

MODIFICATION OF EARTH'S GRAVITY SPHERE

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MODIFICATION OF EARTH'S GRAVITY SPHERE

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Abstract. *The standard radius of the Earth's gravity sphere is 917.000 km. Here we present that the radius is 1.400.000 km.*

Key words: *Celestial mechanics, gravitation, Earth, Sun.*

1. INTRODUCTION

Earth's gravity sphere is a space around the Earth (as a material point) where in the Earth's force of attraction is stronger than the gravitational forces of other bodies, including the gravitational force of Sun. The formula that determines the radius ρ of the so called sphere of influences (gravity sphere) of the Earth's gravity in this case is ([1], p. 196),

$$(1) \quad \rho = r \sqrt[5]{(m_1/M)^2},$$

where r is the distance between the Earth and Sun, $m_1 = M_{\oplus}$ is the mass of the Earth, and $M_{\odot} \approx 333000 m_1$ is the mass of the Sun. The size of this radius of the Earth's sphere amounts approximately to $\rho = 917\,000\text{ km}$ or ([2], p.108) $923\,000\text{ km}$.

$$(3) \quad F = \kappa \frac{m_1 m_2}{\rho^2}.$$

Verification of the formula (1) with the use of the Newton's formula of "universal gravitational force"

$$(2) \quad F = \kappa \frac{m_1 m_2}{\rho^2},$$

led to a paradoxical result. According to formula (2), at the boundary of the Earth's gravity sphere, it should be $F_{\oplus} = F_{\odot}$. However, the calculation shows the opposite. And indeed, let us show this with some more details.

Let it be assumed that: $m_1 = M_{\oplus}$ is the mass of the Earth, $m_2 = M_{\odot}$ is the mass of the Sun, and m is the mass of any body at the boundary $\rho_{\oplus} = x = 917\,000\text{ km}$. For the above mentioned assertions of the book the mass of the Sun is $M_{\odot} \approx 333\,000 M_{\oplus}$, whereas a tabulated distance of the Earth from the Sun is $\rho_{\odot} = a = 149\,600\,000\text{ km}$.

First. The Sun and the Earth act at the same time on a body having the mass m in a critical boundary point at the distance $\rho_{\oplus} = x$ with the forces according to Newton's formula (2):

$$(3) \quad F_{\oplus} = \kappa \frac{M_{\oplus} m}{x^2}, \quad F_{\odot} = \kappa \frac{M_{\odot} m}{(r-x)^2}.$$

Therefore, in a critical point $\rho_{\oplus} = x = 917\,000$, it should be

$$(4) \quad F_{\odot} = \kappa \frac{M_{\odot} m}{(149\,600\,000 - x)^2} = 1,5063 \cdot 10^{-11} \kappa M_{\odot} m.$$

and

$$(5) \quad F_{\oplus} = \kappa \frac{M_{\oplus} m}{x^2} = 0,11892 \cdot 10^{-11} \kappa M_{\oplus} m.$$

This shows that, according to Newton's formula, the gravitational force of the Sun at the distance of 917 000 km from the center of the Earth is more than 12 times greater than the value of the Earth's gravitational force, i.e.

$$F_{\odot} = 12,666\,611 F_{\oplus} \iff F_{\oplus} = 0,0789478 F_{\odot}.$$

However, this is not in compliance either with the definition of gravity sphere, or with the phenomena in the nature. The Moon moves around the Earth at an average distance of 384 400 km, under the dominant attraction of the Earth, not the Sun.[7]

The second. Let's determine the boundary of the Earth's gravity sphere with the use of a strict procedure, by means of the universal gravity formula (2). According to the Newton's gravity theory (2) would follow, so that it should be:

$$\frac{M_{\oplus} m}{x^2} = \frac{M_{\odot} m}{(r-x)^2},$$

or for $M_{\odot} = 333\,000 M_{\oplus}$ follow $(\rho - x)/x)^2 = M_{\odot}/M_{\oplus} = 333\,000$.

