Theoret. Appl. Mech., Vol.39, No.3, pp. 233–254, Belgrade 2012

Lie group analysis of heat and mass transfer effects on steady mhd free convection dissipative fluid flow past an inclined porous surface with heat generation

M. Gnaneswara Reddy*

Abstract

In this paper, an analysis has been carried out to study heat and mass transfer effects on steady two-dimensional flow of an electrically conducting incompressible dissipating fluid past an inclined semi-infinite porous surface with heat generation. A scaling group of transformations is applied to the governing equations. The system remains invariant due to some relations among the parameters of the transformations. After finding three absolute invariants, a third-order ordinary differential equation corresponding to the momentum equation, and two second-order ordinary differential equations corresponding to energy and diffusion equations are derived. The coupled ordinary differential equations along with the boundary conditions are solved numerically. Many results are obtained and a representative set is displayed graphically to illustrate the influence of the various parameters on the dimensionless velocity, temperature and concentration profiles. Comparisons with previously published work are performed and the results are found to be in very good agreement.

Keywords: Lie group analysis, Natural convection, MHD, Viscous dissipation, mass transfer, Inclined porous surface, Heat generation

*Department of Mathematics, Acharya Nagarjuna University Ongole Campus, Ongole,A.P. (India) - 523 001, e-mail: mgrmaths@gmail.com

Nomenclature

B_0−applied magnetic field
C−species concentration in the boundary layer
C_∞ −the species concentration in the fluid far away from the plate
c_p−specific heat at constant pressure
D−mass diffusivity
Ec−Eckert number
f−dimensionless stream function
g−acceleration due to gravity
Gr−local temperature Grashof number
Gm−local mass Grashof number
K' −the permeability of the porous medium
K−permeability parameter
k−thermal conductivity of the fluid
M−magnetic field parameter
Pr −Prandtl number
Q_0−heat generation constant
Q−heat generation parameter
Sc−Schmidt number
T −the temperature of the fluid in the boundary layer
T_∞ −the temperature of the fluid far away from the plate
u, v−velocity components in x, y directions

Greek Symbols
η−similarity variable
α−angle of inclination
β−coefficient of thermal expansion
β^*−coefficient of concentration expansion
σ−electrical conductivity
ρ−density of the fluid
ν−kinematic viscosity
θ−dimensionless temperature
φ−dimensionless concentration

Subscripts
w−condition at wall

$\infty-$condition at infinity

Superscript
$()'$ $-$differentiation with respect to η

1 Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport. An analysis is performed to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration by Chen [1]. Convective heat and mass transfer along a semi-infinite vertical flat plate in the presence of a strong non-uniform magnetic field and the effect of Hall currents is analyzed by using the scaling group of transformations, see in Megahed et al. [2]. Beithou et al. [3] studied the effect of porosity on the free convection flow along a vertical plate embedded in a porous medium is investigated. Ibrahim et al. [4] investigated the similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using Lie group method. They also presented the numerical results for the non-uniform magnetic field.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Soundalgekar et al. [5] analysed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field. Elbashbeshy [6] studied the heat and mass transfer along a vertical plate under the combined buoyancy effects of thermal and species diffusion, in the presence of magnetic field. Helmy [7] presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by an infinite vertical plane surface of constant temperature.

Kalpadides and Balassas [8] studied the free convective boundary layer problem of an electrically conducting fluid over an elastic surface by group theoretic method. Their results agreed with the existing result for the group of scaling symmetry. They found that the numerical solution also does so. The Navier-Stokes and boundary layer equations for incompressible flows were derived using a convenient coordinate system by Pakdemirli [9]. The results showed that the boundary layer equations accept similarity solutions for the constant pressure gradient case. The importance of similarity transformations and their applications to partial differential equations was studied by Pakdemirli and Yurusoy [10]. They investigated the special group transformations for producing similarity solutions. They also discussed spiral group of transformations. Using Lie group analysis, three dimensional, unsteady, laminar boundary layer equations of non-Newtonian fluids are studied by Yurusoy and Pakdemirli [11, 12]. They assumed that the shear stresses are arbitrary functions of the velocity gradients. Using Lie group analysis, they obtained two different reductions to ordinary differential equations. They also studied the effects of a moving surface with vertical suction or injection through the porous surface. They further studied exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. They found that the boundary layer thickness increases when the non-Newtonian behaviour increases. They also compared the results with that for a Newtonian fluid. Yurusoy et al. [13] investigated the Lie group analysis of creeping flow of a second grade fluid. They constructed an exponential type of exact solution using the translation symmetry and a series type of approximate solution using the scaling symmetry.

Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. In most of the studies mentioned above, viscous dissipation is neglected. Gebhart [14] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Gebhart and Mollendorf [15] considered the effects of viscous dissipation for the external natural convection flow over a surface. Gnaneswara Reddy and Bhaskar Reddy [16] studied the radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate with viscous dissipation by using finite element method. Recently, Gnaneswara Reddy and Bhaskar Reddy [17] investigated mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium. Sivasankaran et al. [18] analyzed lie group analysis of natural convection heat and mass transfer in an inclined surface. Gnaneswara Reddy and Bhaskar Reddy [19] have presented soret and dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation.

In this article, application of scaling group of transformation for heat and mass transfer effects on steady free convection flow in an inclined plate in the presence of MHD, heat generation and viscous dissipation have been employed. This reduces the system of nonlinear coupled partial differential equations governing the motion of fluid into a system of coupled ordinary differential equations by reducing the number of independent variables. The system remains invariant due to some relations among the parameters of the transformations. Three absolute invariants are obtained and used to derive a third-order ordinary differential equation corresponding to momentum equation and two second-order ordinary differential equations corresponding to energy and diffusion equations. With the use of Runge-Kutta fourth order along shooting method, the equations are solved. Finally, analysis has been made to investigate the effects of thermal and solutal Grashof numbers, magnetic field parameter, permeability parameter, heat generation parameter, Prandtl number, Viscous dissipation parameter, and Schmidt number on the motion of fluid using scaling group of transformations, viz., Lie group transformations.

