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Size effects for micro-fractured bodies under compressive loading

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Abstract

A two-scale damage model for micro-fractured media is constructed using the asymptotic homogenization method. At the small-scale level, we consider locally periodic microstructures of two-types: micro-cracks nucleating from pores and wing-type micro-cracks.

Based on an energy criterion for micro-crack propagation we deduce the macroscopic damage model, without supplementary assumptions on the overall behavior. We show that the resulting two-scale model has the property of capturing a micro-structural length - the distance between neighbor micro-cracks. The influence of the micro-structural parameters on the effective behavior is studied. We illustrate the capacity of the model to predict size effects under compression loadings.

Keywords: micro-cracks, wing-type, pores, homogenization, asymptotic developments, damage, size effects, compression loading

1 Introduction

Experimental observations show that, in brittle specimens under uniaxial compression, macroscopic cracks nucleate and grow in the direction parallel to that of the axial loading. At the origin of such macroscopic crack formation are small-scale heterogeneities, like wing cracks or pore-like flaws. Under compression loading, such micro-heterogeneities lead to tensile micro-crack formation, growth and coalescence to macroscopic cracks.

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Since the origin of this particular type of failure is a micro-mechanical one, the proper way to describe these phenomena is a multi-scale approach. Our aim, in this paper, is to construct a damage model for compressive loadings.

Previously, Dascalu and co-workers ([6], [7], [8], [10]) proposed a damage model for straight micro-cracks based on a change of scale linking the microscopic energy dissipated by the micro-fracture and the macroscopic energy release rate. A material length characterizing the size of the micro-structure was present in the deduced damage equations, therefore the model was able to describe size effects.

In this paper we first propose a model which considers the complex case of small-scale geometry with cracks propagating from pores and evolving symmetrically with respect to them in the direction of the loading. The damage evolution laws are deduced based on the method developed in [6], [7], [8], [10] and taking into account the porous microstructure ([1], [17]). The pore size is considered as a parameter of the model and the damage variable is defined as the normalized length of the flaw composed by the pore and the two symmetric micro-cracks connected with it. The influence of the porosity and the microstructural size of the material on the macroscopic response is emphasized.

The second part of the paper is devoted to the case of wing-type microcracks. Several models have been developed to describe the mechanism leading to the wing-crack propagation. The first characterization of this mechanism is found in the work of Brace and Bombolakis ([5]). Since then, many researchers studied and modeled wing cracks (Bobet [4], Hoek and Bienawski [11], Horii and Nemat-Nasser [13], [12], Fanella and Krajcinovic [9], ...). Some researchers investigated the wing-type crack and the brittle failure in solids using the continuum theory, also called interaction field theory, for short- and long-term behavior of hard rock under compression (e.g. Miura et al. [18] who considered the mechanisms of crack growth given by the interaction field theory for predicting the creep failure of rocks in compression).

More recently, the previous models were extended to take into account dynamics effects. Nemat-Nasser and Deng [19] considered an array of interacting and dynamically growing wing crack to estimate the rate-dependent dynamic damage evolution in brittle solid. The effect of strain rate is included through the dependence of dynamic stress intensity factor on the speed of the crack growth. Huang et al.[14], [15] proposed an approach that combines damage theory with dynamic growth of the wing cracks, in order to model the dynamic fracture process of rock specimens subjected to high strain rate uniaxial compressive loading. Their model assumed dilute pre-existing crack distributions with no interaction, but Paliwal et al. [20] tried to overcome this inconvenient and developed a methodology based on a complex variable approach to obtain an approximate local effective stress field as a manifestation of micro-crack interactions.

The model presented in this paper is obtained by considering a locally periodic distribution of wing-type micro-cracks and by the use of the asymptotic developments based homogenization method. We start from the model developed by Fanella and Krajcinovic [9] which is used at the micro-scale, within the framework of homogenization as proposed in Dascalu and co-workers ([10], [8], [6], [7]). We will show that the obtained damage model is able to predict size effects related to failure.

2 Cracks emerging from pores

Consider a two-dimensional isotropic elastic medium containing a large number of small pores and micro-cracks developed from pores. The distribution is assumed to be locally periodic, so that one can locally find a periodicity cell, of length ε, containing one pore with two symmetric cracks (see Fig.1). The length ε, also representing the mutual distance between centers of neighbor pores, is a characteristic size of the micro-structure. The two cracks are assumed to be straight and of total length $d^{\varepsilon} - \phi$, where ϕ is the diameter of the pore.

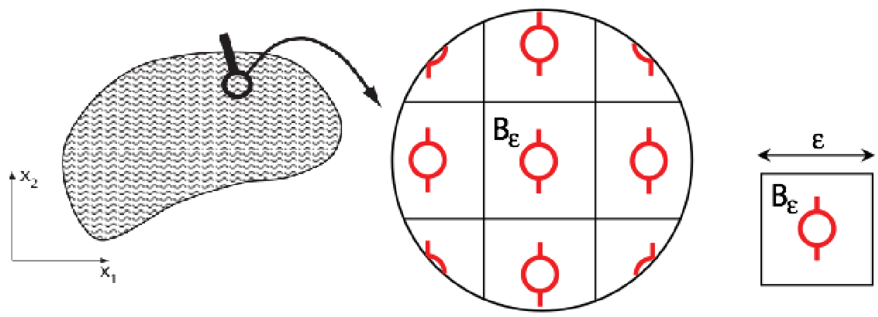

Figure 1: Fissured porous medium with locally periodic micro-structure.