Further calculation gives: $(\rho - x)^2 = (577,6152 x)^2$, i.e. $\rho - x = 577,6152 x$, or $\rho = 578,0652 x$, and from there, for $\rho = 149\,600\,000$ km, it follows that

$$x = 258\,795,993 \text{ km.}$$

This is contradictory to the fundamental laws of dynamics, as well as the actual state of the motion of the Moon around the Earth at an average distance of 384 400 km, and particularly the formula (1), which demonstrates the radius of the sphere of the Earth's gravity. Doubt about the validity of the Newton's formula is increased by a fact from the above mentioned book. According to the Newton's formula (1) it follows that the acceleration of gravity depends not only on the distance, but it is asserted that at the first cosmic velocity of 7.91 km/s, a body will escape from the Earth's attraction and will rotate around the planet Earth under an assumption that the resistance of the medium is ignored. At the second cosmic velocity $v_{or} = 11,19$ km/s, a missile will leave the area of the Earth's gravity sphere.

2. One modification theory of gravity

In the papers [1, 2, 3, 4] author is demonstrated that our formula of mutual action of two bodies has the form

$$(8) \quad F_{\rho} = \frac{\dot{\rho}^2 + \rho\ddot{\rho} - v_{or}^2}{m_1 + m_2} \frac{m_1 m_2}{\rho} = M^* \frac{\dot{\rho}^2 + \rho\ddot{\rho} - v_{or}^2}{\rho} = F^* + F^{**},$$

where we introduced notations:

$$M^* = \frac{m_1 m_2}{m_1 + m_2}, \quad F^* = M^* \frac{\dot{\rho}^2 + \rho\ddot{\rho}}{\rho}, \quad F^{**} = M^* \frac{v_{or}^2}{\rho}.$$

For the escaping boundary of the attraction of a body having a mass of m and the body having a mass of M , it will be

$$M^* \frac{\dot{\rho}^2 + \rho\ddot{\rho} - v_{or}^2}{\rho} = 0,$$

or in Simić's form

$$\frac{d}{dt}(\rho\dot{\rho}) - v_{or}^2 = 0.$$

For the purpose of clearer and more straightforward comprehension of this assertion, let us mention that formula (6), in relation to the natural coordinate system, can be reduced to a simpler form. It is sufficient to observe that it is $v^2 = \dot{\rho}^2 + \rho^2\dot{\theta}^2$ so as to reduce the formula (6) to a form

$$F_{\rho} = M^*(\ddot{\rho} - \rho\dot{\theta}^2).$$

In the state of motion where $F_{\rho} = 0$, the known formula for normal acceleration follows

$$\ddot{\rho} = \rho\dot{\theta}^2 = \frac{v^2}{\rho},$$

as well as formula for the force of mutual attraction

$$(7) \quad F^{**} = M^* \frac{v^2}{\rho},$$

where $\rho = R = \text{const.}$

Table. It has been shown what the radial accelerations of the satellites are at different altitudes H above the Earth according to the standard formula $\gamma = gR^2/\rho^2$, as well as the formula $\gamma^* = v^2/\rho$, which follows from the formula (6).

Altitude	Velocity	Acceleration	
H km	v km/s	γ	γ^*
0	7,91	981,0	982,3
100	7,84	948,9	950,0
1000	7,35	732,1	733,0
10000	4,93	148,4	148,4
100000	1,94	3,5	3,5
384400	1,02	0,002693	0.002706

Let's note that the last type of table refers to the average speed of the Moon's motion around the Earth and its average distance from the center of the Earth.

By the application of formula (8) to the motion of the Moon in relation to the Sun and in relation to the Earth, it has been proven that the gravitational force of the Earth, which acts on the Moon, is greater than the corresponding force of the Sun. In this way, dynamical paradox in the theory of the Moon's motion has been removed, [8]. It is logical that it is possible to determine the boundary of the Earth's gravity sphere in the same way.

Using this procedure, we obtain a significant modification of the Earth's gravitation sphere. Starting from the aforementioned definitional of the gravity sphere of two bodies, let us find the boundary x of the gravity sphere of the Earth in relation to the gravitational force of the Sun for that same body. By the very nature of things and by mathematical logics, initial relation of that task is that the gravitational force of the Earth is greater than, and at the boundary of the sphere $\rho = x$ is equal to, the Sun's gravitational force, i.e.,

$$F_{\oplus} = F_{\odot},$$

where :

$$F_{\oplus} = \frac{M_{\oplus} m}{M_{\oplus} + m} \frac{v_{or\oplus}^2}{x}, \quad v_{or\oplus} < 1 \text{ km/s.}$$

$$F_{\odot} = \frac{M_{\odot} m}{M_{\odot} + m} \frac{v_{or\odot}^2}{a - x}, \quad v_{or\odot} = 29,8 - (19,5 + 0,3) = 10 \text{ km/s.}$$

