Theoret. Appl. Mech., Vol.37, No.4, pp. 263–287, Belgrade 2010

MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation

Hemant Poonia* **R.C.Chaudhary†**

Abstract

An unsteady, two-dimensional, hydromagnetic, laminar mixed convective boundary layer flow of an incompressible and electrically-conducting fluid along an infinite vertical plate embedded in the porous medium with heat and mass transfer is analyzed, by taking into account the effect of viscous dissipation. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. Numerical evaluation of the analytical results is performed and graphical results for velocity, temperature and concentration profiles within the boundary layer are discussed. The results show that increased cooling ($G_r > 0$) of the plate and the Eckert number leads to a rise in the velocity profile. Also, an increase in Eckert number leads to an increase in the temperature. Effects of S_c on velocity and concentration are discussed and shown graphically.

Keywords: MHD, heat and mass transfer, free convection flow, porous medium, viscous dissipation.

*Department of Mathematics, University of Rajasthan, Jaipur-302004, India, e-mail: pooniahemant@yahoo.co.in

†Principal, Yagyavalkya Institute of Technology, Jaipur-302022, India, e-mail: rcchaudhary@rediffmail.com

1 Introduction

Combined buoyancy-generated heat and mass transfer due to temperature and concentration variations, in fluid- saturated porous media, have several important applications in variety of engineering processes including heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, solar energy porous wafer collector systems, ceramic materials, migration of moisture through air contained in fibrous insulations and grain storage installations and the dispersion of chemical contaminants through water-saturated soil, super convecting geothermic etc. The vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer has been investigated by Bejan and Khair [2]. Lai and Kulacki [1] used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous medium due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Soundalgekar [3] analysed the effects of variable suction and the horizontal magnetic field on the free convection flow past infinite vertical porous plate and made a comparative discussion of different parameters and the free convection flow of mercury and ionized air. Many works on heat and mass transfer have focused mainly on regular geometries, the recent studies of them such as heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of the magnetic field studied by Elbashbeshy [4]. Similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption studied by Chamkha and Khaled [5]. Soundalgekar [6] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate with mass transfer. Muthucumaraswamy and

Ganesan [7] have studied numerical solution of flow past an impulsively started semi-infinite isothermal vertical plate with uniform mass diffusion. Das et al. [8] have considered the effects of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Chaudhary et al. [9] & [10] have considered the effect of radiation on MHD heat transfer past vertical plate. A study of Hall effects over the heat and mass transfer flow of visco-elastico fluid is made by Chaudhary et al. [11]. Recently Singh and Kumar [12] have investigated the heat and mass transfer MHD flow through porous medium.

The objective of the present paper is to analyze the heat and mass transfer effects on an unsteady two dimensional laminar mixed convective boundary layer flow of viscous, incompressible, electrically conducting fluid, along a vertical plate with suction, embedded in porous medium, in the presence of transverse magnetic field, by taking into account the effects of the viscous dissipation. The equation of continuity, motion, energy and mass transfer, which govern the flow field are solved by using a regular perturbation method. The behaviour of velocity, temperature, concentration has been discussed for variations in the governing parameters.

2 Mathematical analysis

An unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous incompressible electrically conducting fluid past an infinite vertical flat plate in a uniform porous medium, in the presence of thermal and concentration buoyancy effects has been considered. The x'-axis is taken in the upward direction along the plate and y'-axis normal to it. A uniform magnetic field is applied in the direction perpendicular to plate. Assume the suction velocity to be time dependent. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are:

Equation of Continuity:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Equation of Motion:

$$\upsilon\frac{\partial^2 u'}{\partial y'^2}-v'\frac{\partial u'}{\partial y'}= \frac{\partial u'}{\partial t'}-g\beta\left(T'-T'_\infty\right)-g\beta^*\left(C'-C'_\infty\right)+\frac{\sigma B_0^2 u'}{\rho}+\frac{\upsilon u'}{K'} \quad (2)$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=a\frac{\partial^2 T'}{\partial y'^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \quad (3)$$

Equation of Mass Transfer:

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}, \quad (4)$$

where, $u', v'-$ denote the components of velocity in the boundary layer in x' and y' direction respectively; $T'-$ the temperature in the boundary; $T'_\infty-$ the temperature of the free stream; $t'-$ the time; β and β^*- the volumetric coefficient of thermal and concentration expansion respectively; $\rho-$ the density of the fluid; $\mu-$ the coefficient of viscosity; $g-$ the acceleration due to the gravity; $\upsilon-$ the kinematics viscosity; $\sigma-$ the electrical conductivity; C_p- the heat capacity of the fluid; $a=\frac{\kappa}{\rho C_p}$ (the thermal diffusivity); $\kappa-$ the coefficient of thermal conductivity; B_0- the magnetic induction; $C'-$ the concentration in the boundary layer; $C'_\infty-$ the concentration in the fluid far away from the plate; $D-$ the mass diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} y'=0,\quad u'=0,\ T'=T'_\infty+T_0(t)\left(T'_0-T'_\infty\right),\\ C'=C'_\infty+C_0(t)\left(C'_0-C'_\infty\right)\\ y'\to\infty,\ u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \end{aligned} \quad (5)$$

Non-dimensional quantities are defined as:

$$\begin{gathered}
u=\frac{u'}{\upsilon_0},\ y=\frac{\upsilon_0 y'}{\upsilon},\ t=\frac{\upsilon_0^2 t'}{4\upsilon},\ S_c=\frac{\upsilon}{D},\ K=\frac{K'\upsilon_0^2}{\upsilon^2},\\
w=\frac{4\upsilon w'}{\upsilon_0^2},\ T_0\left(t\right)=1+\varepsilon e^{i\omega t},\ \theta=\frac{T'-T'_\infty}{T'_0-T'_\infty},\\
\varphi=\frac{C'-C'_\infty}{C'_0-C'_\infty},\ M=\frac{\sigma B_0^2\upsilon}{\rho\upsilon_0^2},\ P_r=\frac{\mu C_p}{\kappa},\\
G_r=\frac{g\beta\upsilon\left(T'_0-T'_\infty\right)}{\upsilon_0^3},\ G_c=\frac{g\beta^*\upsilon\left(C'_0-C'_\infty\right)}{\upsilon_0^3}
\end{gathered} \tag{6}$$

From equation of continuity (1), it is clear that the suction velocity normal to the plate is either a constant or a function of the time. Hence, it is assumed in the form

$$\upsilon'=-\upsilon_0\left(1+\varepsilon\alpha e^{i\omega t}\right), \tag{7}$$

where, α is a real positive constant, ε and $\varepsilon\alpha$ are small less than unity and υ_0 is a non-zero positive constant suction velocity, the negative sign indicates that the suction is towards the plate.

In terms of (6), equations (2), (3) and (4) become

$$\frac{\partial^2 u}{\partial y^2}+\left(1+\varepsilon\alpha e^{i\omega t}\right)\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial u}{\partial t}-G_r\theta-G_c\theta+\left(M+\frac{1}{K}\right)u \tag{8}$$

$$\frac{\partial^2 \theta}{\partial y^2}+P_r\left(1+\varepsilon\alpha e^{i\omega t}\right)\frac{\partial \theta}{\partial y}=\frac{1}{4}P_r\frac{\partial \theta}{\partial t}-EP_r\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

$$\frac{\partial^2 \varphi}{\partial y^2}+S_c\left(1+\varepsilon\alpha e^{i\omega t}\right)\frac{\partial \varphi}{\partial y}=\frac{1}{4}S_c\frac{\partial \varphi}{\partial t} \tag{10}$$

and the boundary conditions are

$$\begin{gathered}
y=0,\ u=0,\ \theta=T_0\left(t\right),\ \varphi=C_0\left(t\right)\\
y\to\infty,\ u\to 0,\ \theta\to 0,\ \varphi\to 0,
\end{gathered} \tag{11}$$

where, $T_0(t)-$ the temperature at the wall; $M-$ the Magnetic parameter; P_r- the Prandtl number; $K-$ the Porosity parameter; $G_r-$the thermal Grashof number; G_c- the solutal Grashof number; $E-$the Eckert number; $\omega-$ the frequency of the suction velocity; S_c- the Schmidt number.

