# LOOPS IN THE SUNS ORBIT 

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[^0]According to: Tib Journal Abbreviations (C) Mathematical Reviews, the abbreviation TEOPM7 stands for TEORIJSKA I PRIMENJENA MEHANIKA.

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#### Abstract

Besides translation, spin around its axis and rotation around center of the Milky Way, the Sun performs relative motion in the solar system Laplacian plane, also. This motion was anticipated by Newton himself, in his Principia.

The form of the Sun's orbit is substantially different from the other solar system bodies' orbits. Namely, the Sun moves along the path composed of the chain of large and small loops $[1,2,6,9]$. This chain is situated within the circular outline with the diameter approximately twice as large as the Sun's is. Under supposition that the solar system is stable, the Sun is going to move along it, in the same region, for eternity, never reitereiting the same path.

It was also shown in this work that velocity and acceleration of the Sun's center of mass are completely defined by the relative velocities and accelerations of the planets with respect to the Sun.


Keywords loops in the Sun's orbit, velocity, acceleration

## 1. Introduction

As said, Isaac Newton was the first who pointed to the fact that the Sun moves around the center of mass of the solar system (Philosophiae Naturalis Principia M athematica): „...since that centre of gravity (the solar system mass centre) is constantly at rest, the sun, according to the various positions of the planets, must continually move every way, but will never recede far from that centre".

If the solar system is treated as a stable, isolated system of the point mass particles moving under mutual gravitational interactions, two dynamic conservation principles may be used for the study of its motion: conservation of the momentum and of the angular momentum of the system.

The consequence of the first rule is uniform motion of the system's mass center C , while the consequence of the second is motion of the system in one, Laplacian, or invariant plane [3].

This plane is within $0,5^{0}$ of the Jupiter's orbital plane and may be regarded as the weighted average of all planetary orbital planes. The point mass particles solar system model, involves necessity of neglecting differences between the orbital planes mainly
originated in the transfer of the (small, but changeable) Sun's and planet's spin angular momenta to its total angular momentum.

Neglecting rotation of the solar system, as a whole, around the center of our galaxy, this plane moves translatory, together with the mass center $C$ through the space.

## 2. Coordinate Sy stems

Existence of the invariant plane permits introduction of an "inertial" reference frame $x C y$ lying in it. A nother Cartesian coordinate system $x^{\prime} 0 y^{\prime}$ ( $0 x^{\prime}$ and $0 y^{\prime}$ parallel with Cx and Cy ) was adopted as the relative, that is, the heliocentric frame of reference (Fig. 1).


Fig. 1. Inertial and Relative Frame of Reference.

## 3. Position, Velocity and Acceleration Vectors of the Sun

It seems that, using astronometrical data positioning the outer planets from 1653. to 2060., P. D. Jose was the first one to determine the Sun's orbit, in his work [1], 1965. This paper was abundantly cited later in the works treating the sunspot activities.

In his work [2] R. Bitsch has determined the Sun's orbit integrating differential equations of motions of this celestial body exposed to the resultant of the planets' attracting gravitational forces. It was supposed that these heavenly bodies are in circular, uniform motions around the Sun. The initial planet's configuration was adopted arbitrary, because the author assumed that, in the long term, it does not affect the shape of the orbit.

One would be tempted to use term K epler's for the model employed in the work [1] and Copernicus' for the model in [2], but it would not be correct, since the Sun moves in both models, of course.

In fact, the choice of either model in determining the trajectory, velocity or acceleration of the Sun is completely irrelevant, since the orbit, velocity and acceleration of the Sun's center of mass are very small compared with the correspondent kinematic parameters characterizing motions of the planets. Correctly defining the initial conditions is all that matters.

For that reason "Copernican" model is the adopted here: the planets move at an average distance, with average angular velocity around the Sun.

Taking the planets' configuration on 21 M arch 1978 [5] as initial conditions and the model in which all planets, excepting Mercury (its orbit is not stabilized in the solar system's invariant plane yet) move uniformly around the Sun, M .M arjanov [9] obtained, practically, the same form of the Sun's path as J ose had. This trajectory covers the time interval of fifty years: 21. 03. 1978. +50 .