We consider the initial heterogeneous porous medium represented by a bounded two-dimensional domain $\mathcal{B}$ with a smooth external boundary. In the solid part we have the equilibrium equations

$$\frac{\partial \sigma_{ij}^{\varepsilon}}{\partial x_j} = 0, \qquad \sigma_{ij}^{\varepsilon} = a_{ijkl} e_{xkl}(\mathbf{u}^{\varepsilon}), \tag{1}$$

where $\mathbf{u}^{\varepsilon}$ and $\boldsymbol{\sigma}^{\varepsilon}$ are the displacement and the stress fields, a_{ijkl} are the elasticity coefficients and where we denoted the strain tensor $e_{xij}(\mathbf{u}^{\varepsilon}) = \frac{1}{2}(\frac{\partial u_i^{\varepsilon}}{\partial x_j} + \frac{\partial u_j^{\varepsilon}}{\partial x_i})$ with respect to x coordinates. We assume that the boundaries of the cracks and of the pores are traction-free :

$$\boldsymbol{\sigma}^{\varepsilon} \mathbf{n} = 0, \tag{2}$$

where $\mathbf{n}$ is the unit normal vector.

2.1 Homogenization by asymptotic developments

Local periodicity is assumed, that is around each point one can find a small neighborhood in which the micro-structure is periodically distributed, with periods of size ε (see Fig.1). Such a distribution can be reproduced from the unit cell $Y = [0,1] \times [0,1]$ by rescaling with the small parameter ε so that the period of the material is εY, as in Fig. 2. The parameter ε, which is assumed to be small enough with respect to the characteristic dimensions of the whole body, is the microscopic length scale. This condition allow us to distinguish between microscopic and macroscopic variations. The two distinct scales are represented by the variables $\mathbf{x}$, which are referred to as *macroscopic variables* and the variables $\mathbf{y} = \mathbf{x}/\varepsilon$, referred to as *microscopic variables*.

In the unit cell Y, we denote the union of the two cracks by CY, the pore boundary by CP and the solid part by Y_s. We introduce the damage parameter $d = d^{\varepsilon}/\varepsilon$, representing the scaled distance between the two crack tips in the cell, and the scaled diameter of the pore $a = \phi/\varepsilon$. For a given pore diameter a, the time evolution of the damage variable d describes the symmetric micro-crack propagation. This evolution will make the object of the next section, here we consider only a spatial distribution $d = d(\mathbf{x}, t)$ of "frozen" micro-crack lengths, at a given instant of time t.

According to the method of asymptotic homogenization (e.g. [3],[21]), we look for expansions of $\mathbf{u}^{\varepsilon}$ and $\boldsymbol{\sigma}^{\varepsilon}$ in the form given by (3) and (4):

$$\mathbf{u}^{\varepsilon}(\mathbf{x},t) = \mathbf{u}^{(0)}(\mathbf{x},\mathbf{y},t) + \varepsilon \mathbf{u}^{(1)}(\mathbf{x},\mathbf{y},t) + \varepsilon^2 \mathbf{u}^{(2)}(\mathbf{x},\mathbf{y},t) + \varepsilon^3 \mathbf{u}^{(3)}(\mathbf{x},\mathbf{y},t) + \ldots \tag{3}$$

$$\boldsymbol{\sigma}^{\varepsilon}(\mathbf{x},t) = \frac{1}{\varepsilon}\boldsymbol{\sigma}^{(-1)}(\mathbf{x},\mathbf{y},t) + \boldsymbol{\sigma}^{(0)}(\mathbf{x},\mathbf{y},t) + \varepsilon \boldsymbol{\sigma}^{(1)}(\mathbf{x},\mathbf{y},t) \ldots \tag{4}$$

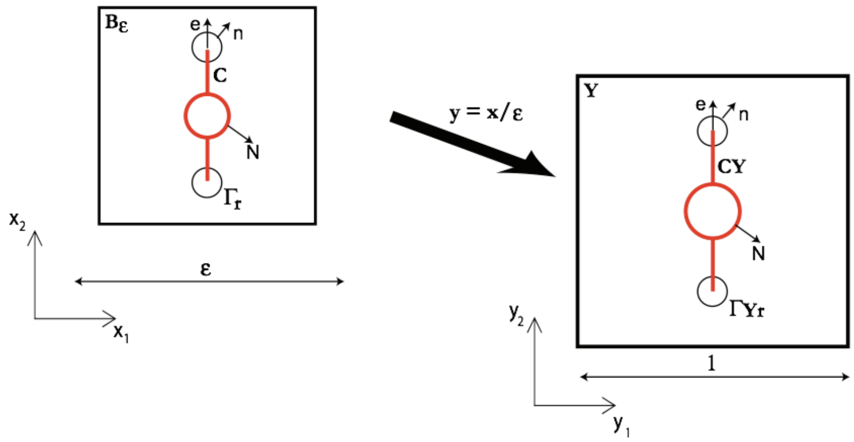

Figure 2: Material period and the unit cell.

where $\mathbf{u}^{(i)}(\mathbf{x},\mathbf{y},t)$, $\sigma^{(i)}(\mathbf{x},\mathbf{y},t)$, $\mathbf{x}\in\mathcal{B}$, $\mathbf{y}\in Y$ are smooth functions and Y-periodic in $\mathbf{y}$.

