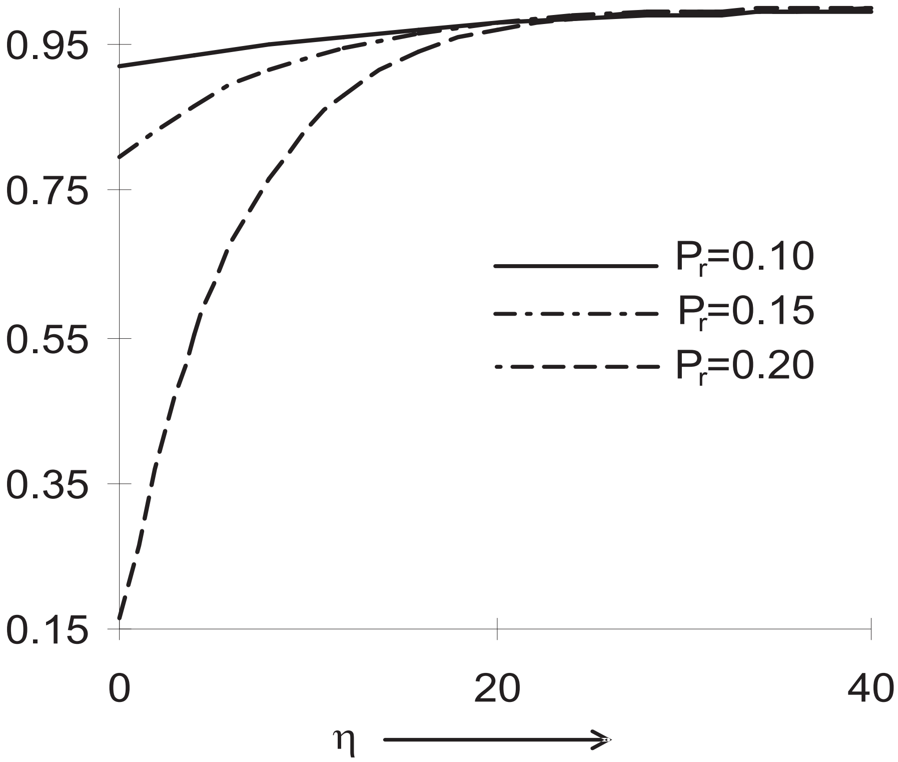

Figure 2: Graph of $\overline{c_1}(\eta)/c_0$ for various values of Prandtl number against the normal distance η from the disc taking $N = 10, M = 0.6,\ S_m = 0.224,\ R_e = 1, a = 1.5,\ \alpha = -0.01$.

this study can be listed as follows:

- the effect of the temperature gradient is to separate the species of the binary fluid mixture i. e. separation of species ceases to take place in absence of the temperature gradient.

- the effect of increase in the values of the product of the thermal diffusion number and the Eckert number is to increase the separation of species of the binary fluid mixture.