2 Mathematical analysis

A steady two-dimensional hydromagnetic flow heat and mass transfer effects of a viscous, incompressible, electrically conducting and dissipating fluid past a semi-infinite inclined plate embedded in a porous medium with an acute angle $\alpha$to the vertical. The flow is assumed to be in the$x$- direction, which is taken along the semi-infinite inclined plate and y- axis normal to it. A magnetic field of uniform strength $B_0$is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The surface is maintained at a constant temperatureT_w, which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w is greater than the constant concentrationC_∞. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Then, under the usual Boussinesq's and boundary layer approximations, the governing equations are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)\cos\alpha+g\beta^*\left(C-C_\infty\right)\cos\alpha-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K'}u \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho\, c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q_0}{\rho c_p}\left(T-T_\infty\right) \tag{3}$$

Species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u=v=0,\quad T=T_w,\;\; C=C_w\;\; at\;\; y=0 \tag{5}$$

$$u\to 0,\;\; T\to T_\infty,\;\; C\to C_\infty\quad as\quad y\to\infty$$

The second and third terms on right hand side of the energy equation (3) represent the viscous dissipation and the heat generation respectively.

On introducing the following non-dimensional quantities

$$\bar{x}=\frac{x\,U_\infty}{\nu}\,,\;\bar{y}=\frac{y\,U_\infty}{\nu}\,,\quad \bar{u}=\frac{u}{U_\infty},\;\;\bar{v}=\frac{v}{U_\infty},\;M=\frac{\sigma B_0^2\nu}{U_\infty^3},\;Gr=\frac{\nu\,g\,\beta(T_w-T_\infty)}{U_\infty^3},\;K=\frac{\nu^3}{K'U_\infty^3} \tag{6}$$

$$Gm=\frac{\nu\,g\,\beta^*(C_w-C_\infty)}{U_\infty^3},\;\theta=\frac{T-T_\infty}{T_w-T_\infty}\,,\;\varphi=\frac{C-C_\infty}{C_w-C_\infty},\quad \Pr=\frac{\nu}{\alpha},\quad Ec=\frac{U_\infty^2}{c_p\left(T_w-T_\infty\right)},\quad Sc=\frac{\nu}{D}$$

$$Q = \frac{Q_0 \nu}{\rho c_p U_\infty^2}$$

Substituting (6) into equations (1) - (4) and dropping the bars, we obtain,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta \, Cos\alpha + Gm \, \varphi \, Cos\alpha - (M + K) \, u \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Q\theta \tag{9}$$

$$u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial y^2} \tag{10}$$

The corresponding boundary conditions take the form

$$u = v = 0, \quad \theta = 1, \quad \varphi = 1 \quad at \;\; y = 0 \tag{11}$$

$$u \to 0, \;\; \theta \to 0, \;\; \varphi \to 0 \;\; as \; y \to \infty$$

By using the stream function $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ we have

$$\left(\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}\right) = \frac{\partial^3 \psi}{\partial y^3} + Gr\theta Cos\alpha + Gm\varphi Cos\alpha - (M + K)\frac{\partial \psi}{\partial y} \tag{12}$$

$$\left(\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 + Q\theta \tag{13}$$

$$\left(\frac{\partial \psi}{\partial y}\frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \varphi}{\partial y}\right) = \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial y^2} \tag{14}$$

Finding the similarity solutions of equations (12) - (14) is equivalent to determining the invariant solutions of these equations under a particular continuous one parameter group. One of the methods is to search for a transformation group from the elementary set of one parameter scaling transformation. We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (Mukhopadhyay et al. [20]),

$\Gamma:\ x^* = xe^{\varepsilon\alpha_1},\ y^* = ye^{\varepsilon\alpha_2},\ \ \psi^* = \psi e^{\varepsilon\alpha_3},\ u^* = ue^{\varepsilon\alpha_4},\ v^* = ve^{\varepsilon\alpha_5}, \theta^* = \theta e^{\varepsilon\alpha_6},\ \varphi^* = \varphi e^{\varepsilon\alpha_7}$ (15) where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and α_7 are transformation parameters andε is a small parameter whose interrelationship will be determined by our analysis.

Equation (15) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, v, \theta, \varphi)$ to the coordinates $(x^*, y^*, \psi^*, u^*, v^*, \theta^*, \varphi^*)$.

Substituting transformations equation (15) in (12), (13) and (14), we get

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*} - \frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial {y^*}^2}\right) = e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial {y^*}^3} + e^{-\varepsilon\alpha_6}Gr\theta Cos\alpha + e^{-\varepsilon\alpha_7}Gm\varphi Cos\alpha - (M+K)\, e^{\varepsilon(\alpha_2-\alpha_3)}\frac{\partial\psi^*}{\partial y^*} \tag{15}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*} - \frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right) = \frac{1}{\Pr}e^{\varepsilon(2\alpha_2-\alpha_6)}\frac{\partial^2\theta^*}{\partial {y^*}^2} + e^{\varepsilon(4\alpha_2-2\alpha_3)}Ec\left(\frac{\partial^2\psi^*}{\partial {y^*}^2}\right)^2 + Q\theta^* e^{-\varepsilon\alpha_6} \tag{16}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\varphi^*}{\partial x^*} - \frac{\partial\psi^*}{\partial x^*}\frac{\partial\varphi^*}{\partial y^*}\right) = \frac{1}{Sc}e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2\varphi^*}{\partial {y^*}^2} \tag{17}$$