Substituting the expansions into Eq. (1) and the boundary conditions (2) we obtain boundary value problems for the different orders of ε, formulated on the unit cell Y. It can be shown (e.g. [21]) that the function $\mathbf{u}^{(0)}=\mathbf{u}^{(0)}(\mathbf{x},t)$ is independent of $\mathbf{y}$ variable, representing the *macroscopic displacement* field.

For given $e_x(\mathbf{u}^{(0)})$ in the case of open traction-free cracks, we deduce the following boundary-value problem for the displacement field $\mathbf{u}^{(1)}$:

$$\frac{\partial}{\partial y_j}\left(a_{ijkl}e_{ykl}(\mathbf{u}^{(1)})\right)=0, \qquad \text{in } \ Y_s \tag{5}$$

$$a_{ijkl}e_{ykl}(\mathbf{u}^{(1)})n_j=-a_{ijkl}e_{xkl}(\mathbf{u}^{(0)})n_j, \qquad \text{on } CY^{\pm}\cup CP \tag{6}$$

and with periodicity boundary conditions on the external boundary of the cell. In the last relation $CY^{\pm}$ denote the two faces of the micro-cracks.

The microscopic correction $\mathbf{u}^{(1)}$ has a linear dependence of the *macroscopic deformations* $e_{xpq}(\mathbf{u}^{(0)})$:

$$\mathbf{u}^{(1)}=\xi^{11}e_{x11}(\mathbf{u}^{(0)})+2\xi^{12}e_{x12}(\mathbf{u}^{(0)})-\xi^{22}e_{x22}(\mathbf{u}^{(0)}) \tag{7}$$

The characteristic functions $\boldsymbol{\xi}^{pq}(\mathbf{y},d,a)$ are elementary solutions of (5-6), for a given length of the crack, for a given size of the pore and for particular macroscopic deformations having the only non-vanishing component $e_{x11} = 1$ or $e_{x12} = 1$ or $e_{x22} = -1$, respectively. Remark that $\boldsymbol{\xi}^{22}(\mathbf{y}, d, a)$ corresponds to a compressive macroscopic deformation applied to the unit cell through the internal boundary conditions (6).

Consider the mean value $\langle\cdot\rangle = \frac{1}{|Y|}\int_{Y_s} \cdot \, dy$, where $|Y|$ is the measure of Y. By applying the mean value operator to the boundary value problem corresponding to the 1^{st}-order of ε, we can deduce (e.g. [21]) the homogenized equilibrium equation

$$\frac{\partial}{\partial x_j}\Sigma_{ij}^{(0)} = 0, \tag{8}$$

where $\Sigma_{ij}^{(0)} = \langle\sigma_{ij}^{(0)}\rangle = \langle a_{ijkl}(e_{xkl}(\mathbf{u}^{(0)}) + e_{ykl}(\mathbf{u}^{(1)}))\rangle$ is the *macroscopic stress.*

The effective elastic law is obtained as

$$\Sigma_{ij}^{(0)} = C_{ijkl}(d, a)e_{xkl}(\mathbf{u}^{(0)}), \tag{9}$$

where $C_{ijkl}(d, a)$ are the homogenized coefficients given generally by the formula

$$C_{ijkl}(d, a) = \frac{1}{|Y|}\int_{Y_s}(a_{ijkl} + a_{ijmn}e_{ymn}(\xi^{kl}))\ dy \tag{10}$$

except for the coefficients calculated computed with $\boldsymbol{\xi}^{22}$ which are given by : $C_{1122} =< a_{1122} - a_{1111}e_{y11}(\xi^{22}) - a_{1122}e_{y22}(\xi^{22}) >$ and $C_{2222} =< a_{2222} - a_{1111}e_{y11}(\xi^{22}) - a_{1122}e_{y22}(\xi^{22}) >$.

These formulae allow for the computation of the homogenized coefficients as functions of the damage variable.

2.2 The damage law

For the modeling of the evolution of damage we adopt a quasi-static description, in which the previous equilibrium problem should be completed with damage evolution equations. In this section we remind the main steps to be followed in obtaining the damage equation through the homogenization of the microscopic balance of energy for propagating micro-cracks. For the details of the procedure the reader is reffered to [6], [7].

For the initial heterogeneous problem, the *fracture energy release rate* during crack extension can be expressed as

$$\mathcal{G}_\varepsilon = \lim_{D_\epsilon \to O}\int_{\partial D_\varepsilon} \mathbf{e}\cdot\mathbf{b}(\mathbf{u}^\varepsilon)\mathbf{n}\ ds \tag{11}$$

where D_ε is a disk of infinitesimal radius, surrounding the crack tip O, with $\mathbf{n}$ the outward normal to the disk D_ϵ, $\mathbf{e}$ is the unit vector in the propagation direction (see Fig. 2) and $b_{ij}(\mathbf{u}^\varepsilon) = \frac{1}{2}a_{mnkl}e_{xkl}(\mathbf{u}^\varepsilon)e_{xmn}(\mathbf{u}^\varepsilon)\delta_{ij} - \sigma^\varepsilon_{jk}u^\varepsilon_{k,i}$ is the Eshelby configurational stress tensor.