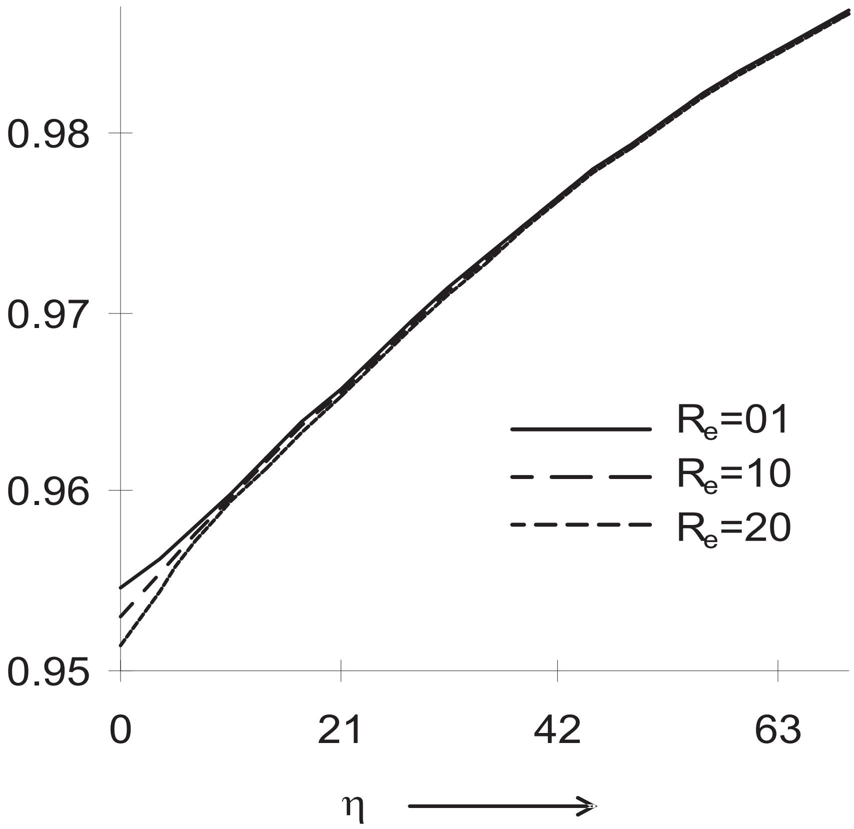

Figure 3: Graph of $\overline{c_1}(\eta)/c_0$ for various values of Reynolds number against the normal distance η from the disc taking $N = 10, P_r$=0.07, $S_m = 0.224$, $M = 0.6, a = 1.5, \alpha = -0.01$.

- the effect of decrease in the values of the magnetic parameter, the Prandtl number and the Reynolds number is to increase the separation of species of the binary fluid mixture.

- the effect of increase in the values of the Schmidt number is to increase the species separation.