3 Solution of the problem

For the solution of equations (8), (9) and (10), we assume

$$\begin{aligned} u(y,t) &= u_1(y) + \varepsilon e^{i\omega t} u_2(y) \\ \theta(y,t) &= 1 + \varepsilon e^{i\omega t} - \theta_1(y) - \varepsilon e^{i\omega t}\theta_2(y) \\ \varphi(y,t) &= 1 + \varepsilon e^{i\omega t} - \varphi_1(y) - \varepsilon e^{i\omega t}\varphi_2(y) \end{aligned} \tag{12}$$

Substituting equation (3) in equations (8), (9) and (10), equating harmonic terms and neglecting coefficient of ε^2, we get

$$u_1''(y) + u_1'(y) - \left(M + \frac{1}{K}\right) u_1(y) = - G_r(1-\theta_1(y)) - G_c(1-\varphi_1(y)) \tag{13}$$

$$u_2''(y) + u_2'(y) - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right) u_2(y) = - G_r(1-\theta_2(y)) - G_c(1-\varphi_2(y)) - \alpha u_1'(y) \tag{14}$$

$$\theta_1''(y) + P_r\theta_1'(y) = EP_r\left(u_1'(y)\right)^2 \tag{15}$$

$$\theta_2''(y) + P_r\theta_2'(y) - \frac{i\omega}{4}P_r\theta_2(y) = - \frac{i\omega}{4}P_r + 2EP_r u_1'(y)\, u_2'(y) - \alpha P_r\theta_1'(y) \tag{16}$$

$$\varphi_1''(y) + S_c\varphi_1'(y) = 0 \tag{17}$$

$$\varphi_2''(y) + S_c\varphi_2'(y) - \frac{i\omega}{4}S_c\varphi_2(y) = -\frac{i\omega}{4}S_c - \alpha S_c\varphi_1'(y)\,, \tag{18}$$

where, primes denote differentiation with respect to y.

The corresponding conditions are

$$\begin{gathered} y = 0, u_1 = 0, u_2 = 0, \theta_1 = 0, \theta_2 = 0, \varphi_1 = 0, \varphi_2 = 0, \\ y \to \infty, u_1 \to 0, u_2 \to 0, \theta_1 \to 1, \theta_2 \to 1, \varphi_1 \to 1, \varphi_2 \to 1 \end{gathered} \tag{19}$$

Solving equations (17) and (18), under the boundary conditions (19), we get

$$\varphi_1 (y) = 1 - e^{-S_c y} \tag{20}$$

$$\varphi_2 (y) = (iI_0 - 1) e^{-\lambda_1 S_c y} + 1 - iI_0 e^{-S_c y}, \tag{21}$$

where $I_0 = \frac{4\alpha S_c}{\omega}$.

The equations (13) to (16) are still coupled and non-linear, whose exact solution are not possible, so we can expand $u_1, u_2, \theta_1, \theta_2$ in terms of E (Eckert no.) in following form, as the Eckert number is very small for incompressible flows.

$$\begin{aligned} u_1 (y) &= u_{11} (y) + E u_{12} (y) \\ u_2 (y) &= u_{21} (y) + E u_{22} (y) \\ \theta_1 (y) &= \theta_{11} (y) + E \theta_{12} (y) \\ \theta_2 (y) &= \theta_{21} (y) + E \theta_{22} (y) \end{aligned} \tag{22}$$

Introducing equations (22) into (13) to (16), we obtain the following systems of equations.

$$\begin{aligned} u''_{11} (y) + u'_{11} (y) - \left(M + \frac{1}{K} \right) u_{11} (y) = \\ - G_r (1 - \theta_{11} (y)) - G_c (1 - \varphi_1 (y)) \end{aligned} \tag{23}$$

$$u''_{12} (y) + u'_{12} (y) - \left(M + \frac{1}{K} \right) u_{12} (y) = G_r \theta_{12} (y) \tag{24}$$

$$\begin{aligned} u''_{21} (y) + u'_{21} (y) - \left(M + \frac{1}{K} + \frac{i\omega}{4} \right) u_{21} (y) = \\ - G_r (1 - \theta_{21} (y)) - G_c (1 - \varphi_2 (y)) - \alpha u'_{11} (y) \end{aligned} \tag{25}$$

$$u''_{22}(y)+u'_{22}(y)-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{22}(y)=G_r\theta_{22}(y)-\alpha u'_{12}(y) \quad (26)$$

$$\theta''_{11}(y)+P_r\theta'_{11}(y)=0 \quad (27)$$

$$\theta''_{12}(y)+P_r\theta'_{12}(y)=P_r\left(u'_{11}(y)\right)^2 \quad (28)$$

$$\theta''_{21}(y)+P_r\theta'_{21}(y)-\frac{i\omega}{4}P_r\theta_{21}(y)=-\frac{i\omega}{4}P_r-\alpha P_r\theta'_{11}(y) \quad (29)$$

$$\begin{aligned}\theta''_{22}(y)+P_r\theta'_{22}(y)-\frac{i\omega}{4}P_r\theta_{22}(y)=&\\ -\alpha P_r\theta'_{12}(y)+2P_ru'_{11}(y)\,u'_{21}(y)&\end{aligned} \quad (30)$$

and the corresponding boundary conditions are

$$\begin{aligned}&y=0, u_{11}=0,\ u_{12}=0,\ u_{21}=0,\ u_{22}=0,\\ &\theta_{11}=0,\ \theta_{12}=0,\ \theta_{21}=0,\ \theta_{22}=0\\ &y\to\infty,\ u_{11}\to0,\ u_{12}\to0,\ u_{21}\to0,\ u_{22}\to0,\\ &\theta_{11}\to1,\ \theta_{12}\to0,\ \theta_{21}\to1,\ \theta_{22}\to0\end{aligned} \quad (31)$$

Solving the equations (23) to (30), under the boundary conditions, we get

$$u_{11}(y)=(I_3+I_4)\,e^{-a_1y}-I_3e^{-P_ry}-I_4e^{-S_cy} \quad (32)$$

$$\begin{aligned}u_{12}(y)=&\ I_{25}e^{-a_1y}+I_{18}e^{-P_ry}+I_{19}e^{-2P_ry}\\ &+I_{20}e^{-2S_cy}+I_{21}e^{-2a_1y}+I_{22}e^{-(P_r+S_c)y}\\ &-I_{23}e^{-(a_1+S_c)y}-I_{24}e^{-(a_1+P_r)y}\end{aligned} \quad (33)$$

$$\begin{aligned}u_{21}(y)=&\ I_{30}e^{-d_1y}-A_{26}e^{-b_1P_ry}-A_{27}e^{-P_ry}\\ &-A_{28}e^{-\lambda_1S_cy}-A_{29}e^{-S_cy}+A_{30}e^{-a_1y}\end{aligned} \quad (34)$$