The system will remain invariant under the group of transformationsΓ, and we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = -\alpha_6 = -\alpha_7 = \alpha_2 - \alpha_3$$

$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_6 = 4\alpha_2 - 2\alpha_3 = -\alpha_6$$

$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 = 2\alpha_2 - \alpha_7$$

These relations gives$\alpha_2 = \frac{1}{4}\alpha_1 = \frac{1}{3}\alpha_3,\ \alpha_4 = \frac{1}{2}\alpha_1,\ \alpha_2 = -\frac{1}{4}\alpha_1,\ \alpha_6 = \alpha_7 = 0$

Thus the set of transformations Γreduce to one parameter group of transformations as

$$x^* = xe^{\varepsilon\alpha_1},\ y^* = ye^{\varepsilon\frac{\alpha_1}{4}},\ \ \psi^* = \psi e^{\varepsilon\frac{3\alpha_1}{4}},\ \ u^* = ue^{\varepsilon\frac{\alpha_1}{2}},\ v^* = ve^{-\varepsilon\frac{\alpha_1}{4}}, \theta^* = \theta,\ \ \varphi^* = \varphi$$

Expanding by Tailors method in powers of ε and keeping terms up to the order ε we get

$$x^* - x = x\varepsilon\alpha_1,\ y^* - y = y\varepsilon\frac{\alpha_1}{4},\ \ \psi^* - \psi = \psi\varepsilon\frac{3\alpha_1}{4},\ \ u^* - u = u\varepsilon\frac{\alpha_1}{2},\ v^* - v = -v\varepsilon\frac{\alpha_1}{4},$$
$$\theta^* - \theta = 0,\ \ \varphi^* - \varphi = 0$$

The characteristic equations are
$\frac{dx}{x\alpha_1} = \frac{dy}{y\frac{\alpha_1}{4}} = \frac{d\psi}{\psi\frac{3\alpha_1}{4}} = \frac{du}{u\frac{\alpha_1}{2}} = \frac{dv}{-v\frac{\alpha_1}{4}} = \frac{d\theta}{0} = \frac{d\varphi}{0}$(19)
Solving the above equations, we find the similarity transformations

$$\eta = x^{-\frac{1}{4}}y,\ \psi^* = x^{\frac{3}{4}}f(\eta),\ \theta^* = \theta(\eta),\ \varphi^* = \varphi(\eta) \tag{18}$$

Substituting these values in Equations (15) - (17), we finally obtain the system of nonlinear ordinary differential equations

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + Gr\theta Cos\alpha + Gm\varphi Cos\alpha - (M+K)f' = 0 \tag{19}$$

$$\theta'' + \frac{3}{4}\Pr f\theta' + \Pr Ecf''^2 + \Pr Q\theta = 0 \tag{20}$$

$$\varphi'' + \frac{3}{4}Scf\varphi' = 0 \tag{21}$$

The corresponding boundary conditions take the form

$$f = 0,\quad f' = 0,\quad \theta = 1,\quad \varphi = 1\ \ at\ \eta = 0$$

$$f' \to 0,\quad \theta \to 0,\quad \varphi \to 0\ \ as\ \eta \to \infty \tag{22}$$

3 Results and discussion

The set of nonlinear ordinary differential equations (19) - (21) with boundary conditions (22) have been solved by using the Runge-Kutta fourth order along with Shooting method. First of all, higher order non-linear differential Equations (19) - (21) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al. [21]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.01$is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. To analyze the results, numerical computation has been carried out using the method described in the previous section for variations in the governing parameters viz., the thermal Grashof numberGr, solutal Grashof numberGm, magnetic field parameter$M$, permeability parameter$K$, angle of inclinationα, Prandtl number$\Pr$, Eckert number Ec, heat generation parameter Q, and Schmidt numberSc. In the present study following default parameter values are adopted for computations: $Gr = Gm = 2.0$,$\alpha = 30^0$,$M = 2.0$,$K = 1.0 \Pr = 0.71$,$Ec = 0.01$,$Q = 1.0$,$Sc = 0.6$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

In order to assess the accuracy of our computed results, the present result has been compared with Sivasankaran et al. [18] for different values of $Gr$is shown Fig. 1 with$K = 0.0$. It is observed that the agreements with the solution of velocity profiles are excellent.

The influence of the free convection parameter, Grashof number (Gr)on velocity and temperature distributions withηcoordinate is depicted in Fig.2. The thermal Grashof numberGrsignifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. Increasing Gr corresponds to an increase in thermal buoyancy force in the regime. As such the flow is decelerated which causes the velocity to plummet considerably. Peak velocities (as shown in Fig. 2), and fall from 0.068 for $Gr = 1.0$ to 0.012 for$Gr = 4.0$. Here, the positive values of Gr correspond to cooling of the plate. There is a sharp rise in velocity near the sphere surface after which velocities peak and then decrease continuously to zero far from the surface.

Figure 3 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gm, while all other parameters are kept at some fixed values. The solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

For various values of the magnetic parameter M, the velocity profiles are plotted in Figure 4. It can be seen that as M increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow.

Figure 5 shows the effect of the porosity parameter on the dimensionless velocity profiles. It is observed that the velocity decreases as the porosity increases. The reason for this behavior is that the wall of the surface provides an additional effect to the fluid flow mechanism, which causes the fluid to move at a retarded rate with reduced temperature. These behaviors are shown in Fig. 5. Also, it is observed that the concentration of the fluid is almost not affected with increase of the porosity parameter.

Figure 6 shows the effect of angle of inclination to the vertical direction on the velocity profiles. From this figure we observe that the velocity is decreased by increasing the angle of inclination. The fact is that as the angle of inclination increases the effect of the buoyancy force due to thermal diffusion decreases by a factor of$\cos\alpha$. Consequently the driving force to the fluid decreases as a result velocity profiles decrease.

