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Study of flow past an exponentially accelerated isothermal vertical plate in the presence of chemical reaction

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Abstract

Theoretical study of unsteady flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion has been presented in the presence of homogeneous chemical reaction of first order. The plate temperature is raised to T_w and species concentration level near the plate is made to rise linearly with time. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, a and time. It is observed that the velocity increases with increasing values of a or t. But the trend is just reversed with respect to K

Keywords: chemical reaction, accelerated, isothermal, vertical plate, exponential, heat transfer, mass diffusion.

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Nomenclature

C'	concentration $kg.m^{-3}$
C	dimensionless concentration $kg.m^{-3}$
D	mass diffusion coefficient $m^2.s^{-1}$
g	acceleration due to gravity $m.s^{-2}$
Gr	thermal Grashof number
Gc	mass Grashof number
Pr	Prandtl number
k	thermal conductivity of the fluid $W.m^{-1}.K^{-1}$
K_l	chemical reaction parameter
K	dimensionless chemical reaction parameter
Sc	Schmidt number
T	temperature K
t'	time s
t	dimensionless time s
u_0	amplitude of the oscillation
u	velocity component in x-direction $m.s^{-1}$
U	dimensionless velocity component in x-direction $m.s^{-1}$
x	spatial coordinate along the plate
y	spatial coordinate normal to the plate
Y	dimensionless spatial coordinate normal to the plate

*G*reek symbols

α	thermal diffusivity
β	coefficient of volume expansion
β^*	volumetric coefficient of expansion with concentration
η	similarity parameter
μ	coefficient of viscosity $Pa.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid
σ	electric conductivity
θ	dimensionless temperature K

Subscripts

w conditions on the wall
∞ free stream conditions

1 Introduction

The effect of chemical reactions depends whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is either heterogeneous, if it takes place at an interface or homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be of first order, if the rate of reaction is directly proportional to concentration itself.

Chambre and Young[1] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al*[2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction were studied by Das *et al*[3]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level. Gupta[4] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar[5]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [6]. Basant Kumar Jha *et al* [7] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

It is proposed to study the effects of on flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2 Analysis

Here the unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion, in the presence of homogeneous chemical reaction of first order is studied. Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and and the concentration level near the plate is also raised linearly with time. It is also assumed that there exists first order chemical reaction between the fluid and the species concentration. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \, \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \, \frac{\partial^2 C'}{\partial y^2} - K_l \, C' \tag{3}$$

with the following initial and boundary conditions:

$$
\begin{array}{lll}
 & u = 0, & T = T_\infty, \\
t' > 0: & u = u_0 \exp(a't'), & T = T_w, \\
 & u \to 0 & T \to T_\infty,
\end{array}
$$

$$
\begin{array}{lll}
C' = C'_\infty & \text{for all} & y, t' \leq 0 \\
C' = C'_\infty + (C'_w - C'_\infty)\ A\ t' & \text{at} & y = 0 \\
C' \to C'_\infty & \text{as} & y \to \infty
\end{array} \tag{4}
$$

where $A = u_0^2/\nu$.

On introducing the following non-dimensional quantities:

$$
\begin{array}{l}
U = \frac{u}{u_0}, \quad t = \frac{t_0'^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\
Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\
Gc = \frac{\nu g \beta^*(C'_w - C'_\infty)}{u_0^3}, Pr = \frac{\mu C_p}{k}, \\
a = \frac{a'\nu}{u_0^2}, \quad K = \frac{\nu K_l}{u_0^2}, \quad Sc = \frac{\nu}{D}
\end{array} \tag{5}
$$

into equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\ C + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\ \frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\ \frac{\partial^2 C}{\partial Y^2} - K\ C \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$
\begin{array}{llllll}
 & U = 0, & \theta = 0, & C = 0 & \text{for all} & Y, t \leq 0 \\
t > 0: & U = \exp(at), & \theta = 1, & C = t & \text{at} & Y = 0 \\
 & U \to 0, & \theta \to 0, & C \to 0 & \text{as} & Y \to \infty
\end{array} \tag{9}
$$