$$\begin{aligned}u_{22}(y) = {} & I_{56}e^{-d_1 y} + A_9 G_r I_{46} e^{-b_1 P_r y} - A_{84} e^{-(a_1+S_c)y} \\ & - A_{85} e^{-(a_1+P_r)y} + A_{63} e^{-P_r y} + A_{86} e^{-2P_r y} \\ & + A_{87} e^{-2S_c y} + A_{88} e^{-2a_1 y} e^{-2S_c y} + A_{89} e^{-(P_r+S_c)y} \\ & + A_{75} e^{-(d_1+P_r)y} + A_{76} e^{-(d_1+S_c)y} - A_{77} e^{-(a_1+d_1)y} \\ & + A_{78} e^{-(1+b_1)P_r y} + A_{79} e^{-(b_1 P_r+S_c)y} - A_{80} e^{-(a_1+b_1 P_r)y} \\ & + A_{81} e^{-(P_r+\lambda_1 S_c)y} + A_{82} e^{-(1+\lambda_1)P_r y} - A_{83} e^{-(a_1+\lambda_1 S_c)y}\end{aligned} \tag{35}$$

$$\theta_{11}(y) = 1 - e^{-P_r y} \tag{36}$$

$$\begin{aligned}\theta_{12}(y) = {} & I_{11} e^{-P_r y} + I_5 e^{-2P_r y} + I_6 e^{-2S_c y} + I_7 e^{-2a_1 y} \\ & + I_8 e^{-(P_r+S_c)y} - I_9 e^{-(a_1+S_c)y} - I_{10} e^{-(a_1+P_r)y}\end{aligned} \tag{37}$$

$$\theta_{21}(y) = 1 - e^{-b_1 P_r y} + i I_{26}\left(e^{-b_1 P_r y} - e^{-P_r y}\right) \tag{38}$$

$$\begin{aligned}\theta_{22}(y) = {} & I_{46} e^{-b_1 P_r y} - P_r [A_{44} e^{-(a_1+S_c)y} - A_{45} e^{-(a_1+P_r)y} \\ & + A_{33} e^{-P_r y} + A_{46} e^{-2P_r y} + A_{47} e^{-2S_c y} \\ & + A_{48} e^{-2a_1 y} + A_{49} e^{-(P_r+S_c)y} - A_{50} e^{-(d_1+P_r)y} \\ & - A_{51} e^{-(d_1+S_c)y} + A_{52} e^{-(a_1+d_1)y} - A_{53} e^{-(1+b_1)P_r y} \\ & - A_{54} e^{-(b_1 P_r+S_c)y} + A_{55} e^{-(a_1+b_1 P_r)y} - A_{56} e^{-(P_r+\lambda_1 S_c)y} \\ & - A_{57} e^{-(1+\lambda_1)P_r y} + A_{58} e^{-(a_1+\lambda_1 S_c)y}]\end{aligned} \tag{39}$$

The constants are given in Appendix.

4 Results and discussion

The formulation of the problem that accounts for the effect of viscous dissipation on the flow of an incompressible viscous fluid along an infinite vertical flat plate embedded in a porous medium in the presence of an uniform magnetic field was accomplished. The governing equations of the flow field were solved analytically, using a perturbation method, and the expressions for the velocity, temperature, concentration were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters viz. the thermal Grashof number (G_r), the solutal Grashof number (G_c), the Prandtl number (P_r), the Schmidt number (S_c), the Eckert number (E), the Magnetic parameter (M) and the Porosity parameter (K).

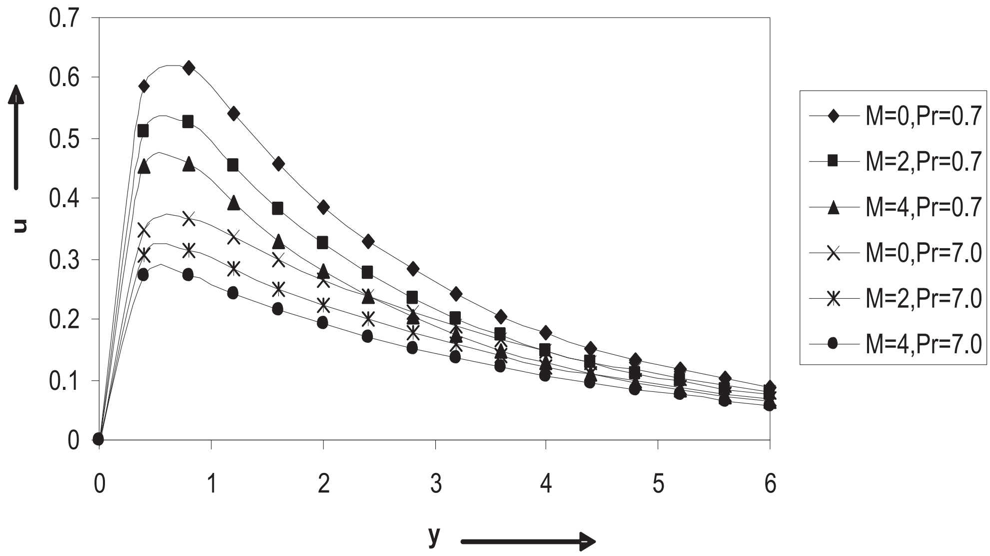

Figure 1: Velocity distribution for various values of M and P_r. ($K = 0.1$, $G_r = 5$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

Fig. 1 shows the typically velocity profiles in the boundary layer for various values of the parameters M and Prandtl number (P_r). Fig. 1 illustrates the influence of magnetic parameter on the velocity u for the case of air ($P_r = 0.7$) and water ($P_r = 7.0$). It is observed that the velocity increases rapidly near to the wall of the porous plate , reaches a maximum and then decays to the free stream value of y. It is concluded that in case of air ($P_r = 0.7$), the fluid velocity u decreases with increasing the Magnetic parameter M and in case of water ($P_r = 7.0$), the fluid velocity decreases with increasing the parameter M as shown in fig. 1. This is because of the fact that the application of the transverse magnetic field to an electrically conducting fluids gives rise to a respective type of force known as Lorentz force. This force has the tendency to slowdown the motion of the fluid in the boundary layer.

Fig. 2 presents the typical velocity profiles in the boundary layer for various values of the thermal Grashof number (G_r). It is observed that an increase in G_r leads to a rise in the values of velocity due to enhancement in buoyancy force. Here, the positive values of G_r correspond to cooling of the plate. In addition, it is observed that the velocity increases sharply near the wall of the porous plate as G_r increases and then decays to the free stream value. For the case of different values of the solutal Grashof number, the velocity profiles in boundary layer are shown in fig. 3. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by G_c.

The effects of the viscous dissipation parameter i.e. the Eckert number on the velocity and temperature are shown in fig. 4 and 5. It is observed that the fluid velocity increases sharply and attains a distinctive maximum value near to the wall of the porous plate and then decays continuously with increasing y as shown in fig. 4. In fig. 5, the temperature decreases exponentially with increasing y. It is also concluded that greater viscous dissipative heat causes a rise in the velocity as well as the temperature.