Figures 7 & 8 display the velocity and temperature distributions for different values of the heat generation parameter$Q$. It is seen from Figure 7 that the velocity profile is influenced considerably and increases when the value of heat generation parameter increases. From Figure 8, when the value of heat generation parameter increases, the temperature distribution also increases along the boundary layer.

Figures 9 and 10 illustrate the velocity and temperature profiles for different values of the Prandtl number Pr. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Figure 9). From Figure 10, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of the viscous dissipation parameter i.e., the Eckert number Ec on the velocity and temperature are shown in Figures 11 and 12 respectively. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figures 11 and 12.

The influence of the Schmidt number Sc on the velocity and concentration profiles are plotted in Figures 13 and 14 respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The

Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figures 13 and 14.

4 Conclusions

By using the Lie group analysis, we first find the symmetries of the partial differential equations and then reduce the equations to ordinary differential equations by using scaling and translational symmetries. Exact solutions for translational symmetry and a numerical solution for scaling symmetry are obtained. From the numerical results, it is seen that the effect of increasing thermal Grashof number or solutal Grashof number is manifested as an increase in flow velocity. It is interesting to note that the temperature decreases much faster than the air temperature. In the presence of a magnetic field parameter, the permeability of porous medium, viscous dissipation is demonstrated to exert a more significant effect on the flow field and, thus, on the heat transfer from the plate to the fluid. It is seen that the velocity profile is influenced considrably and increases when the value of heat generation parameter increases, and when the value of heat generation parameter increases, the temperature distribution also increases along the boundary layer. The velocity and concentration is found to decrease gradually as the Schmidt number is increased.

The results of the study are of great interest because flows on a vertical stretching surface play a predominant role in applications of science and engineering, as well as in many transport processes in nature.

References

[1] Chen CH (2004). Heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration. *Acta Mechanica* **172**, pp.219–235.