Since, in the inertial plane of reference must be

$$
\sum_{k=1}^{8} m_{k} \vec{r}_{k}=\sum_{k=1}^{8} m_{k} \vec{v}_{k}=\sum_{k=1}^{8} m_{k} \vec{a}_{k}=0
$$

and

$$
m_{0} \vec{r}_{0}+\sum_{k=1}^{8} m_{k}\left(\vec{r}_{0}+\vec{R}_{k}\right)=0, \quad m_{0} \vec{v}_{0}+\sum_{k=1}^{8} m_{k}\left(\vec{v}_{0}+\dot{\overrightarrow{R_{k}}}\right)=0, m_{0} \vec{a}_{0}+\sum_{k=1}^{8} m_{k}\left(\overrightarrow{\mathrm{a}}_{0}+\ddot{\vec{R}}_{k}\right)=0
$$

it follows out that

$$
\vec{r}_{0}=-\frac{\sum_{k=1}^{8} m_{k} \vec{R}_{k}}{M}, \quad \vec{v}_{0}=-\frac{\sum_{k=1}^{8} m_{k} \dot{\vec{R}_{k}}}{M} \text { and } \vec{a}_{0}=-\frac{\sum_{k=1}^{8} m_{k} \ddot{\vec{R}_{k}}}{M} \text {, where } M=\sum_{k=1}^{8} m_{k}
$$

Thus, position, velocity and acceleration of the Sun's center of mass are completely defined by the relative positions, velocities and accelerations of the planets with respect to the Sun.

Fig. 2 shows the orbit of the Sun from 21.03.2000. to 21.03.2040. It, as mentioned, corresponds to the orbit that got Jose /1/, but is substantially different from the one given by Bish /2/, as the latter had wrongly assumed that initial conditions do not affect the form of a path.

The contour of the Sun is given as referential and the dot marks denote years.
Concerning the influence of the inner planets on the form of the path, Jose was right: it is quite negligible. Their influence would become visible only if a part of seemingly smooth path was magnified thousand to ten thousand times. The influence of the inner planets actions is far more evident when speed and acceleration of the Sun are considered.

The maximal distance from Sun to the center of mass of $C \sim 1.5 \cdot 10^{\circ} \mathrm{km}$ is obtainable when all planets are lined up on the same side of the star and the minimal $\sim 21000 \mathrm{~km}$, when Jupiter is on one side and all the other planets on the other side of the Sun, in the same direction.

Of course, chances for exactly such alignments of the celestial bodies are reduced to zero /11/.

The anticipated path of the Sun over the next 2000 years is represented in the Figure 3. It is situated within the circular outline with the diameter at least twice size of the Sun's. A gain, the Sun's disk is given for comparison.

Provided that the solar system is stable, Sun is going to move in this region for eternity, never reitereiting the same path.


Fig.2. Sun's Path from 21.03.2000. - 21. 03.2040.
4. Orbit
6. Velocity

Sun's velocity from 21 03.2000. to 21 03.2040. is represented in the Figure 4. Here we can see again that the outer, giant planets, especially Jupiter and Saturn have dominant influence on this kinematical quantity. When these two planets are in conjunction ( ~ every 20 years), the speed of the Sun is maximal $\approx 16 \mathrm{~m} / \mathrm{s}(15.7 \mathrm{~m} / \mathrm{s})$, and when in opposition, the velocity is minimal, about $9 \mathrm{~m} / \mathrm{s}(8,8 \mathrm{~m} / \mathrm{s})$.

## 6. Acceleration

The acceleration diagram from 21. 03.2000. to 21. 03.2040. is given in the Figure 5. It shows much more irregularities and roughness than the previous diagram and represents, of course, a measure of the resulting planets' gravitational attractions. No need to say that the peaks in this diagram correspond to the different conjunctional combinations of two or more planets.


Fig. 3. Sun's Orbit Over 2000 Y ears.
$\vec{v}_{0}=-\frac{\sum_{k=1}^{8} m_{k} \vec{R}_{k}}{M} \Rightarrow$


Fig. 4. Sun's V elocity from 21. 03.2000. to 21. 03.2040.

If all the planets fell in the same direction, on the same side of the Sun, therefore, when they were all in conjunction, the Sun would have a maximal acceleration $\sim 0,27 \mu / s^{2}$.

W hen J upiter is on one side and all the other planets on the other side of the Sun, in the same direction, acceleration of the Sun is minimal $\sim 0,14 \mu / s^{2}$.

$$
\overrightarrow{\mathrm{a}}_{0}=-\frac{\sum_{\mathrm{k}=1}^{8} \mathrm{~m}_{\mathrm{k}} \overrightarrow{\mathrm{R}}_{\mathrm{k}}}{\mathrm{M}} \Rightarrow
$$



Fig. 5. Sun's A cceleration from 21 03.2000. to 21 03.2040.