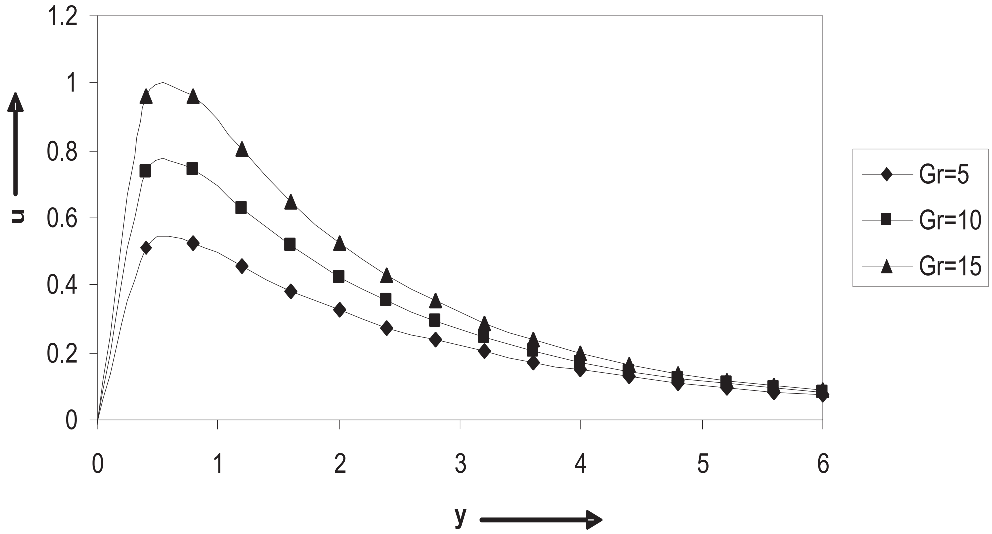

Figure 2: Velocity distribution for various values G_r. ($K = 0.1$, $M = 2$, $P_r = 0.7$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

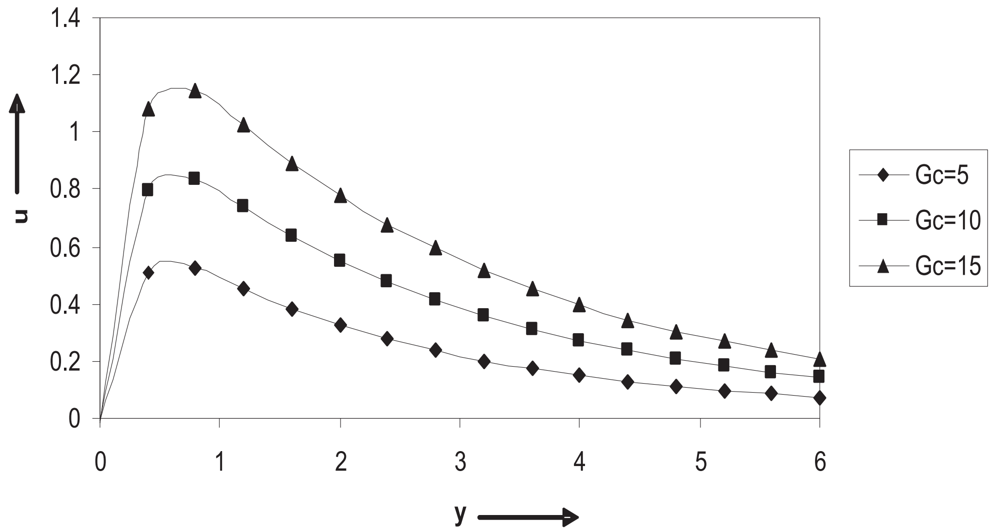

Figure 3: Velocity distribution for various values of G_c. ($K = 0.1, M = 2, P_r = 0.7, G_r = 5, E = 0.001, \omega = 10, \alpha = 0.5, \varepsilon = 0.01, S_c = 0.30$)

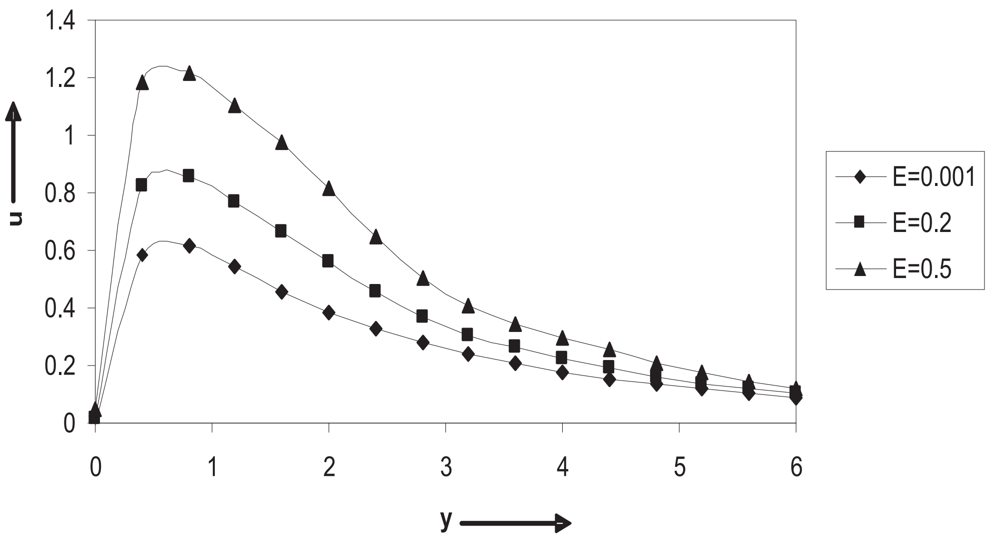

Figure 4: Velocity distribution for various values of E. ($K = 0.1$, $M = 0$, $P_r = 0.7$, $G_r = 5$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

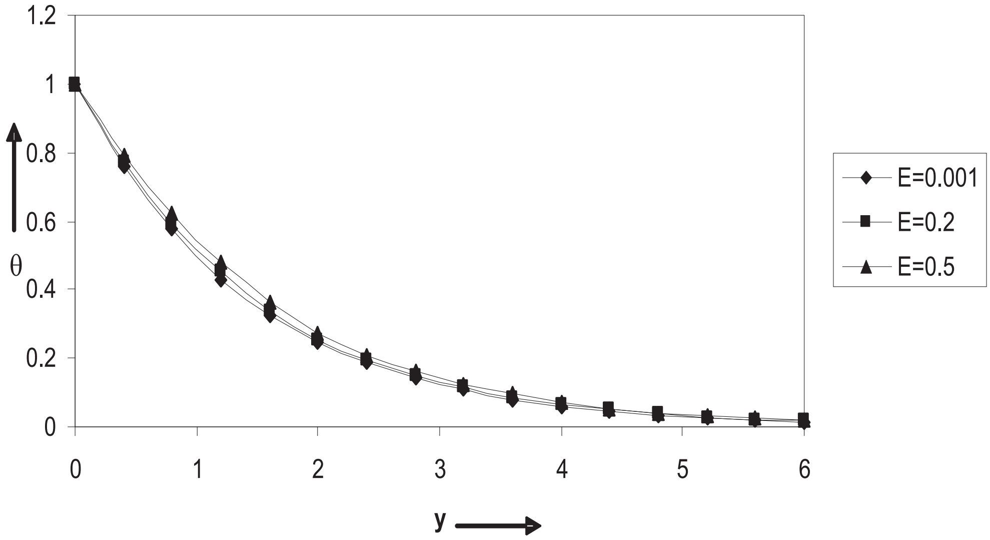

Figure 5: Temperature distribution for various values of E. ($K = 0.1$, $M = 0$, $P_r = 0.7$, $G_r = 5$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

Fig. 6, shows velocity profiles for different values of the porosity parameter K. It is observed that the fluid velocity increases sharply and a peak value near to the plate and decays continuously as increasing y. It is also observed that the fluid velocity increases with increasing the porosity parameter K.