The propagation of each micro-crack in the elastic body is governed by the following laws:

$$\mathcal{G}_\varepsilon \le \mathcal{G}_f \ ; \ \dot{d^\varepsilon} \ge 0 \ ; \ \dot{d^\varepsilon}(\mathcal{G}_\varepsilon - \mathcal{G}_f) = 0 \tag{12}$$

where a superimposed dot denotes time derivative and $\mathcal{G}_f$ is the critical fracture energy of the material. These relations should be completed with the reduced dissipation inequality:

$$\mathcal{D}_f \equiv \mathcal{G}_\varepsilon \dot{d^\varepsilon} \ge 0 \tag{13}$$

Assuming the symmetric extension of micro-cracks, from (5)-(6) we deduce ([6], [7]) for $\dot{d} \neq 0$, the global balance of energy on the unit cell :

$$\frac{\mathcal{G}_\varepsilon}{\varepsilon} = -\frac{1}{2}\frac{dC_{ijkl}(d,a)}{dd}e_{xkl}(\mathbf{u}^{(0)})e_{xij}(\mathbf{u}^{(0)}) \tag{14}$$

where the right member $Y_d \equiv -\frac{1}{2}\frac{dC_{ijkl}(d,a)}{dd}e_{xkl}(\mathbf{u}^{(0)})e_{xij}(\mathbf{u}^{(0)})$ is the *damage energy release rate*. We note that this relation is entirely deduced from micro-structural assumptions, without any assumptions on the scaling of energy. This scaling with ε is naturally appearing in the derivation of the damage equation (14). For evolving damage, the previous relation shows that the micro-structural length ε makes the link between the surface energy dissipated during micro-crack propagation and damage energy dissipated per unit volume. This energy scaling property will assure the presence of the internal length ε in the damage law.

Using (14) from the micro-crack evolution laws (12) we deduce the *damage laws :*

$$Y_d \le \frac{\mathcal{G}_f}{\varepsilon} \ ; \ \dot{d} \ge 0 \ ; \ \dot{d}(Y_d - \frac{\mathcal{G}_f}{\varepsilon}) = 0 \tag{15}$$

$$\mathcal{D}_d \equiv Y_d \dot{d} \ge 0 \tag{16}$$

These relations are coupled with the equilibrium equation (8). For brittle damage, $\mathcal{G}_f$ is a constant. Generally, it may depend on the crack length d and its velocity $\dot{d}$.

2.3 Numerical implementation - size effects

In this section we give numerical results for the case of cracks emerging from pores. We consider that the elastic matrix is isotropic, of Young's modulus

$E = 2\ GPa$ and Poisson's ratio $\nu = 0.1$. The normalized pore diameter is taken $a = 0.2$. The fracture energy was taken $\mathcal{G}_f = 20J/m^2$.

In section 2.1 formulae for the computation of the homogenized coefficients starting from the elementary deformation modes were deduced. For the numerical implementation of the effective coefficients we used the finite element program FEAP, developed by Berkeley University ([22]). Triangular finite elements with three Gauss points for the displacements were used. Computation technique has two steps: first, one needs to compute finite element solutions for the characteristic functions on unit cells containing micro-cracks of different lengths; then polynomial interpolation to construct the functions $C_{ijkl}(d, a)$ of the variable d is used.

In Fig. 3 we represented the homogenized coefficients vs. the damage variable d. We note that the presence of micro-cracks induces an anisotropic effective response and that the homogenized coefficients depend nonlinearly on the damage variable.

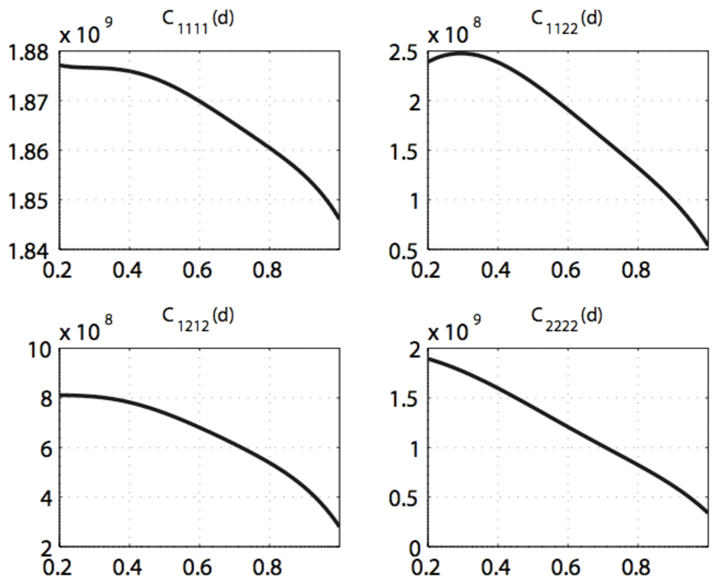

Figure 3: Homogenized coefficients vs. damage variable d.

