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TENSOR MODELING OF THE GEOMAGNETIC FIELD A REVIEW

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Tensor modeling of the geomagnetic field
A review

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From the cover of the book

Blurb/Shorttext:

Mathematical modeling of the dynamics of Artificial Earth Satellites (AES), interacting with Earth's Magnetic Field (EMF), requires a prior solution to the problem of modeling of the EMF, characterized by a complex structure and the absence in the final form of the dependence of the magnetic induction on the coordinates of the near-Earth space. Analysis of this issue and the subject of this monograph offers a tensor-based analytical modeling methodology for capacity building, and the gradient induction EMF induction and subsequent choice of a suitable model for a successful analytical inventories and / or numerical study of dynamic problems of the satellite. Along with the possibility of justification of the tensor approach to multipole modeling inventories, they are concise writing and ease of use in the computer algebra systems, the work contains a ready-made algorithms and programs. The results obtained in equal measure can be applied to describe the Earth's gravitational field and gravitational and magnetic fields of other planets. Therefore, the monograph is intended not only to readers concerned with analysis of the dynamics of the satellite, but also for all those interested in mathematical modeling of planetary physical fields.

1. Classical models of Earth's magnetic field,
2. Quadrupole approximation of Earth's magnetic field,
3. Multipole structure of the Earth's magnetic field in an arbitrary N-th approximation,
Appendix A. Programs in to construct multipole of the magnetic field of the Earth,
Appendix B. The program for construction of the magnetic induction and selection of the correct approximation of the magnetic field of the Earth.

About Monograph

Solving of modern cosmic dynamic problems takes into consideration orbital motion of AES (artificial Earth satellites), as well as their motion around a specific center of mass, and requires a detailed analysis of forces and moments that affect AES in Earth's field.

All of this analysis refers to setting up an adequate mathematical model of AES dynamics which enables, with sufficient degree of accuracy, to consider and predict the motion of AES using analytical and numerical methods.

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During its motion in space around the Earth, AES is subjected to various forces and moments caused by its interaction with Earth's magnetic field (EMF), such as magnetic forces, due to the existence of solenoids, hysteresis rods and natural magnets (in terms of AES itself being magnetized in EMF) on the AES, Lorentz forces due to the motion of the AES "infected" by the EMF, Coulomb's forces caused by AES interaction with the plasma in space around the Earth, forces caused by Foucault eddy currents and others. Mathematical description of the forces and moments mentioned above requires information about EMF's induction (sometimes not only induction but also its gradient) as a function of a radius vector in Earth's vicinity. Due to EMF structure complexity and a lack of its functional dependence in analytical form, the problem of mathematical modeling of EMF is an unavoidable stage in formulating of a most adequate mathematical model for AES dynamics occurs. Based on this problem, further analytical and/or numerical consideration of this or other cosmic problem can be performed.

This monograph is dedicated to analyzing and solving of such problems, written by the author based on years of his experience in studying problems of AES motion related to a center of mass, under the influence of EMF.

In Chapter 1, the author presents the question of EMF modeling, using classic models for studying of AES motion dynamics under the influence of EMF. Main attention was given to the EMF model named "straight dipole", which is most used in analytical considerations, along with "oblique dipole" models. For EMF, modeled with "straight dipole", an accurate expression for magnetic induction was obtained using EFM gradient. At the same time it was shown that certain, simple according to literature, approach to the use of EMF gradient is incorrect, as opposed to the use of curvatures of EMF force lines around the AES. The use of a well-known, simpler EMF model, the "straight magnetic dipole" enabled the solving of a greater number of important practical problems. Also, using such models does not always lead to qualitatively better results, in case that they don't enable the full use of Earth's day-night rotation effects on EMF induction (vector **B**). For example, during the analysis of the rotating motion dynamics of the "infected" AES, principle necessity of using day-night EMF rotation arises, due to its essential effect on AES rotating motion. It is shown that this effect is not only expressed quantitatively, but also qualitatively, e.g. in change in nature of motion stability during the occurrence of new linear and non-linear parametric resonances, which can lead to increasing oscillations of AES. Unfortunately, the "straight dipole" model does not allow the effects of day-night rotation of EMF on "infected" AES rotating motion to be fully expressed, due to its symmetry related to the day-night Earth rotation axis, which leads to the need to replace said model with another, more general one.

The next in complexity EFM model, the "oblique magnetic dipole", is indeed devoid of the above mentioned flaw. In addition, simply replacing the "straight dipole" model with the "oblique dipole" does not take into account all adjustments made to the EMF induction of the same magnitude. This suggests that, from the standpoint of AES dynamics problems, "oblique dipole" EMF models do not improve the "straight dipole" ones. Therefore, in order to set up the mathematical model of AES rotation, coupled with EMF by means of, e.g. magnetic or Lorentz moments, which are proportional to the magnetic induction B, while correctly using all members of the same order, required for the expression for vector **B**, it is necessary to take into account the following effect, i.e. the superposition of two dipoles that determine the geomagnetic potential, and possibly even, higher order components (depending on the given problem).

In Chapter 2, during the process of solving the problems formulated above, a quadrupole approximation of EMF is taken into consideration, and a mathematical apparatus that enables analytical determining of vector B for a quadrupole approximation is suggested. For this purpose, a general from of solution was found for the problem of determining the gradient of a random harmonic function U and the gradient of the vector of field $\mathbf{B} = -\text{grad}U$ by its known scalar potential $U = U(x^1, x^2, x^3)$, determined in relation to a random curved orthogonal coordinate system with x^1, x^2, x^3 coordinates. The comparison of quadrupole EMF model and simpler ones is given. It was shown that determining the vector **B** by using a quadrupole member of EMF potential, results in adjustments of such a low order, similar to those achieved by replacing the EMF magnetic dipole with the "straight dipole", and thereby brings a much greater essential harmony to the calculation of vector B when compared to the use of EMF gradient within the dipole model.

In chapter 3, the development (elaboration) of the mathematical apparatus presented in chapter 2 is given. The possibility of multipole representation of geomagnetic potential, along with magnetic induction and its gradient with arbitrary degree of accuracy, is explained from a programming and algorithm point of view. Expressions obtained are characterized by their short track and convenience of use during the analytical, as well as the computer consideration with the use of computer algebra or numerical methods. For practical use in the applied problems of cosmic dynamics, analytical expressions have been determined for the first seven multipole tensors of EMF, presented by Gauss coefficients.

The algorithm was also taken into consideration, and a program was developed (see appendix A) for the mathematical software MAPLE, in order to realize symbolic calculations, thus enabling the determining of analytical expressions for the components of a multipole tensor of arbitrary range. In addition, a program was made (see appendix B) to enable the analytical determining of vector **B** for an arbitrary final approximation.

Some technical details

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