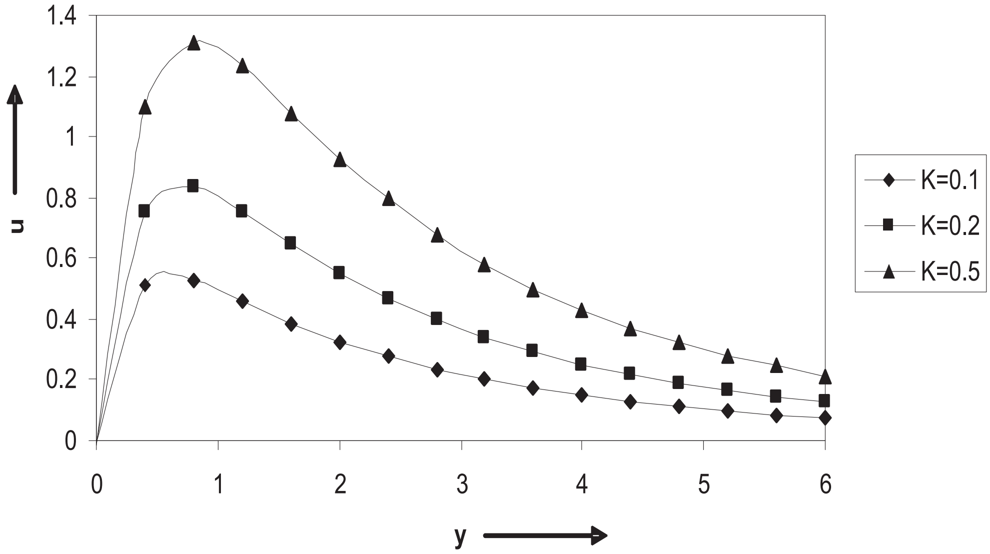

Figure 6: Velocity distribution for various values of K. ($M = 2$, $P_r = 0.7$, $G_r = 5$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

Fig. 7, shows the behaviour of temperature for different values of Prandtl number. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature with in the boundary layer. The reason is that smaller values of P_r are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of P_r. Hence, in the case of smaller Prandtl number as the thermal boundary layer is thicker and the rate of heat transfer is reduced.

Fig. 8 and 9, show the effects of Schmidt number on the velocity and concentration respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy

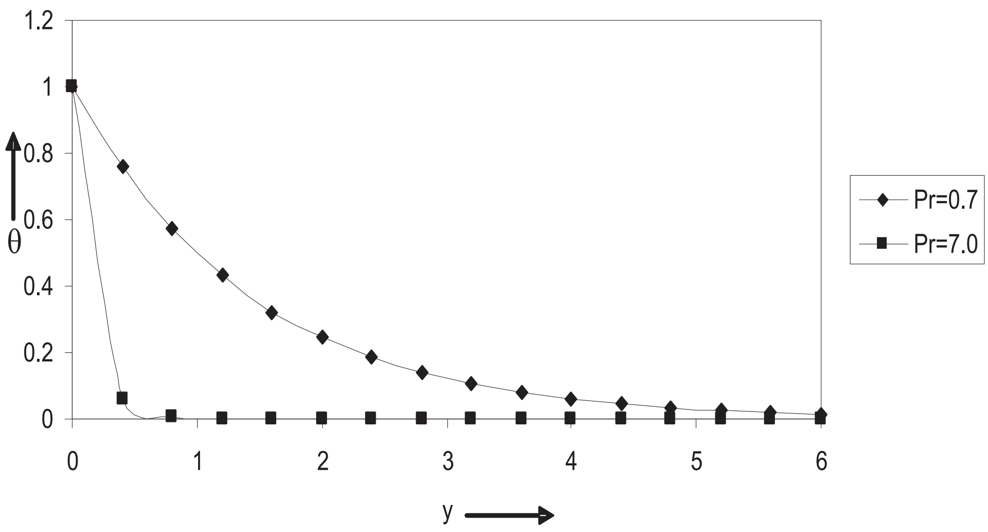

Figure 7: Temperature distribution for various values of P_r. ($K = 0.1$, $M = 2$, $G_r = 5$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$, $S_c = 0.30$)

effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration buoyancy layers.

In Fig. 10 and 11, it is observed that the value of temperature and concentration remain same for increasing the frequency of the suction velocity ω.

5 Conclusion

The governing equations for unsteady MHD convective heat and mass transfer flow past an infinite vertical plate embedded in a porous medium was formulated. Viscous dissipation effects were also included in the present work. The plate velocity is maintained at constant value and the flow was subjected to a transverse magnetic field. The resulting partially differential equations were transformed into a set of ordinary

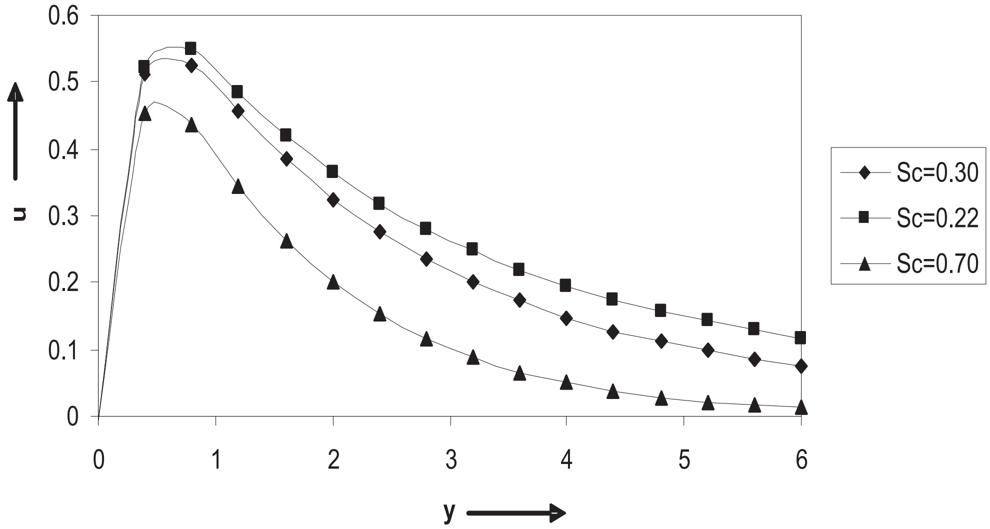

Figure 8: Velocity distribution for various values of S_c. ($K = 0.1$, $M = 0$, $P_r = 0.7$, $G_r = 5$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$)

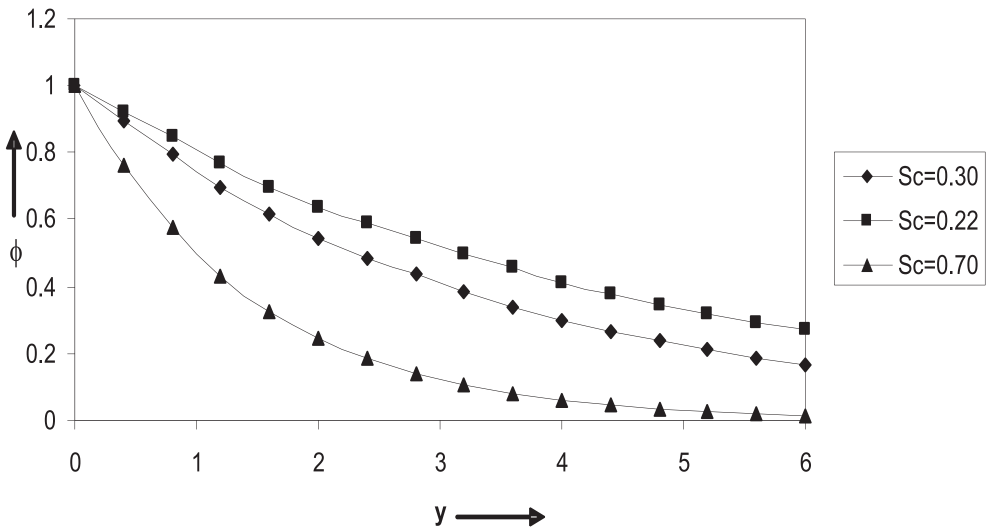

Figure 9: Concentration distribution for various values of S_c. ($K = 0.1$, $M = 0$, $P_r = 0.7$, $G_r = 5$, $E = 0.001$, $\omega = 10$, $\alpha = 0.5$, $\varepsilon = 0.01$, $G_c = 5$)