Ratio of the gravitational forces F_{\oplus} and F_{\odot} at the boundary of the Earth's gravity sphere is:

$$\frac{F_{\oplus}}{F_{\odot}} \equiv \frac{v_{or\oplus}^2}{x} : \frac{v_{or\odot}^2}{a - x} = 1.$$

From here, it follows that

$$(8) \quad x = \frac{a}{1 + (v_{or\odot}/v_{or\oplus})^2}.$$

Value of the fraction which is derived, depends, as we can see, on the ratio of the orbital speeds of bodies in relation to the Sun and the Earth at the boundary x of the Earth's gravity sphere. Let us analyze that for our needs.

First: $v_{or\odot} \neq v_{or\oplus}$, because it is $v_{or\odot} = v_{\oplus} \pm v_{or\oplus} - v_{\odot}$; $v_{or\oplus} \neq v_{\odot}$.

The second: For $v_{\oplus} = 1$ is $x = a/(1 + v_{or\odot}^2)$.

The third: for $v_{or} > 1$ the value of the fraction is decreased, and already for $v_{or} > 1$ the fraction (10) is decreased, and for $v_{or} < 1$ it is increased. In view of the fact, let us choose $v_{or} = 1$. As it can be seen, the boundary of the Earth's gravity sphere depends on the ratio of the speeds of two bodies in relation to the Earth $v_{or\oplus}$ and in relation to the Sun $v_{or\odot}$. Usually the velocity $v_{or\oplus}$ is not known, so that we are left only with a hypothetical analysis on the basis of the average standard data. The velocity of the Sun $v_{or\odot}$ is even less known. Speeds of the Sun in relation

to various groups of stars [4]. The standard velocity of the Sun is usually taken to be $v_{\odot} = 20\,000$ km/s. Since the mean velocity of the Earth's motion around the Sun is $v_{\oplus} \approx 30\,000$ km/s. In this state of motion, it is

$$v_{or\odot} \approx v_{\oplus} - v_{\odot} = 10\text{ km/s.}$$

For this logical choice and numerical values of the standard quantities (see for example [3]):

$$\begin{aligned} \frac{M_{\oplus} m}{M_{\oplus} + m} &= 0,987; & \frac{M_{\odot} m}{M_{\odot} + m} &= 0,999; \\ a &= 149\,600\,000\text{ km}, & M_{\odot} &= 333\,000 M_{\oplus}, \end{aligned}$$

it is obtained that the radius of the gravitation sphere of the Earth is $x = 1\,481\,188$ km, or

$$(9) \quad x \approx 1\,481\,000\text{ km.}$$

Therefore, for the standard data which are taken, the radius of the gravitation sphere of the Earth is significantly greater than the radius $x = 917\,000$ km, and expressly than $x = 258\,795$ km.

Conclusion

In the first part of this paper it is proven that the formula of the gravitational sphere of the Earth (1) has not been derived on the basis of the Newton's formula (3). By direct calculation with the use of the formula (3) it is shown that the formula leads to the results, which are not in accordance with the nature of the motion between the Sun and the Earth. Convincing example is the motion of the Moon, for which the formula (3) leads to paradoxical dynamic result of the Newton's gravity theory.

With the use of the formula (6) for the mutual attraction of two bodies, the above mentioned paradox in the theory of the Moon's motion is removed and one solution to the problem of three bodies (Sun-Earth-Moon) is obtained. That was a reason to consider the boundary (2) of the gravity sphere of the Earth in this paper. Approximately correct result for the radius of the Earth's gravity sphere on the basis of the formula (11) amounts to **1 400 000** km, which is considerably different from the value (2).

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MODIFIKACIJA ZEMLJINE GRAVITACIONE SFERE

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Apstrakt. Standardni radijus Zemljine gravitacione sfere je 917.000 km. Ovde se prikazuje je radijus 1.400.000 km.

Key words: *Nebeska mehanika, gravitacija, Zemlja, Sunce.*

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