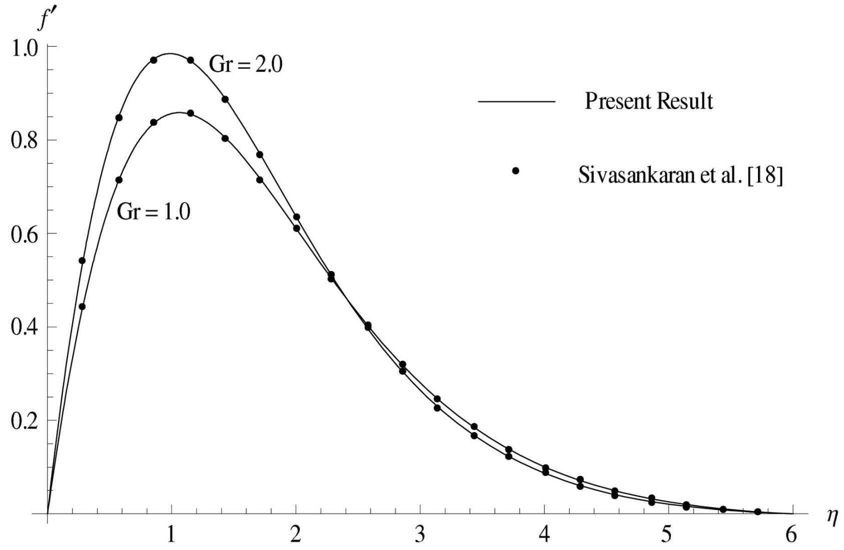

Figure 1: Comparison of velocity profiles

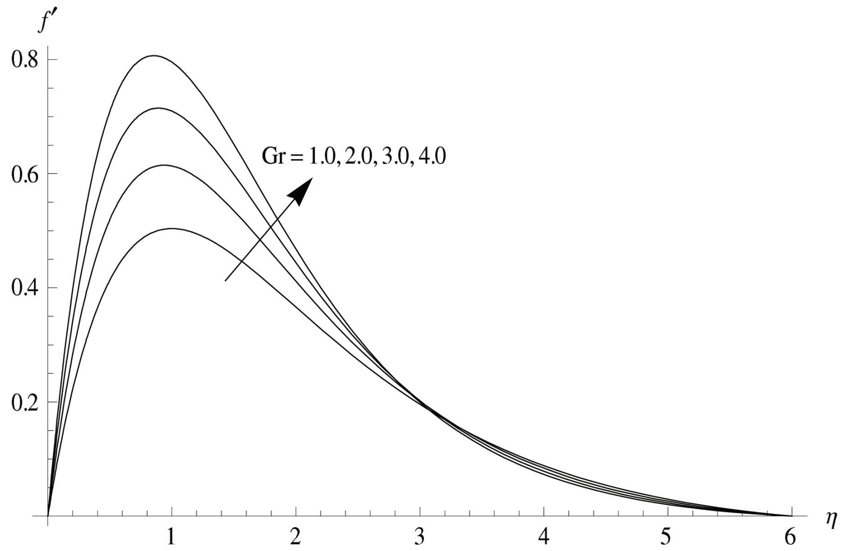

Figure 2: Velocity profiles for different values of Gr

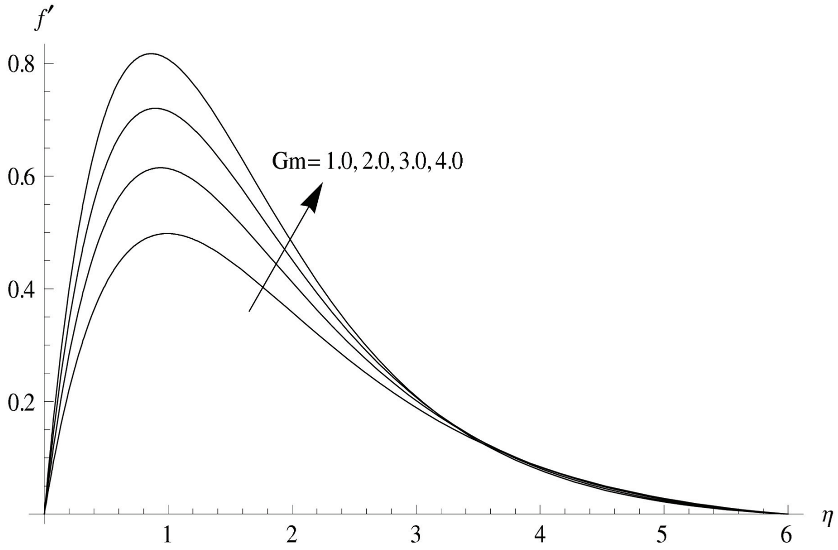

Figure 3: Velocity profiles for different values of Gm

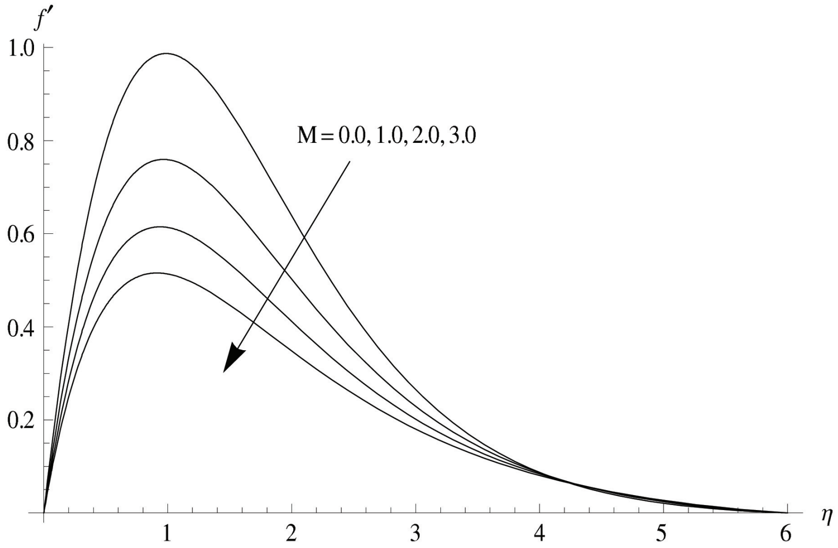

Figure 4: Velocity profiles for different values of M

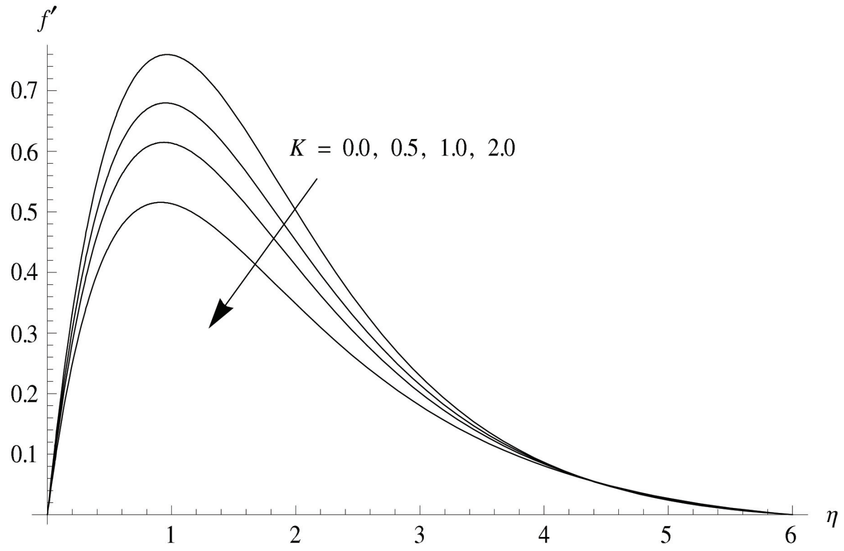

Figure 5: Velocity profiles for different values of K

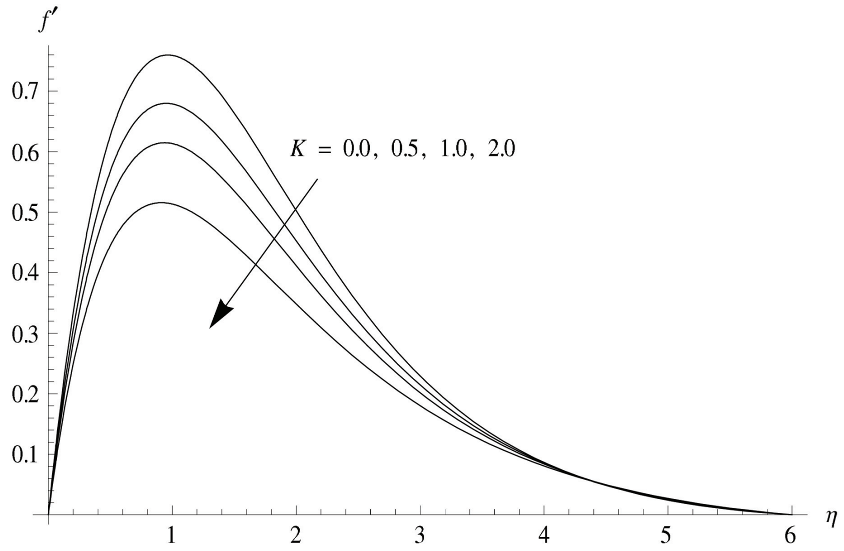

Figure 6: Velocity profiles for different values of α

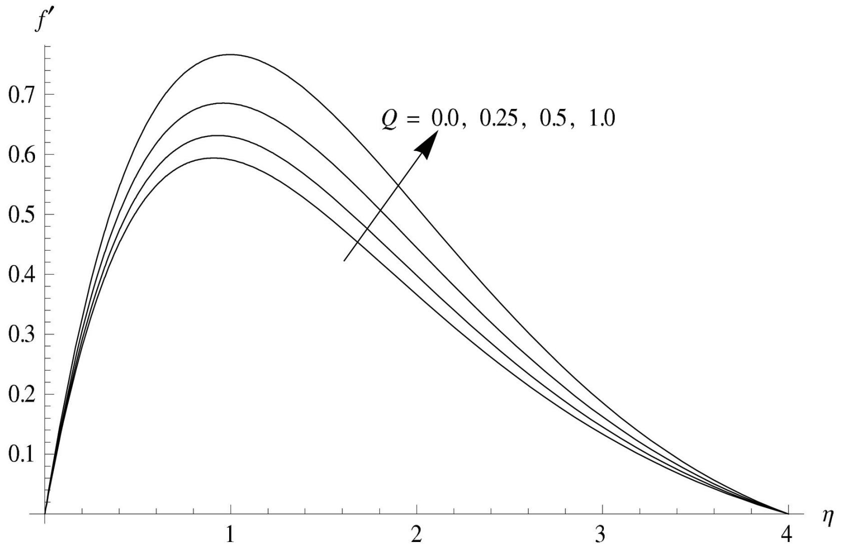

Figure 7: Velocity profiles for different values of Q

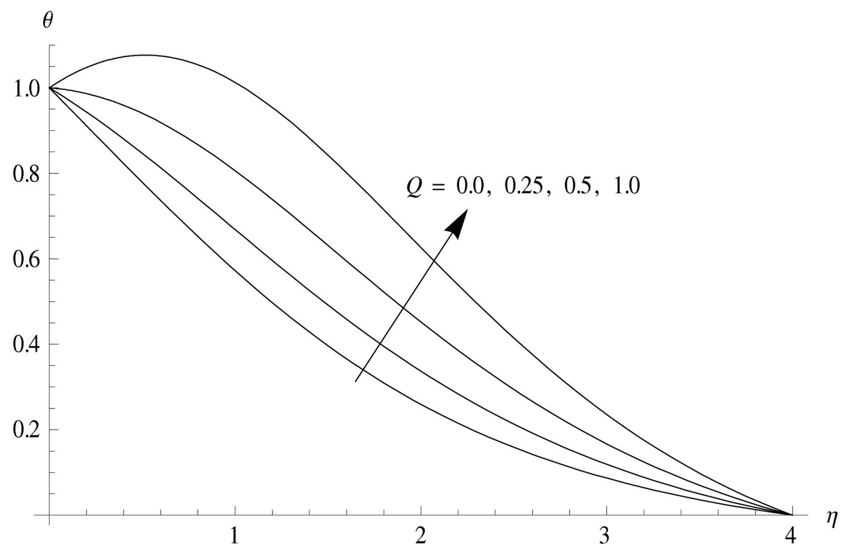

Figure 8: Temperature profiles for different values of Q

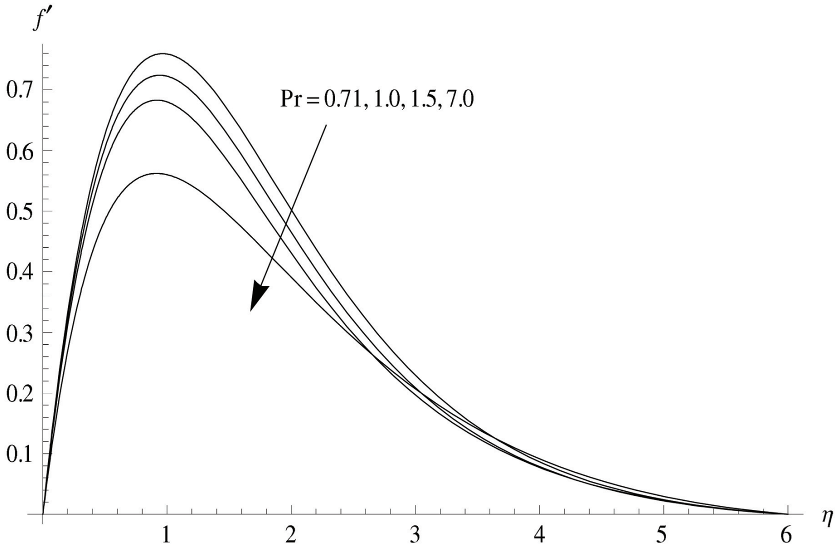

Figure 9: Velocity profiles for different values of Pr

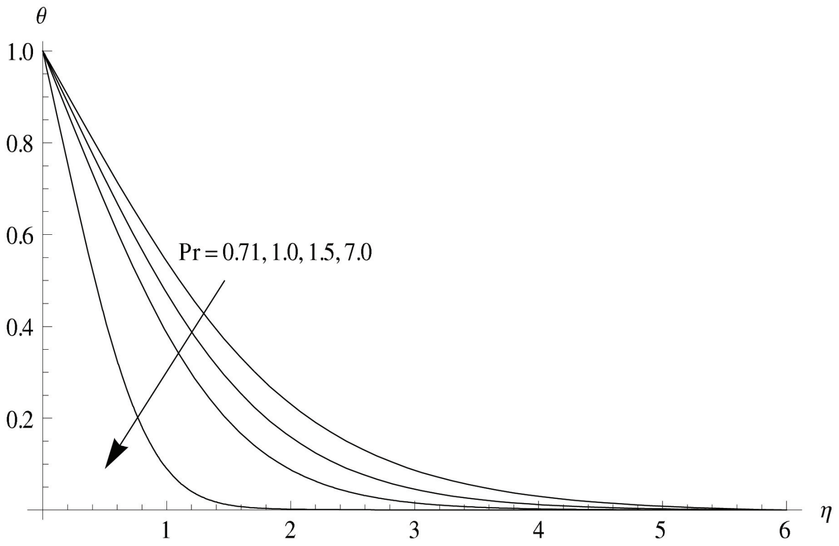

Figure 10: Temperature profiles for different values of Pr

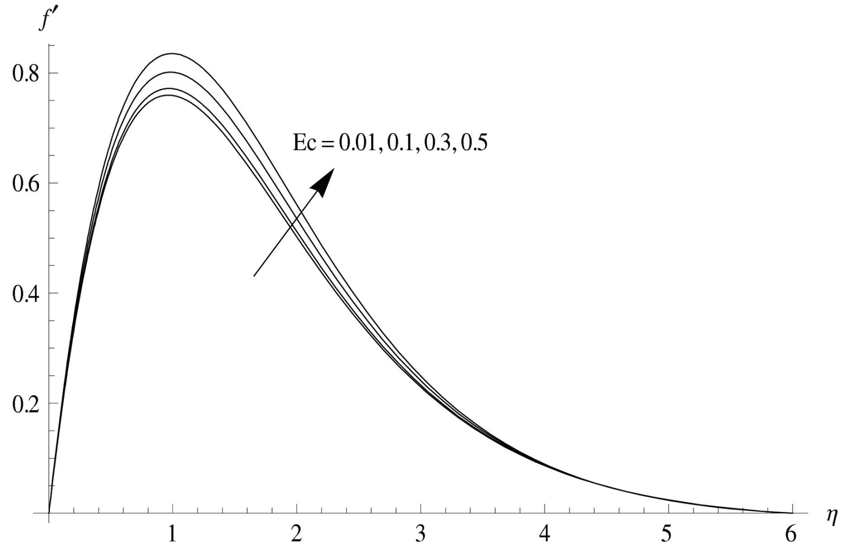

Figure 11: Velocity profiles for different values of Ec

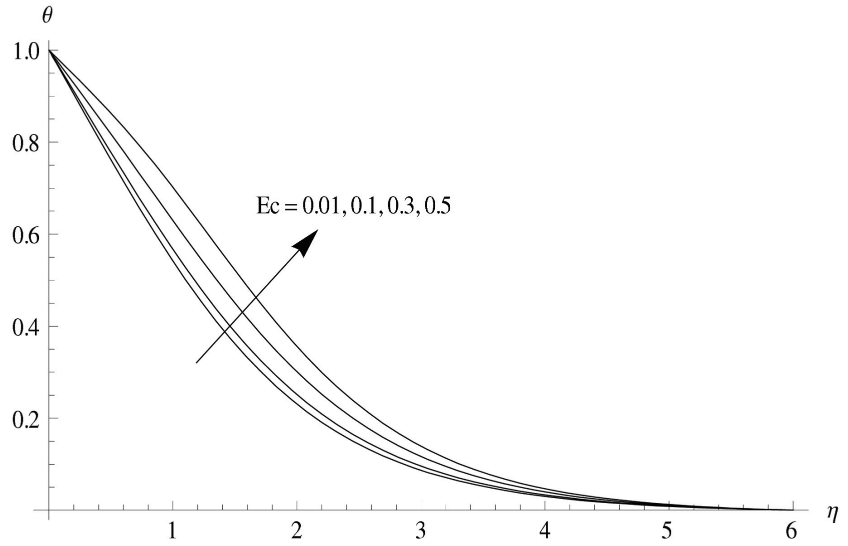

Figure 12: Temperature profiles for different values of Ec

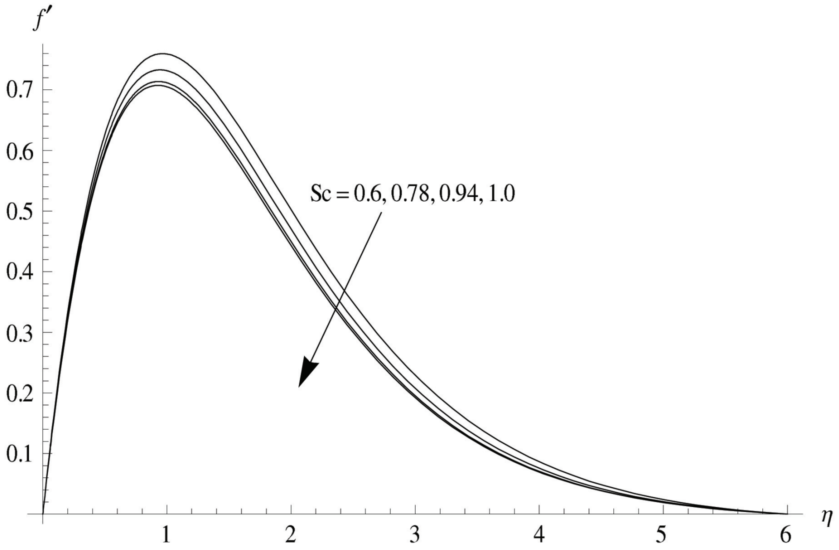

Figure 13: Velocity profiles for different values of Sc

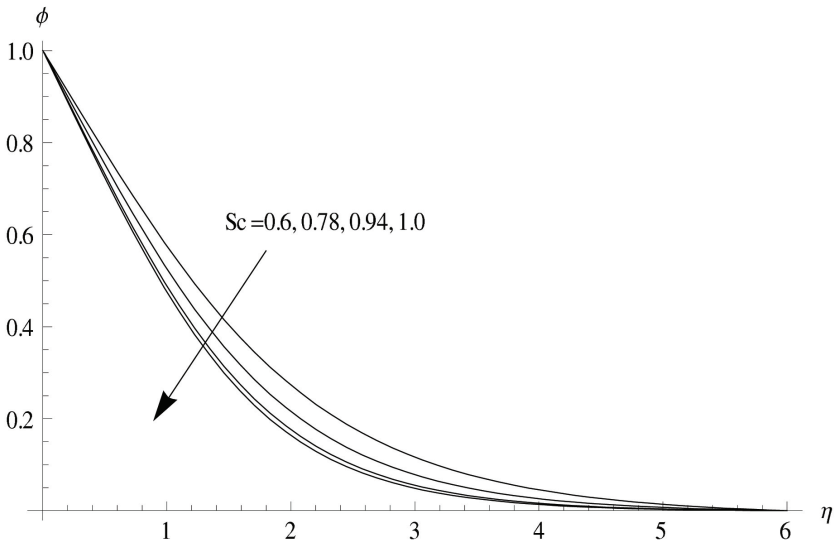

Figure 14: Concentration profiles for different values of Sc

[2] Megahed AA, Komy SR, and Afify AA (2003). Similarity analysis in magnetohydrodynamics: Hall effects on free convection flow and mass transfer past a semi-infinite vertical flat plate. *International Journal of Non-linear Mechanics* **38**, pp. 513–520.