Since the damage evolution law was obtained from a brittle micro-fracture criterion, our model predicts brittle damage. For an increasing vertical compressive loading, starting from an undamaged state $d = a$, the macroscopic stress do not induce damage until it reaches a critical value Σ_{22} for which the complete failure of the cell occurs in a brittle way. In Fig. 4, we plot the critical failure stress as a function of the micro-structural size ε. The two curves correspond to two different normalized pore sizes $a = 0.2$ and $a = 0.3$. We remark that the failure compressive stress increases for smaller inter-distances between pores, for proportional pore sizes, and for smaller pore diameters when the mutual distance between centers is fixed. These results clearly show the influence of the micro-structural parameters: the distance between centers of two neighbor pores ε and the pore size $a \cdot \varepsilon$ on the effective elasto-damage response. In this way, the obtained damage model is able to predict size effects.

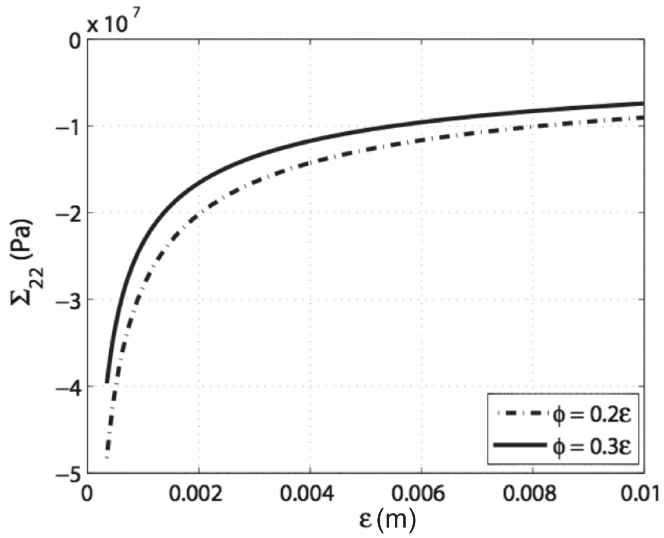

Figure 4: Size effects: critical failure stress Σ_{22} vs. microscopic size ε, for pore diameters $0.2 \cdot \varepsilon$ and $0.3 \cdot \varepsilon$

3 Wing type micro-cracks

In this second part of the paper, we consider a 2D isotropic elastic medium containing wing-type micro-cracks. As before, the distribution is assumed to be locally periodic and each micro-crack (composed by a main inclined crack with two branches) is considered in one periodicity cell of length ε (Fig. 5). The length ε also represents the distance between centers of two neighbor micro-cracks.

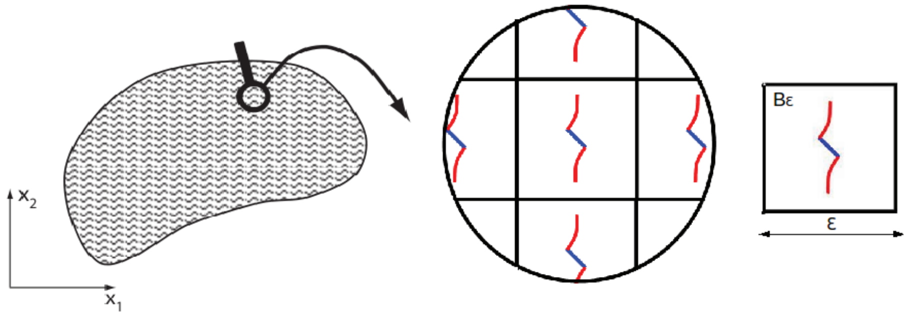

Figure 5: Fissured medium with locally periodic micro-structure containing wing-type micro-cracks.

Starting from the initial model of Brace et al. ([5]), many researchers proposed models for the wing-type crack. These models have in common the fact that the extension of the branches is controlled by the shear of the initial inclined crack. We adopt here the model of Fanella and Krajcinovic ([9]) and we implement their idea in our framework of homogenization through asymptotic developments. In the model we assume a sliding crack of length $2a$ (Fig. 6). Using a Coulomb type criterion, the shear stress denoted τ_s, reduced by the presence of the friction, is given by ([9]):

$$\tau_s = (\sigma_{11}^{\varepsilon} - \sigma_{22}^{\varepsilon})\frac{sin(2\phi)}{2} - \mu_f(\sigma_{11}^{\varepsilon}\cos^2(\phi) + \sigma_{22}^{\varepsilon}\sin^2(\phi)), \tag{17}$$

where μ_f is the friction coefficient, ϕ is the angle made by the inclined crack with the horizontal axis and $\boldsymbol{\sigma}^{\varepsilon}$ is the stress field (Eq. 1).

The shear on the crack induces a traction zone at the crack tips, the consequence being the appearance of branches (wings) that progressively align to the maximum loading direction.