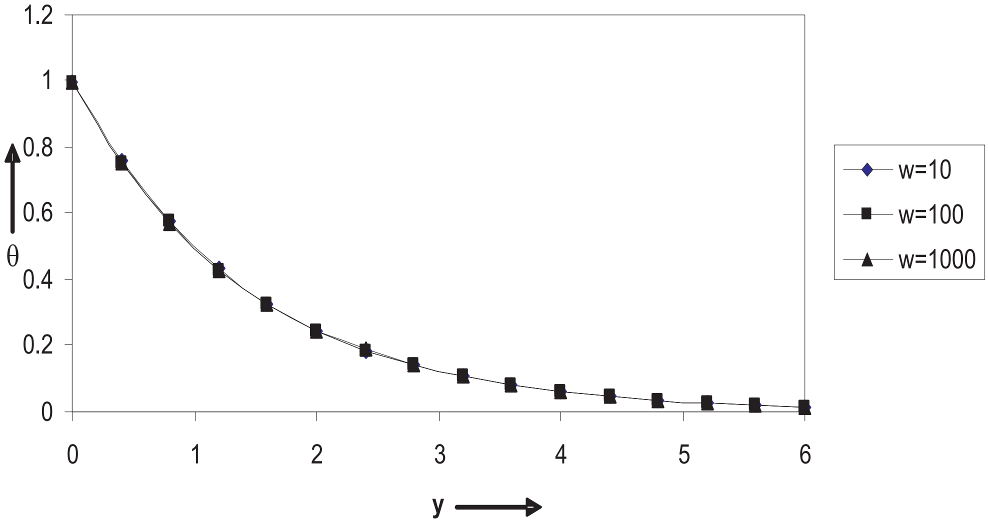

Figure 10: Temperature distribution for various values of ω. ($K = 0.1$, $M = 4$, $P_r = 0.7$, $G_r = 5$, $E = 0.001$, $\alpha = 0.5, \varepsilon = 0.01$, $S_c = 0.30$, $G_c = 5$)

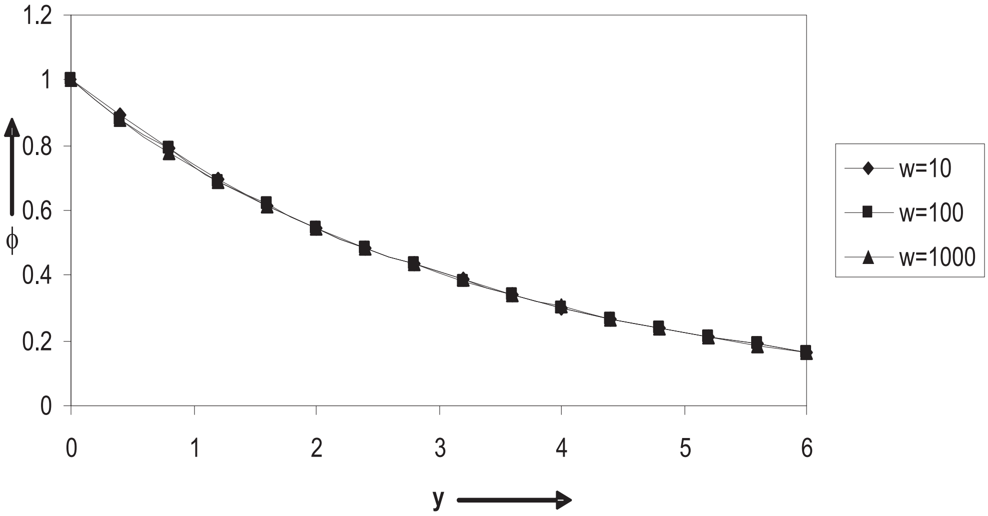

Figure 11: Concentration distribution for various values of ω. ($K = 0.1$, $M = 4$, $P_r = 0.7$, $G_r = 5$, $E = 0.001$, $\alpha = 0.5$, $\varepsilon = 0.01$, $S_c = 0.30$, $G_c = 5$)

differential equations using two- term series and solved in closed form. Numerical evaluations of the closed form results were performed and graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some physical parameters. It was found that when thermal and solutal Grashof numbers were increased, the thermal and concentration buoyancy effects were enhanced and thus, the fluid velocity increased. Also, when the Schmidt number was increased, the concentration level was decreased resulting in decreased fluid velocity. In addition, it found that the temperature as well as velocity increased due to increase in viscous dissipative parameter.

Acknowledgment

One of the author Hemant Poonia is highly indebted to the UGC for awarding a JRF.

References

[1] Lai, F. C. and Kulachi, F. A., The effects of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, Int. J. Heat and Mass Transfer, 33, (1990), 1028-1031.

[2] Bejan, A. and Khair, K. R., Heat and Mass transfer by natural convection in a porous medium, Int. J. Heat and Mass Transfer, 28, (1985), 909-918.

[3] Soundalgekar, V. M., Unsteady MHD free convection flow past an infinite vertical flat plate with variable suction, Indian J. Pure Appl. Math., 3, (1972), 426-436.

[4] Elbashbeshy, E. M. A., Heat and Mass Transfer along a vertical plate with variable surface temperature and concentration in presence of the magnetic field, Int. J. Eng. Sci., 34, (1997), 512-522.

[5] Chamkha, A. J. and Khaled, A. R. A., Similarity solutions for hydro magnetic simultaneous Heat and Mass Transfer by natural convection

from an inclined plate with internal heat generation or absorption, Heat Mass Transfer, 37, (2001), 117-123.

[6] Soundalgekar, V. M., Effects of Mass Transfer and free convection on the flow past an impulsively started vertical plate, ASME J. of Appl. Mechanics, 46, (1979), 757-760.

[7] Muthucumaraswamy, R. and Ganesean, P., Unsteady flow past an impulsively started vertical plate with heat and mass transfer, Forschung im Ingenieurwesen, 66, (2000), 17-23.

[8] Das, U. N.; Deka, R. K. and Soundalgekar, V. M., Effects of Mass Transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen, 60, (1994), 284-287.

[9] Chaudhary, R. C.; Sharma, Bhupendra Kumar and Jha, Abhay kumar, Radiation effect with simultaneous thermal and mass diffusion MHD mixed convection flow from a vertical surface with ohmic heating, Rom. Jorn. Phys., 51, (2006), 715-727.

[10] Jha, A. Kumar, Sharma; Bhupendra Kumar and Chaudhary, R. C., Heat transfer by free convection flow with radiation along a porous hot vertical plate in the presence of transverse magnetic field, South East Asian J. Math. & Math. Sc., 6, (2007), 17-26.

[11] Chaudhary, R. C. and Jha, A. Kumar, Heat and Mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with Hall effect, Latin American Applied Research(Argentina), 38, (2008), 17-26.

[12] Singh, R. D. and Kumar, Rakesh, Heat and Mass transfer in MHD flow of a viscous fluid through porous medium with variable suction and heat source, Proc. Indian Natnl Sci. Acad., 75, (2009), 7-13.