[3] Beithou N, Albayrak K, and Abdulmajeed A (1998). Effects of porosity on the free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium. *Turkish Journal of Engineering & Environmental Sciences 22*, pp. 203–209.

[4] F. S. Ibrahim, M.A. Mansour, M.A.A. Hamad (2005), Lie-group analysis of radiative andmagnetic field effects on free convection and mass transfer flow past a semi-infinitevertical flat plate, *Electronic J. Differential Equations,* **39**, pp. 1–17.

[5] Soundalgekar V.M., Gupta S.K. and Birajdar N.S.(1979), Effects of mass transfer and free effects on MHD Stokes problem for a vertical plate, *Nucl.Eng.Des.*, Vol.53, pp.309-346.

[6] Elbashbeshy M.A.(1997), Heat and mass transfer along a vertical plate surface tension and concentration in the presence of magnetic field, *Int. J. Eng. Sci.*, **34**(5), pp.515-522.

[7] Helmy K.A.(1998), MHD unsteady free convection flow past a vertical porous plate,*ZAMM*, **78**, pp.255-270

[8] V.K. Kalpakides, K.G. Balassas (2004), Symmetry groups and similarity solutions for a freeconvective boundary-layer problem, *Int. J. Non-linear Mech.,* **39**, pp.1659-1670.

[9] M. Pakdemirli (1992), Similarity analysis of compressible boundary layers for arbitrary profiles, *Mech. Res. Comm.,* **19**(5), pp. 399–406.

[10] M. Pakdemirli and M. Yurusoy, Similarity transformations for partial differential equations, *SIAM Rev.,* **40**(1), pp. 96–101, 1998.