- the effect of increase in the values of the suction parameter is

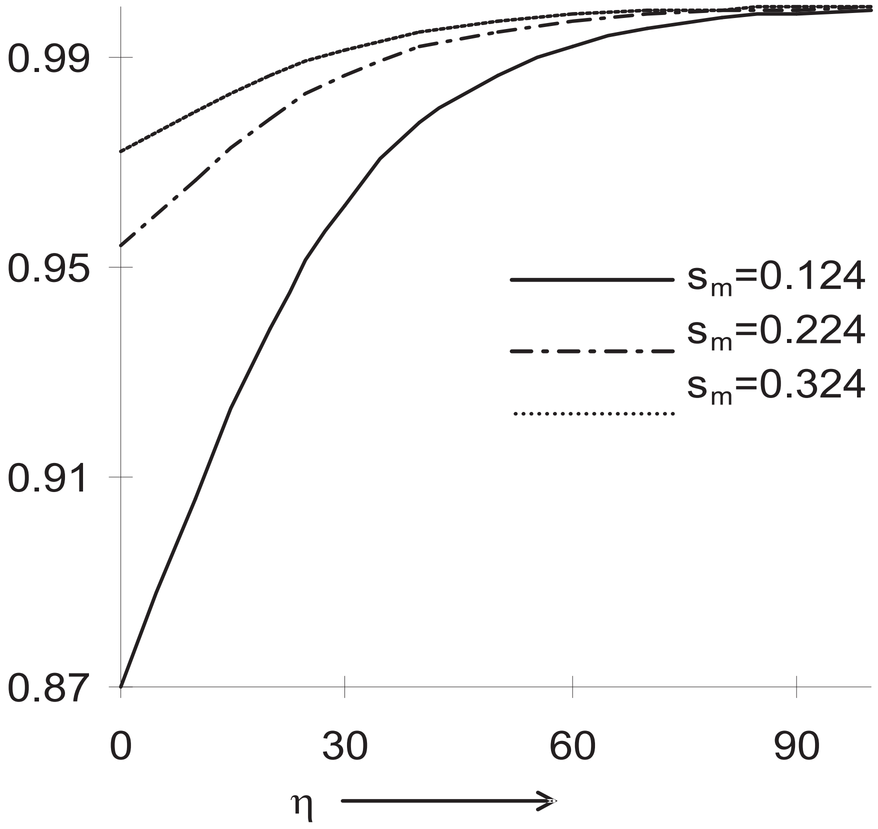

Figure 4: Graph of $\overline{c_1}(\eta)/c_0$ for various values of Schmidt number against the normal distance η from the disc taking $N = 10, P_r = 0.07,\ R_e = 1, M = 0.6, a = 1.5,\ \alpha = -0.01$.

to increase the species separation in between the disc and the infinity from the disc but the concentration of the rarer and lighter component of the binary fluid mixture remains constant at the disc as well as at the infinity from the disc.

- there is no separation of species of the lighter and rarer component of the binary mixture far away from the disc.

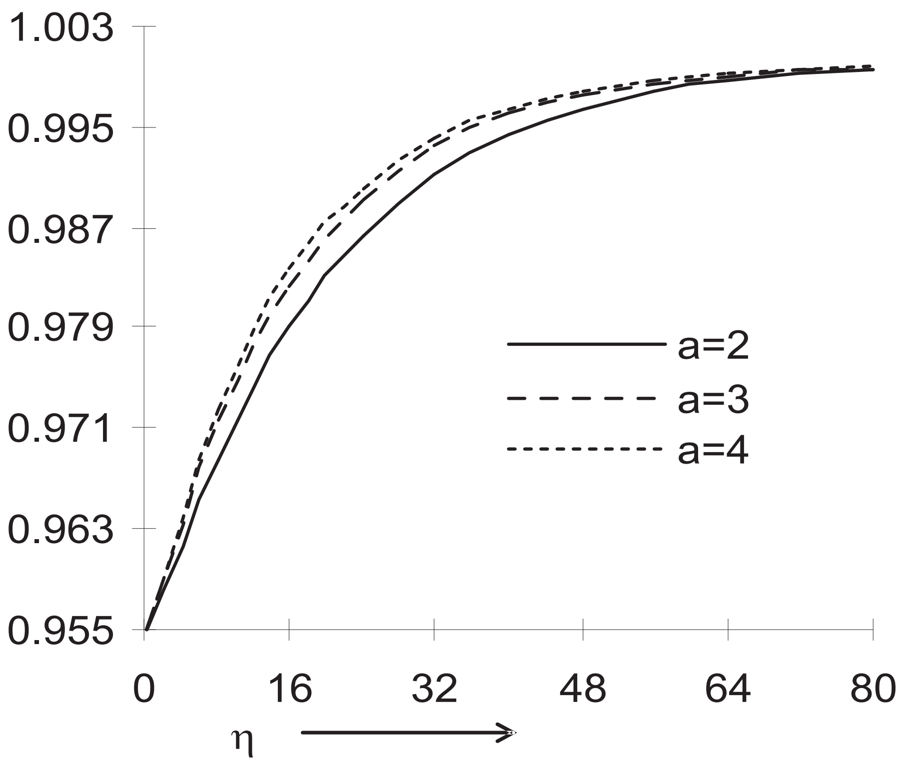

Figure 5: Graph of $\overline{c_1}(\eta)/c_0$ various values of suction parameter against the normal distance η from the disc taking $N = 10, P_r$=0.07, $S_m = 0.224$, $M = 0.6, a = 1.5, \alpha = -0.01$.

Thus the effect of temperature gradient is to separate the components of the binary fluid mixture by throwing the lighter component away from the rotating heated disc and collect the heavier component towards the rotating disc and thus affect the process of separation. The influence of the axial magnetic field is to retard the process of separation. Taking into account the conclusions derived in this paper gas separating instruments can be installed, as an engineering application, in big cities where the harmful gases are present in very small quantities that can be sucked after separating them and thus pollutants can be removed.