Following Fanella and Krajcinovic ([9]) we replace the sliding crack and the branches by an equivalent vertical crack. On the central part of the equivalent crack I^{ε}, of length $2a\alpha\sin\phi$, we apply a concentrated pressure $P(e_x(\mathbf{u}^{\varepsilon}))$ on the normal direction (Fig. 6 (b)). The value of the pressure $P(e_x(\mathbf{u}^{\varepsilon}))$ is given by the relation below:

$$P(e_x(\mathbf{u}^\varepsilon)) = \begin{cases} 2\int_0^{a\alpha\sin\phi} \frac{\tau_s}{\alpha}\cot\phi dx, & e_{x22}(\mathbf{u}^\varepsilon) \neq \mathbf{0}, \\ 0, & e_{x22}(\mathbf{u}^\varepsilon) = \mathbf{0}. \end{cases} \tag{18}$$

We considered $\mu_f = 0.3$, $\phi = 45$ degrees and $\alpha = 0.25$. The correction factor α was introduced by the authors of [9] in order to recover the correct stress intensity factor for wing cracks as abtained in the numerical study of Horii and Nemat-Nasser ([13]).

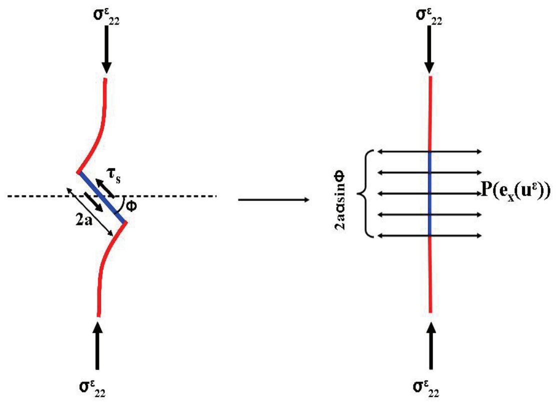

Figure 6: Micro-crack model under compression: a) sliding crack model; b) straight equivalent model.

As for the porous materials, in the solid part, we have the equilibrium equations and the elasticity law (Eq. 1).

On the central part of the crack I^ε the concentrated pressure $P(e_x(u^\varepsilon))$ is acting, due to the replacement of the original sliding crack. The rest of the crack boundaries are traction-free:

$$\boldsymbol{\sigma}^\varepsilon\mathbf{n} = P(e_x(\mathbf{u}^\varepsilon))\mathbf{n} \text{ on } I^\varepsilon, \tag{19}$$

$$\boldsymbol{\sigma}^\varepsilon\mathbf{n} = 0 \text{ on } \mathcal{C} \text{ - } I^\varepsilon. \tag{20}$$

We denoted by $\mathbf{n}$ the normal unit vector on the crack faces.

3.1 Homogenization by asymptotic developments

Similar to the case of cracks emerging from pores, we assume that we can reproduce the locally periodic microstructure of the body through a unit cell

$Y = [0,1] \times [0,1]$, by rescaling with the small parameter ε. In this way the period of the material is εY, as in Fig. 7. The two distinct scales are represented by the variables $\mathbf{x}$ and $\mathbf{y} = \frac{\mathbf{x}}{\varepsilon}$ defined previously. In the unit cell Y, we denote the lips of the two cracks by CY, the central zone (which replace the sliding crack) by I and the solid part by Y_s. We introduce the normalized damage parameter $d = \frac{d^\varepsilon}{\varepsilon}$ representing the scaled distance between the two crack tips in the cell.

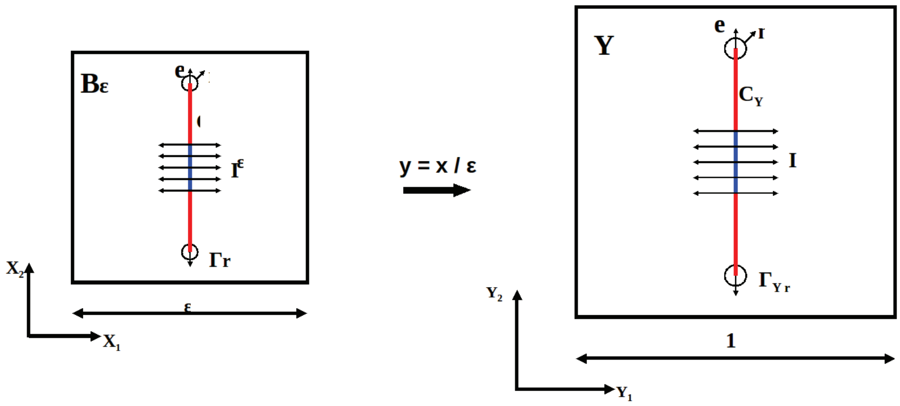

Figure 7: Material period and the unit cell.

Following the method of asymptotic homogenization and using the expansion of $\mathbf{u}^\varepsilon$ and $\boldsymbol{\sigma}^\varepsilon$ (Eq. 3 - 4) into Eq. 1 and the boundary conditions (19), we obtain the boundary value problems for the different orders of ε, formulated on the unit cell Y. As for porous materials, we prove that $\mathbf{u}^{(0)} = \mathbf{u}^{(0)}(\mathbf{x}, t)$ being a trully macroscopic displacement field.