[11] M. Yurusoy, and M. Pakdemirli (1997), Symmetry reductions of unsteady three-dimensionalboundary layers of some non-Newtonian Fluids, *Int. J. Engg. Sci.,* **35**(8), pp. 731–740.

[12] M. Yurusoy and M. Pakdemirli (1999), Exact solutions of boundary layer equations of a specialnon-Newtionian fluid over a stretching sheet, *Mech. Res. Comm.,* **26**(2),pp. 171–175.

[13] M. Yurusoy, M. Pakdemirli (2001), Lie group analysis of creeping flow of a second grade fluid, *Int. J. Non-linear Mech.,* **36**, pp. 955–960.

[14] Gebharat B (1962). Effects of viscous dissipation in natural convection. *J. Fluid Mech.* **14**, pp.225-232.

[15] Gebhart B. and Mollendorf J. (1969) Viscous dissipation in external natural convection flows. *J. Fluid Mech.*, **38**, pp.97-107.

[16] Gnaneswara Reddy M and Bhaskar Reddy N (2010), Radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate with viscous dissipation, *Int. J. Appl. Math and Mech.*, **6**(6), pp. 96-110.

[17] Gnaneswara Reddy M and Bhaskar Reddy N (2011), Mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium, Journal of Applied Fluid Mechanics, **4** (3), pp. 7-11.

[18] S. Sivasankaran, M. Bhuvaneswari, P. Kandaswamy, E.K. Ramasami (2006), Lie roup analysis of natural convection heat and mass transfer in an inclined surface, Nonlinear Analysis: Modelling and Control, **11 (**1), pp. 201–212.

[19] Gnaneswara Reddy M and Bhaskar Reddy N (2010), Soret and dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation, *Int. J. Appl. Math and Mech.*, **6**(1), pp.1-12.

[20] Mukhopadhyay, S., Layek, G.C., Samad, S.A. (2005), Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. *Int. J. Heat Mass Transf.***48**, pp.4460–4466

[21] Jain M K, Iyengar S R K and Jain R K (1985). *Numerical Methods for Scientific and Engineering Computation.* Wiley Eastern Ltd., New Delhi, India.

Submitted in ...

Lie group analysis of heat and mass transfer effects on steady mhd free convection dissipative fluid flow past an inclined porous surface with heat generation

In this paper, an analysis has been carried out to study heat and mass transfer effects on steady two-dimensional flow of an electrically conducting incompressible dissipating fluid past an inclined semi-infinite porous surface with heat generation. A scaling group of transformations is applied to the governing equations. The system remains invariant due to some relations among the parameters of the transformations. After finding three absolute invariants, a third-order ordinary differential equation corresponding to the momentum equation, and two second-order ordinary differential equations corresponding to energy and diffusion equations are derived. The coupled ordinary differential equations along with the boundary conditions are solved numerically. Many results are obtained and a representative set is displayed graphically to illustrate the influence of the various parameters on the dimensionless velocity, temperature and concentration profiles. Comparisons with previously published work are performed and the results are found to be in very good agreement.

doi:10.2298/TAM12030233R ....