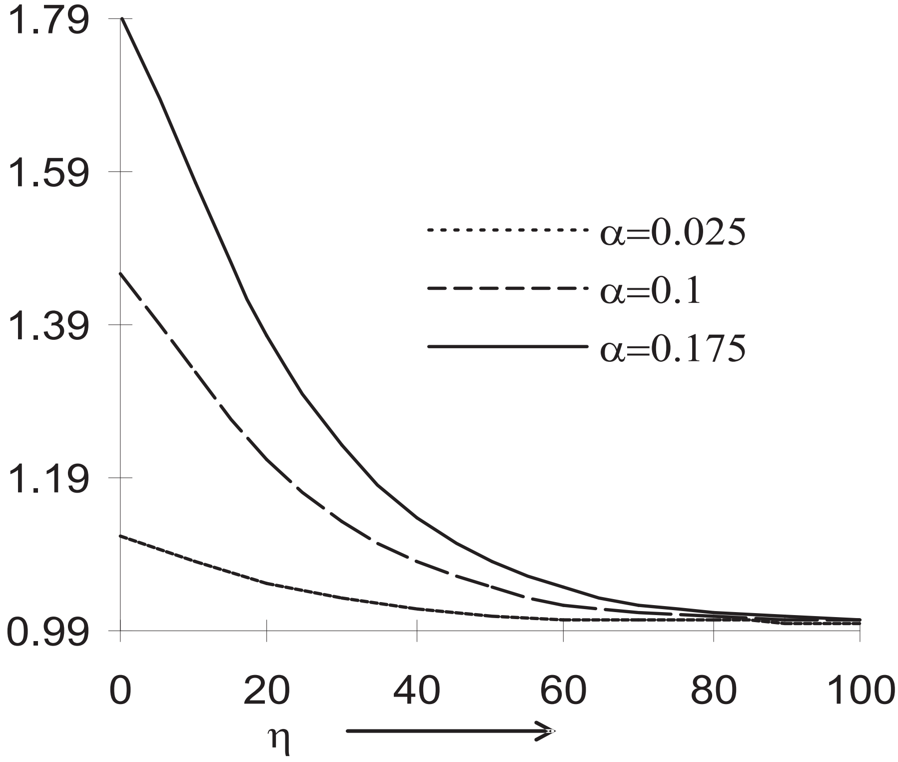

Figure 6: Graph of $\overline{c_1}(\eta)/c_0$ various values of the product of Eckert and thermal diffusion number against the normal distance η from the disc taking $N = 10, P_r = 0.07$, $S_m = 0.224$, $M = 0.6, a = 1.5, R_e = 1$.

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Termalna difuzija u binarnoj mešavini fluida tekućih zbog obrtnog diska sa jakim uniformnim usisavanjem u prisustvu slabog uzdužnog magnetnog polja

Proučava se efekat nekog slabog uniformnog uzdužnog magnetnog polja na razdvajanje binarne mešavine nestišljivih viskoznih termo i elektroprovodnih fluida čije tečenje potiče od obrtnog diska sa inuformnim snažnim usisavanjem. Zanemaruje se indukovano električno polje pa se jednačine koje krakterišu kretanje, temperaturu i koncentraciju rešavaju u cilindričnim polarnim koordinatama razvijanjem parametara tečenja kao i temperature pa i koncentracije u stepene redove po prametru usisavanja. Rešenje dobijeno za raspored koncentracije je grafički prikazano za različita uzdužna rastojanja od diska pri raznim vrenostima bezdimenzionih parametara. Nadjeno je da na razdvajanje vrsta značajno utiču tempearturski gradijent, uzdužno magnetno polje, Reynolds-ov broj, Schmidt-ov broj, Prandtl-ov broj i parametar usisavanja.

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