3 Solution procedure

The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc(\eta\sqrt{Pr}) \tag{10}$$

$$\begin{aligned} C &= \frac{t}{2}\Big[\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \\ &+\exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big] \\ &-\frac{\eta\sqrt{Sct}}{2\sqrt{K}}\Big[\exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\ &-\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})\Big] \end{aligned} \tag{11}$$

$$\begin{aligned} U &= \frac{\exp(at)}{2}\ \big[\exp(2\eta\ \sqrt{at})\ erfc(\eta+\sqrt{at}) \\ &+\exp(-2\eta\sqrt{at})\ erfc(\eta-\sqrt{at})\big] \\ &+2e\ erfc(\eta)+(2ce-d)t\left[(1+2\eta^2)\ erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\ \exp(-\eta^2)\right] \\ &+dt\left[(1+2\ \eta^2\ Pr)\ erfc(\eta\sqrt{Pr})-\ \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\ \exp(-\eta^2 Pr)\right] \\ &-e\exp(ct)\big[\exp(2\eta\sqrt{ct})\ erfc(\eta+\sqrt{ct}) \\ &+\exp(-2\eta\sqrt{ct})\ erfc(\eta-\sqrt{ct})\big] \\ &-e(1+ct)\ \Big[\exp(2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \\ &+\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big] \\ &+\frac{ec\eta\ \sqrt{Sc\ t}}{\sqrt{K}}\Big[\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \end{aligned} \tag{12}$$

$$- \exp(2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} + \sqrt{Kt})\Big]$$
$$+ e\exp(ct)\Big[\exp(2\eta\sqrt{Sc\ (K+c)t})\ erfc(\eta\sqrt{Sc} + \sqrt{(K+c)t})$$
$$+ \exp(-2\eta\sqrt{Sc\ (K+c)t})\ erfc(\eta\sqrt{Sc} - \sqrt{(K+c)t})\Big] \quad \text{(12 contd)}$$

where, $c = \dfrac{KSc}{1-Sc}$, $d = \dfrac{Gr}{1-Pr}$, $e = \dfrac{Gc}{2c^2(1-Sc)}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

4 Results and discussion

In order to get the physical insight into the problem, the numerical computations are carried out for different physical parameters a, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air($Pr = 0.71$). The numerical values of the velocity are computed for different physical parameters like a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of velocity for different time ($t = 0.2, 0.4, 0.6, 0.8$),$K = 5$, $a = 0.5$, $Gr = Gc = 2$ are shown in Figure 1. In this case, the velocity increases gradually with respect to time t. Figure 2 illustrates the effect of velocity for different values of the chemical reaction parameter $(K = 4, 5, 10), a = 0.5, Gr = 5 = Gc$ and $t = 0.2$. The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the relative variation of the velocity with the magnitude of the time and the chemical reaction parameter.

The velocity profiles for different $(a = 0, 0.2, 0.5, 0.8), K = 5, Gr = Gc = 2$ at $t = 0.2$ are studied and presented in figure 3. It is observed that the velocity increases with increasing values of a. Figure 4. demonstrates the effects of different thermal Grashof number $(Gr = 2, 5, 10)$ and mass Grashof number $(Gc = 2)$ on the velocity when $K = 10, a = 0.5$ and $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number.