Table: A verage Sun - Planets \& M aximal Planet- Planets Interactions

| Sun |  | Me | $\mathbf{V}$ | $\mathbf{E}$ | $\mathbf{M a}$ | $\mathbf{J}$ | $\mathbf{S}$ | $\mathbf{U}$ | $\mathbf{N}$ |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sun |  | 3,13 | 13,20 | 8,52 | 0,39 | 100,00 | 8,90 | 0,34 | 0,16 |
| Me | 3,13 |  | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $\mathbf{V}$ | 13,20 | 0,00 |  | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| E | 8,52 | 0,00 | 0,00 |  | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| Ma | 0,39 | 0,00 | 0,00 | 0,00 |  | 0,00 | 0,00 | 0,00 | 0,00 |
| $\mathbf{J}$ | 100,00 | 0,00 | 0,00 | 0,00 | 0,00 |  | 0,04 | 0,00 | 0,00 |
| $\mathbf{S}$ | 8,90 | 0,00 | 0,00 | 0,00 | 0,00 | 0,04 |  | 0,00 | 0,00 |
| $\mathbf{U}$ | 0,34 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |  | 0,00 |
| $\mathbf{N}$ | 0,16 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |  |

## 7. Solar Sy stem Bodies' Gravitational Interactions

Previous diagram will be more understandable if one looks at the following Table of the of the mean Sun - planets and the maximal planet- planets interactions.

The exposed table contains two kinds of data. The first row and the first column represent mean $F_{0-k}, k=1,2, \ldots, 8$, that is, the average Sun - planets interactions. These forces are average because of the adopted "Copernicus' " and not of the "Kepler's" model.

Of course, the Sun-Jupiter interaction is the greatest one and it was taken to be the referential: its value is 100 . An interesting fact is that the second one is the Sun - V enus interaction and that the Sun-Earth is of the same order of magnitude as the Sun-Saturn interaction, although the Saturn's mass is 95 times greater than the E arth's.

All the other values in the Table 1 represent $\sup F_{i-j}, k, j=1,2, \ldots .8$ : maximal, that is, conjunctional interactions of the planets. In the range of the here adopted five digit precision, the only existing is the one between Jupiter and Saturn.

It is evident that motions of the inner planets, especially that of $V$ enus and Earth are the main cause of the fluctuations around a smooth acceleration curve representing the influence of the outer planets, and above all, the influence of J upiter Saturn.

## 8. CONCLUSION

The Sun and the planets in the paper were simulated by the model of stable, isolated system of the point mass particles, moving under mutual gravitational interactions in the averaged Laplacian plane.

Shape of the solar path is entirely different from the orbits of the other bodies moving around it. Namely, the Sun moves along the path in the form of chain consisting of large and small loops. This chain is situated within the circular outline with the diameter approximately twice as Iarge as the Sun's is. Under supposition that the solar system is stable, the Sun is going to move along it, in the same region, for eternity, never reitereiting the same path.

Then the velocity and acceleration diagrams of center of mass of the Sun were given.
At last, for better understanding of the obtained results, Table of the of the mean Sun - planets and the maximal planet- planets interactions was contributed.

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## PETLJ E U SUNČEVOJ PUTANJ I Milutin Marjanov

Pored translatornog kretanja, rotacije oko centra M lečnog Puta i oko svoje ose, Sunce obavlja i relativno kretanje u Laplasovoj ravni Sunčevog sistema. Na to kretanje je ukazao još Njutn, u svome delu Principia.

Oblik Sunčeve putanje bitno se razlikuje od orbita ostalih tela koje oko njega kruže. Sunce se, naime, kreće duž putanje u formi lanca sačinjenog od velikih i malih petlji. Taj lanac je smešten u okvir kružne konture čiji je prečnik približno dva puta veći od prečnika Sunca. Uz pretpostavku da je Sunčev sistem stabilan, Sunce će se večno kretati duž njega, u istoj oblasti, nikada ne ponavljajući isti put.

Zatim su dati dijagrami brzina i ubrzanja centra mase Sunca.
Na kraju je priložena tablica prosečnih interakcija Sunce - planete, kao i maksimalnih interakcija medu planetama, s tačnošću od pet brojčanih jedinica. Ona je dobar pokazatelj zbog čega na oblik orbite, kao i na dijagram brzina najviše utiču spoljne, džinovske planete, dok na izrazito neujednačenost forme dijagrama ubrzanja utiču, pre svega, unutrašnje planete.

KIjučnereči: petlje u Sunčevoj orbiti, brzina i ubrzanje centra mase Sunca

Submitted on April 2009, accepted on June 2012


[^0]:    *doi:10.2298/TAM1301127M Math. Subj. Class.: 70F15; 70M20;70M99.