For a given $e_x(\mathbf{u}^{(0)})$ corresponding to a compression loading ($e_{x22}(\mathbf{u}^{(0)}) < 0$), the boundary-value problem for the first microscopic correction $\mathbf{u}^{(1)}$ is deduced as:

$$\frac{\partial}{\partial y_j}(a_{ijkl} e_{ykl}(\mathbf{u}^{(1)})) = 0 \qquad \text{in } Y_s, \tag{21}$$

$$a_{ijkl}(e_{ykl}(\mathbf{u}^{(1)}) + e_{xkl}(\mathbf{u}^{(0)}))n_j = 0 \text{ on } CY^{\pm} - \mathrm{I}^{\pm}, \tag{22}$$

$$a_{ijkl}(e_{ykl}(\mathbf{u}^{(1)}) + e_{xkl}(\mathbf{u}^{(0)}))n_j = -P(e_x(u^{(0)}))n_i \text{ on } \mathrm{I}^{\pm}. \tag{23}$$

where $\pm$ denote the values on the two faces of the micro-cracks.

The microscopic correction $\mathbf{u}^{(1)}$ has a linear dependence of the *macroscopic deformations* $e_{xpq}(\mathbf{u}^{(0)})$:

$$\mathbf{u}^{(1)} = \xi^{11} e_{x11}(\mathbf{u}^{(0)}) + 2\xi^{12} e_{x12}(\mathbf{u}^{(0)}) - \xi^{22} e_{x22}(\mathbf{u}^{(0)}). \tag{24}$$

The characteristic functions $\boldsymbol{\xi}^{pq}(\mathbf{y}, d, a)$ are elementary solutions of (21-23), for a given length of the crack and for particular macroscopic deformations having the only non-vanishing component $e_{x11} = 1$ or $e_{x12} = 1$ or $e_{x22} = -1$, respectively. As before, $\boldsymbol{\xi}^{22}(\mathbf{y}, d, a)$ corresponds to a compressive macroscopic deformation applied to the unit cell through the internal boundary conditions (23).

By applying the mean value operator to the boundary value problem corresponding to the 1^{st}-order of ε, we can deduce the homogenized equilibrium equation (Eq. 8) and the effective elastic law (Eq. 9) where $C_{ijkl}(d, a)$ are the effective homogenized coefficients. The general formula is given by Eq. (10), except for the homogenized coefficients corresponding to $\boldsymbol{\xi}^{22}$ given by: $C_{1122} =< a_{1122} - a_{1111} e_{y11}(\xi^{22}) - a_{1122} e_{y22}(\xi^{22}) >$ and $C_{2222} =< a_{2222} - a_{1111} e_{y11}(\xi^{22}) - a_{1122} e_{y22}(\xi^{22}) >$.

3.2 The damage law

In the previous work ([6], [7]), the damage law was deduced in the form:

$$\begin{aligned} \dot{d}(\frac{1}{2}\frac{dC_{ijkl}}{dd} e_{xkl}(\mathbf{u}^{(0)}) e_{xij}(\mathbf{u}^{(0)}) + \frac{\mathcal{G}_c}{\varepsilon} + \\ I_{mnpq} e_{xmn}(\mathbf{u}^{(0)}) e_{xpq}(\mathbf{u}^{(0)})) = 0 \end{aligned} \tag{25}$$

as the third loading/unloading condition and where

$$\begin{aligned} I_{mnpq} = \frac{d}{dd}\left(\frac{1}{2}\int_{CY} a_{ijkl}(\delta_{mk}\delta_{nl} + e_{ykl}(\xi^{mn})) n_j [\xi_i^{pq}] ds_y\right) - \\ \int_{CY} a_{ijkl}\left(\delta_{mk}\delta_{nl} + e_{ykl}(\xi^{mn})\right) n_j \left[\frac{d\xi_i^{pq}}{dd}\right] ds_y. \end{aligned} \tag{26}$$

Usually, the integrals $I_{mnpq}(d)$ are computed on the entire crack lips ([6]), but in our specific case, are computed only on the central part, I. The non null integrals entering the damage laws are given by

$$I_{22pq}(\xi^{22}) = \frac{d}{dd}(\frac{1}{2}\int_I P^{22} n_j [\xi_i^{pq}] dS_y). \tag{27}$$

where

$$P^{22} = 2\int_0^{a\alpha\sin\phi} \frac{\mu + 0.3(\lambda+\mu)}{\alpha} \cot\phi dx. \tag{28}$$

In the previous formula $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$, with E = Young modulus and ν = Poisson ratio.

3.3 Numerical implementation - size effects

In this section we give numerical results we obtained using the homogenization by asymptotic developments technique on the special case of wing-type cracks. Elastic isotropic material described by Young's modulus $E = 2GPa$ and Poisson's ratio $\nu = 0.1$ was used.

Starting from the elementary deformation modes, we compute the homogenized coefficients which are functions of the damage parameter, d. For the computation of C_{ijkl} and I_{ijkl} we used the finite element program FEAP, developed by the Berkeley University [22]. For the homogenized coefficients we used triangular finite elements with three Gauss points for the displacements. The modeling of the wing-type micro-cracks demands the computation of C_{ij11}, in tension, for ξ^{11}, and those corresponding to the $\boldsymbol{\xi}^{22}$ (C_{ij22}) in compression. For these coefficients we used Lagrange Multipliers method for the contact between the crack faces.