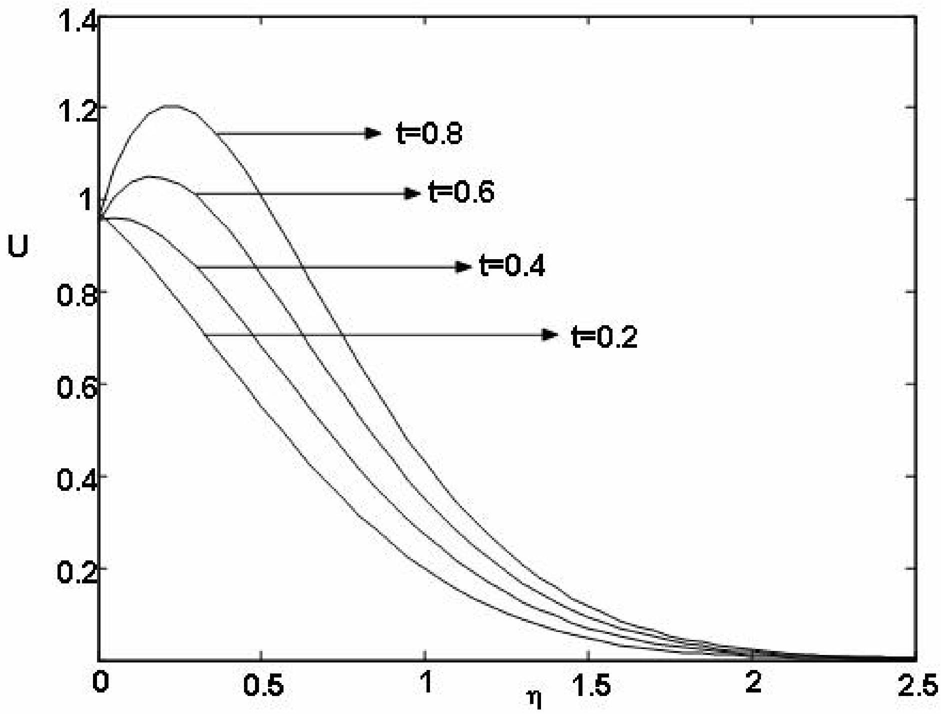

Figure 1. Velocity profiles for different values of t

5 Conclusion

The theoretical solution of flow past an exponentially accelerated infinite isothermal vertical plate in the presence of variable mass diffusion have been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, a and t are studied graphically. It is observed that the velocity increases with

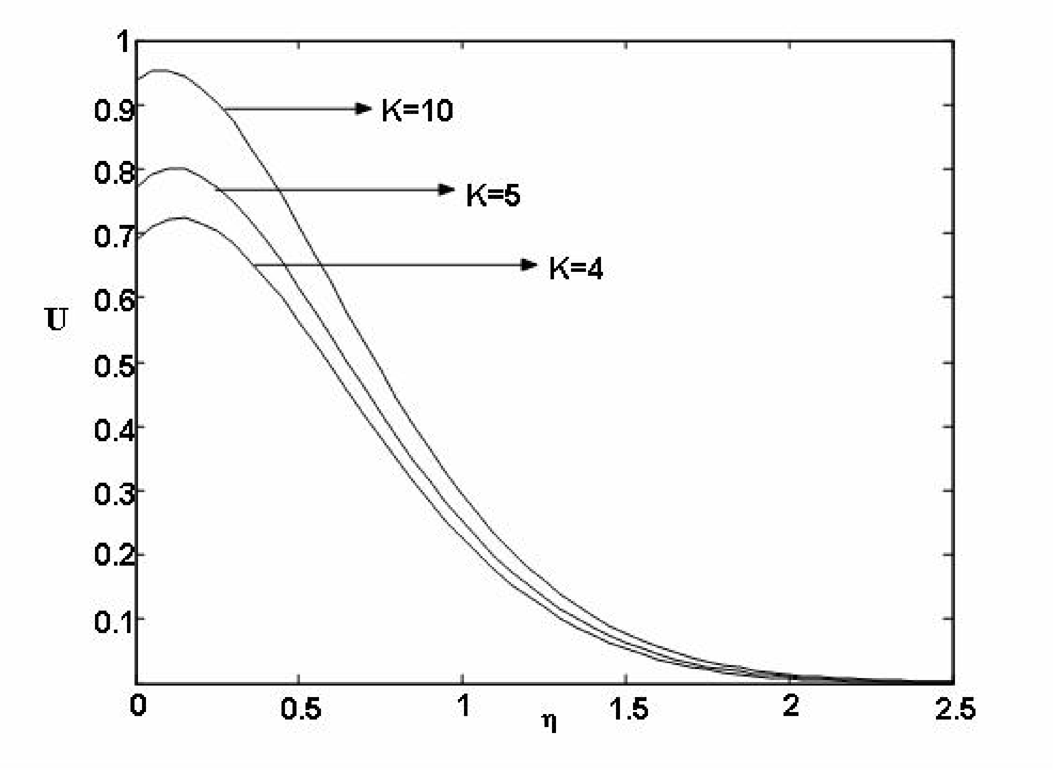

Figure 2. Velocity profiles for different values of K

increasing values of Gr, Gc,a and t. But the trend is just reversed with respect to the Schmidt number