Appendix

$$\lambda_1 = \frac{1}{2}\left(1+\sqrt{1+\frac{i\omega}{S_c}}\right), \quad n = M + \frac{1}{K}, \quad a_1 = \tfrac{1}{2}\left(1+\sqrt{1-4n}\right),$$
$$b_1 = \frac{1}{2}\left(1+\sqrt{1+\frac{i\omega}{P_r}}\right), \quad d_1 = \frac{1}{2}\left(1+\sqrt{1+n+\frac{i\omega}{4}}\right),$$

$$I_0 = \frac{4\alpha S_c}{\omega}, \quad I_1 = P_r^2 - P_r - n, \quad I_2 = S_c^2 - S_c - n, \quad I_3 = \frac{G_r}{I_1},$$

$$I_4 = \frac{G_c}{I_2}, \quad I_5 = \frac{I_3^2 P_r}{2}, \quad I_6 = \frac{I_4^2 S_c P_r}{4S_c - 2P_r}, \quad I_7 = \frac{a_1 P_r \left(I_3 + I_4\right)^2}{4a_1 - 2P_r},$$

$$I_8 = \frac{2I_3 I_4 P_r^2}{P_r + S_c}, \quad I_9 = \frac{2a_1 I_4 S_c P_r \left(I_3 + I_4\right)}{\left(a_1 + S_c\right)^2 - P_r\left(a_1 + S_c\right)}, \quad I_{10} = \frac{2I_3\left(I_3 + I_4\right) P_r^2}{a_1 + P_r},$$

$$I_{11} = I_9 + I_{10} - I_5 - I_6 - I_7 - I_8, \quad I_{12} = 4P_r^2 - 2P_r - n, \quad I_{13} = 4S_c^2 - 2S_c - n,$$

$$I_{14} = 4a_1^2 - 2a_1 - n, \quad I_{15} = \left(P_r + S_c\right)^2 - \left(P_r + S_c\right) - n,$$

$$I_{16} = \left(a_1 + S_c\right)^2 - \left(a_1 + S_c\right) - n, \quad I_{17} = \left(a_1 + P_r\right)^2 - \left(a_1 + P_r\right) - n,$$

$$I_{18} = I_3 I_{11}, \quad I_{19} = \frac{G_r I_5}{I_{12}}, \quad I_{20} = \frac{G_r I_6}{I_{13}},$$

$$I_{21} = \frac{G_r I_7}{I_{14}}, \quad I_{22} = \frac{G_r I_8}{I_{15}}, \quad I_{23} = \frac{G_r I_9}{I_{16}}, \quad I_{24} = \frac{G_r I_{10}}{I_{17}},$$

$$I_{25} = I_{23} + I_{24} - I_{18} - I_{19} - I_{20} - I_{21} - I_{22}, \quad I_{26} = \frac{4\alpha P_r}{\omega},$$

$$I_{27} = b_1^2 P_r^2 - b_1 P_r - n, \quad I_{28} = \lambda_1^2 S_c^2 - \lambda_1 S_c - n,$$

$$I_{29} = a_1^2 - a_1 - n, \quad I_{30} = A_{26} + A_{27} + A_{28} + A_{29} - A_{30},$$

$$I_{31} = \frac{\omega P_r}{4}, \quad I_{32} = \left(a_1 + S_c\right)^2 - P_r\left(a_1 + S_c\right), \quad I_{33} = a_1\left(a_1 + P_r\right),$$

$$I_{34} = 4S_c^2 - 2S_cP_r, \quad I_{35} = 4a_1^2 - 2a_1P_r, \quad I_{36} = S_c\left(S_c + P_r\right),$$

$$I_{37} = d_1\left(d_1 + P_r\right), \quad I_{38} = \left(d_1 + S_c\right)^2 - \left(d_1 + S_c\right)P_r,$$

$$I_{39} = \left(a_1 + d_1\right)^2 - \left(a_1 + d_1\right)P_r, \quad I_{40} = b_1P_r^2\left(1 + b_1\right),$$

$$I_{41} = \left(b_1P_r + S_c\right)^2 - \left(b_1P_r + S_c\right)P_r, \quad I_{42} = \left(a_1 + b_1P_r\right)^2 - \left(a_1 + b_1P_r\right)P_r,$$

$$I_{43} = \lambda_1S_c\left(P_r + \lambda_1S_c\right), \quad I_{44} = \left(1 + \lambda_1\right)^2 S_c^2 - \left(1 + \lambda_1\right)S_cP_r,$$

$$I_{45} = \left(a_1 + \lambda_1S_c\right)^2 - \left(a_1 + \lambda_1S_c\right)P_r,$$

$$I_{47} = \left(d_1 + P_r\right)^2 - \left(d_1 + P_r\right) - \left(n + \frac{i\omega}{4}\right),$$

$$I_{48} = \left(d_1 + S_c\right)^2 - \left(d_1 + S_c\right) - \left(n + \tfrac{i\omega}{4}\right),$$

$$I_{46} = P_r\left(A_{44} - A_{45} + A_{33} + A_{46} + A_{47} + A_{48} + A_{49} - A_{50} - A_{51} \right.$$
$$\left. + A_{52} - A_{53} - A_{54} + A_{55} - A_{56} - A_{57} + A_{58}\right),$$

$$I_{49} = \left(a_1 + d_1\right)^2 - \left(a_1 + d_1\right) - \left(n + \frac{i\omega}{4}\right),$$

$$I_{50} = \left(1 + b_1\right)^2 P_r^2 - \left(1 + b_1\right)P_r - \left(n + \frac{i\omega}{4}\right),$$

$$I_{51} = \left(b_1P_r + S_c\right)^2 - \left(b_1P_r + S_c\right) - \left(n + \frac{i\omega}{4}\right),$$

$$I_{52} = \left(a_1 + b_1P_r\right)^2 - \left(a_1 + b_1P_r\right) - \left(n + \frac{i\omega}{4}\right),$$

$$I_{53} = (P_r + \lambda_1 S_c)^2 - (P_r + \lambda_1 S_c) - \left(n + \frac{i\omega}{4}\right),$$

$$I_{54} = (1 + \lambda_1)^2 S_c^2 - (1 + \lambda_1) S_c - \left(n + \frac{i\omega}{4}\right),$$

$$I_{55} = (a_1 + \lambda_1 S_c)^2 - (a_1 + \lambda_1 S_c) - \left(n + \frac{i\omega}{4}\right), \quad A_0 = \frac{1}{2P_r - \frac{i\omega}{4}},$$

$$A_1 = \frac{1}{I_1 - \frac{i\omega}{4}}, \quad A_2 = \frac{1}{I_2 - \frac{i\omega}{4}},$$

$$I_{56} = -A_9 G_r I_{46} + A_{84} + A_{85} - A_{63} - A_{86} - A_{87} - A_{88} - A_{89} - A_{75} - A_{76} + A_{77} - A_{78} - A_{79} + A_{80} - A_{81} - A_{82} + A_{83},$$

$$A_3 = \frac{1}{I_{12} - \frac{i\omega}{4}}, \quad A_4 = \frac{1}{I_{13} - \frac{i\omega}{4}}, \quad A_5 = \frac{1}{I_{14} - \frac{i\omega}{4}},$$

$$A_6 = \frac{1}{I_{14} - \frac{i\omega}{4}}, \quad A_6 = \frac{1}{I_{15} - \frac{i\omega}{4}}, \quad A_7 = \frac{1}{I_{16} - \frac{i\omega}{4}},$$

$$A_8 = \frac{1}{I_{17} - \frac{i\omega}{4}}, \quad A_9 = \frac{1}{I_{27} - \frac{i\omega}{4}}, \quad A_{10} = \frac{1}{I_{28} - \frac{i\omega}{4}},$$

$$A_{11} = \frac{1}{I_{29} - \frac{i\omega}{4}}, \quad A_{12} = \frac{1}{I_{32} - iI_{31}},$$

$$A_{13} = \frac{1}{I_{33} - iI_{31}}, \quad A_{14} = \frac{1}{I_{34} - iI_{31}}, \quad A_{15} = \frac{1}{I_{35} - iI_{31}},$$

$$A_{16} = \frac{1}{I_{36} - iI_{31}}, \quad A_{17} = \frac{1}{I_{37} - iI_{31}},$$