In Fig. 8 we represent the homogenized coefficients and I_{2222}. Nonlinear dependence of the homogenized coefficients on the damage variable d is observed as well as the anisotropy in the effective response, induced by the presence of the micro-crack.

Using the numerical implementation previously done for the standard crack model, some elementary damage tests have been simulated. The most significant result at the local macroscopic level is the size dependence of the damage yield stress on the microscopic cell size ε shown in Fig.9 (a). For each value of ε, the uniaxial tests were controlled through the applied deformation e_{x22}. We note that for smaller cell sizes we have higher thresholds of damage initiation.

In Fig. 9 (b), the critical macroscopic stress Σ_{22} is represented for different microscopic lengths ε which shows the linear dependence of the damage yield stress on $\varepsilon^{-\frac{1}{2}}$. This prove a size effect of the Hall-Petch type.

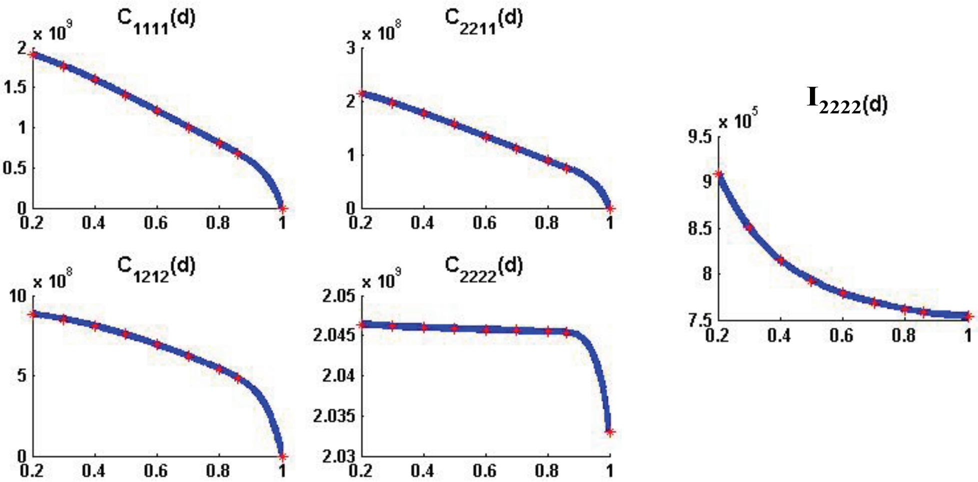

Figure 8: Left) Homogenized coefficients; Right) The integral I_{2222} of the jumps over the crack faces.

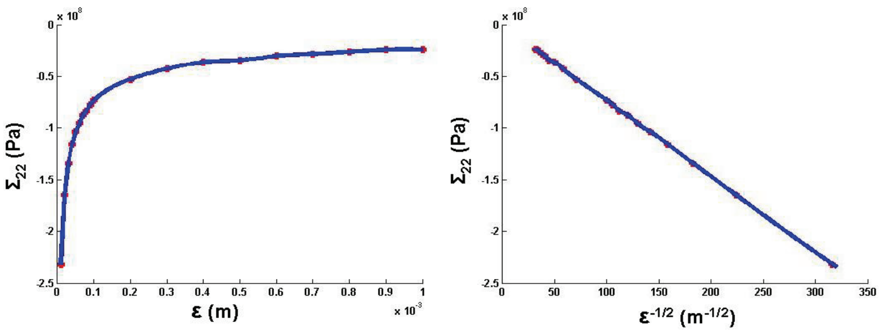

Figure 9: Evolution of the limit of damage initiation as a function of the micro-structural size ε and, respectively, $\varepsilon^{-\frac{1}{2}}$ under uniaxial compression loading

4 Conclusions

We used the asymptotic homogenization technique in order to develop a micro-mechanical damage model specific to microstructures with micro-cracks emerging from pores or with wing-type micro-cracks, under compression loadings. In the case of wing-type micro-cracks, the model of equivalent micro-cracks proposed by Fanella and Krajcinovic was used in the two-scale framework.

The brittle damage behavior was analyzed at the local macroscopic scale. The influence of micro-structural parameters, like the mutual distance between centers of neighbor pores or wing-type micro-cracks as well as the pore size, on the local macroscopic response has been emphasized. We showed that the constructed damage models are able to predict size effects, as a consequence of the presence of a microstructural length parammeter in the damage equations.

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Efekti veličine za materijale sa mikroprslinama pri kompresivnom opterećenju

Koristeći asimptotski metod homogenizacije, formulisan je model oštećenja materijala sa mikroprslinama, baziran na dva materijalna nivoa. Na nižem od ovih nivoa, posmatraju se lokalne periodične mikrostrukture dvije vrste: mikroprsline na površinama materijalnih otvora i račvanje (wing cracks) već postojećih pukotina. Na višem nivou, formulisan je makroskopski model oštećenja u koji je ugradjen energetski kriterijum za prostiranje mikroprslina i rastojanje izmedju mikropukotina kao mikrostrukturalna materijalna dužina. Predloženi model je primenjen na analizu uticaja veličine uzorka na mehaničko ponašanje materijala u uslovima kompresivnog opterećenja.

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