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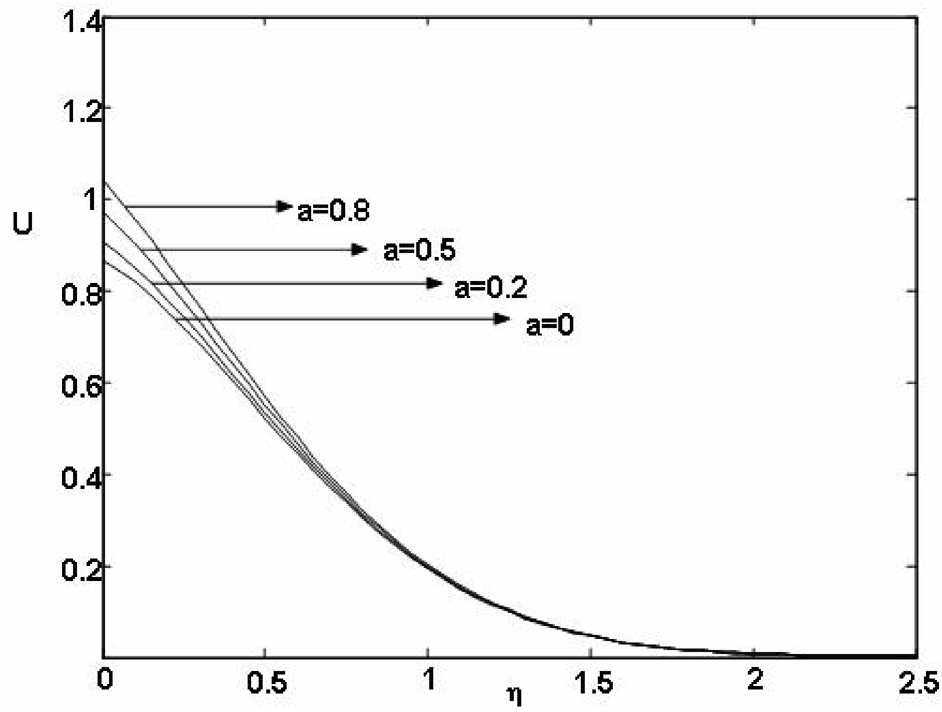

Figure 3. Velocity profiles for different values of a

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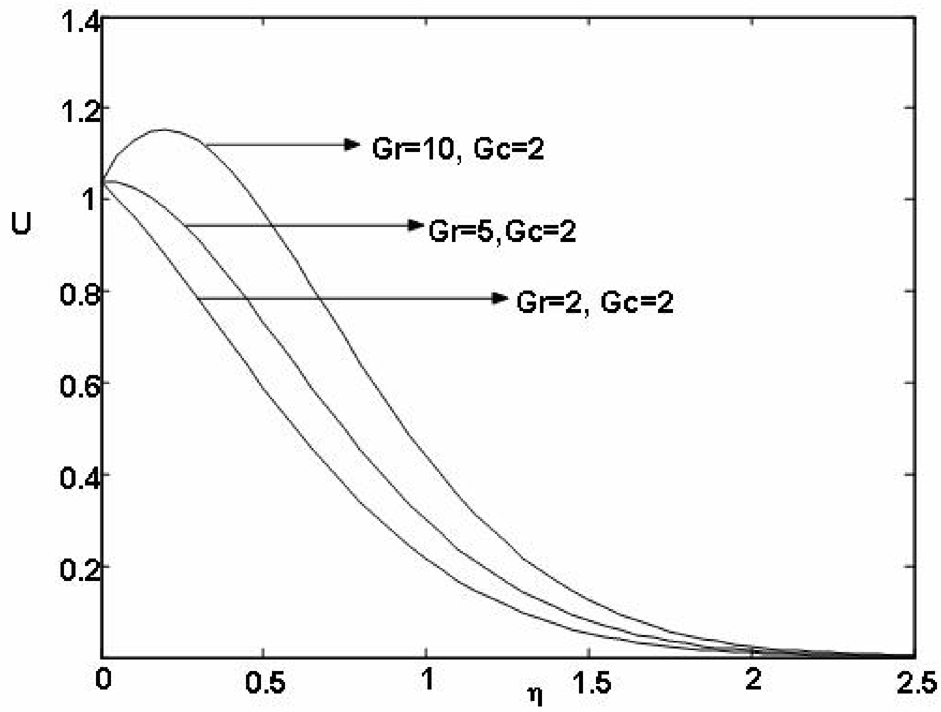

Figure 4. Velocity profiles for different values of Gr,Gc

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Tečenje preko eksponencijalno ubrzane izotermske vertikalne ploče pri hemijskoj reakciji

Prikazano je teorijsko proučavanje nestacionarnog tečenja preko neke eksponencijalno ubrzane izotermske vertikalne ploče sa promenljivom masenom difuzijom pri hemijskoj reakciji prvog reda. Temperatura ploče je podignuta do T_w i nivo koncentracije blizu ploče je propisano linearno rastao sa vremenom. Bezdimenzione jednačine problema su rešene tehnikom Laplasove transformacije. Profili brzine su proučeni za različite fizičke parametre kao što su: hemijski parametar reakcije, termički Grashofov broj, maseni Grashofov broj, a i vreme. Uočeno je da brzina raste pri rastućim vrednostima a i t. Medjutim, trend je obrnut u odnosu na K.

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