$$A_{18}=\frac{1}{I_{38}-iI_{31}},\quad A_{19}=\frac{1}{I_{39}-iI_{31}},\quad A_{20}=\frac{1}{I_{40}-iI_{31}},$$

$$A_{21}=\frac{1}{I_{41}-iI_{31}},\quad A_{22}=\frac{1}{I_{42}-iI_{31}},$$

$$A_{23}=\frac{1}{I_{43}-iI_{31}},\quad A_{24}=\frac{1}{I_{44}-iI_{31}},\quad A_{25}=\frac{1}{I_{45}-iI_{31}},$$

$$A_{26}=G_rA_9\left(1-iI_{26}\right),\quad A_{27}=A_1\left(iI_{26}G_r+\alpha I_3P_r\right),$$

$$A_{28}=G_cA_{10}\left(1-iI_0\right),\quad A_{29}=A_2\left(iI_0G_c+\alpha I_4S_c\right),$$

$$A_{30}=\alpha a_1\left(I_3+I_4\right)A_{11},$$

$$A_{31}=\alpha I_9\left(a_1+S_c\right)+2a_1\left(I_3+I_4\right)\left[iI_0G_cS_cA_2+\alpha I_4S_c\left(S_cA_2+a_1A_{11}\right)\right],$$

$$A_{33}=(4\alpha iI_{11})/\omega,\quad A_{34}=2\alpha I_5+2I_3P_rA_{27},$$

$$A_{32}=\alpha I_{10}\left(a_1+P_r\right)+2a_1P_r\left(I_3+I_4\right)\left[iI_{26}G_rA_1+\alpha I_3\left(A_1P_r+a_1A_{11}\right)\right],$$

$$A_{35}=2\alpha S_cI_6+2I_4S_c^2A_{29},\quad A_{36}=2\alpha a_1\left[I_7+a_1^2\left(I_3+I_4\right)^2A_{11}\right],$$

$$A_{37}=\alpha\left(S_c+P_r\right)I_8+2I_4S_cP_rA_{27}+2I_3P_rS_cA_{29},$$

$$A_{38}=a_1\left(I_3+I_4\right),\quad A_{39}=I_3P_r, A_{40}=I_4S_c,\quad A_{41}=-I_{30}d_1,$$

$$A_{42}=b_1P_rG_rA_9\left(1-iI_{26}\right),\quad A_{43}=\lambda_1G_cS_cA_{10}\left(1-iI_0\right),$$

$$A_{44}=A_{12}A_{31},\quad A_{45}=A_{13}A_{32},\quad A_{46}=A_0A_{34},\quad A_{47}=A_{14}A_{35},$$

$$A_{48}=A_{15}A_{36},\quad A_{49}=A_{16}A_{37},\quad A_{50}=2A_{41}A_{17}A_{39},$$

$$A_{51}=2A_{41}A_{18}A_{40},\quad A_{52}=2A_{41}A_{19}A_{38},\quad A_{53}=2A_{42}A_{20}A_{39},$$

$$A_{54}=2A_{42}A_{21}A_{40},\quad A_{55}=2A_{42}A_{22}A_{38},\quad A_{56}=2A_{43}A_{23}A_{39},$$

$$A_{57}=2A_{43}A_{24}A_{40},\quad A_{58}=2A_{43}A_{25}A_{38},$$

$$A_{59} = A_7A_{12}G_rP_r\left[\alpha\left(a_1+S_c\right)I_9+2iA_2A_{38}I_0G_cS_c \right.$$
$$\left. +2\alpha A_{40}A_{38}\left(A_2S_c+a_1A_{11}\right)\right],$$

$$A_{60} = \alpha\left(a_1+S_c\right)A_7I_{23}, \qquad A_{62} = \alpha\left(a_1+P_r\right)A_8I_{24},$$

$$A_{61} = A_8A_{13}G_rP_r\left[\alpha\left(a_1+P_r\right)I_{10}+2iA_1A_{38}I_{26}G_rP_r \right.$$
$$\left. +2\alpha A_{38}A_{39}\left(A_1P_r+a_1A_{11}\right)\right],$$

$$A_{63} = \alpha P_rA_1\left(4iI_{11}G_r+I_{18}\right), \qquad A_{65} = 2\alpha P_rA_3I_{19},$$

$$A_{64} = 2G_rP_rA_0A_3\left[\alpha I_5+A_1I_3P_r\left(iI_{26}G_r+\alpha A_{39}\right)\right],$$

$$A_{66} = 2S_cG_rP_rA_4A_{14}\left[\alpha I_6+A_2A_{40}\left(iI_0G_c+\alpha A_{40}\right)\right],$$

$$A_{67} = 2\alpha S_cA_4I_{20}, \qquad A_{68} = 2\alpha a_1G_rP_rA_5A_{15}\left(I_7+A_{11}A_{38}^2\right),$$

$$A_{70} = G_rP_rA_6A_{16}\left[\alpha\left(P_r+S_c\right)I_8+2A_1A_{40}P_r\left(iI_{26}G_r+\alpha A_{39}\right) \right.$$
$$\left. +2A_2A_{39}S_c\left(iI_0G_c+\alpha A_{40}\right)\right],$$

$$A_{69} = 2\alpha a_1A_5I_{21}, \qquad A_{71} = \alpha\left(P_r+S_c\right)I_{22}A_6, \quad A_{72} = 2A_{41}G_rP_r,$$

$$A_{73} = 2A_{42}G_rP_r, \qquad A_{74} = 2A_{43}G_rP_r, \qquad A_{75} = \frac{A_{17}A_{39}A_{72}}{I_{47}},$$

$$A_{76} = \frac{A_{18}A_{40}A_{72}}{I_{48}}, \qquad A_{77} = \frac{A_{19}A_{38}A_{72}}{I_{49}}, \qquad A_{78} = \frac{A_{40}A_{39}A_{73}}{I_{50}},$$

$$A_{79} = \frac{A_{40}A_{21}A_{73}}{I_{51}}, \qquad A_{80} = \frac{A_{22}A_{38}A_{73}}{I_{52}}, \qquad A_{81} = \frac{A_{23}A_{39}A_{74}}{I_{53}},$$

$$A_{82} = \frac{A_{24}A_{40}A_{74}}{I_{54}}, \qquad A_{83} = \frac{A_{25}A_{38}A_{74}}{I_{55}}, \qquad A_{84} = A_{59}+A_{60},$$

$$A_{85} = A_{61}+A_{62}, \qquad A_{86} = A_{64}+A_{65}, \qquad A_{87} = A_{66}+A_{67},$$

$$A_{88} = A_{68}+A_{69}, \qquad A_{89} = A_{70}+A_{71},$$

Submitted on January 2010

MHD slobodna konvekcija i prenos mase preko beskonačne vertikalne porozne ploče sa viskoznom disipacijom

Analizirano je 2D nestacionarno hidromagnetsko laminarno konvektivno tečenje u graničnom sloju nestišljivog elektroprovodnog fluida duž beskonačne vertikalne ploče potopljene u poroznu sredinu sa prenosom toplote i mase. Uzima se u obzir viskozna disipacija. Bezdimenzione jednačine problema su analitički rešene korišénjem dvočlanih harmonijskih i neharmonijskih funkcija. Numerička procena analitičkih rezultata je izvedena i diskutovani su grafički rezultati za profile brzine, temperature i koncentracije unutar graničnog sloja. Rezultati pokazuju da povećanje hladjenja ($G_r > 0$) ploče i Ekertovog broja vodi ka porastu u profilu brzine. Takodje, porast Ekertovog broja vodi ka porastu u temperaturi. Uticaji S_c na brzinu i koncentraciju su diskutovani i prikazani grafički.

doi:10.2298/TAM1004263P Math.Subj.Class.: 76E25, 76N20, 76W99