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TRANSIENT FREE CONVECTIVE MHD FLOW PAST AN INFINITE VERTICAL CYLINDER

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According to: *Tib Journal Abbreviations (C) Mathematical Reviews*, the abbreviation TEOPM7 stands for TEORIJSKA I PRIMENJENA MEHANIKA.

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Transient free convective MHD flow past an infinite vertical cylinder

Rudra Kanta Deka* **Ashish Paul†**

Abstract

This paper presents an analytical solution of unsteady one-dimensional natural convective flow of a viscous incompressible and electrically conducting fluid past an infinite vertical cylinder with constant temperature and magnetic field, applied normal to the direction of flow. Exact solutions of dimensionless unsteady linear governing equations are obtained by using Laplace transform technique. Numerical computations for the transient velocity, temperature, skin-friction, Nusselt number are computed and presented in graphs for various set of physical parametric values viz; Grashof number, Prandtl number, magnetic parameter and time.

Keywords: Vertical cylinder, MHD flow, Laplace transform, Free convection

Nomenclature

- B_0 Magnetic induction intensity
- g Acceleration due to gravity
- J_0 Bessel function of first kind and order zero
- J_1 Bessel function of first kind and order one
- K_0 Modified Bessel function of second kind and order zero
- K_1 Modified Bessel function of second kind and order one
- R Dimensionless radial coordinate
- t' Time
- T' Temperature
- T Dimensionless temperature
- u x-component of velocity
- U Dimensionless velocity

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V Dummy real variable used in integrals

Y_0 Bessel function of second kind and order zero

Y_1 Bessel function of second kind and order one

Greek symbols

α Thermal diffusivity of fluid

β Coefficient of thermal expansion of fluid

ν Kinematic viscosity

ρ Density

σ Electrical conductivity of the fluid

1 Introduction

Unsteady free convection flow of a viscous incompressible fluid over a vertical cylinder with constant temperature have attracted attention of many researchers because of their wide applications in the field of engineering and geophysics, such as start-up of a chemical reactor and emergency cooling of a nuclear fuel element by forced circulation in case of power failure. Studies of free convection flow along a vertical cylinder are important in the field of geothermal power generation and drilling operations. In glass and polymer industries, hot filaments, which are considered as a vertical cylinder, are cooled as they pass through the surrounding environment. Sparrow & Gregg [1] first studied the heat transfer from vertical cylinders. Goldstein & Briggs [2] presented an analysis of the transient free convection heat transfer problem from vertical flat plates and vertical circular cylinders to a surrounding initially quiescent fluid by employing Laplace transform technique. Bottemanne [3] presented an experimental results of simultaneous heat and mass transfer by free convection about a vertical cylinder placed in still air for Pr=0.71 and Sc=0.63.

Magnetohydrodynamic flows and heat transfer processes occur in many industrial applications such as the geothermal system, aerodynamic processes, chemical catalytic reactors and processes etc. The earth's magnetic field is thought to be produced by electric currents extends outward from the earth's core into inter-planetary space, wherein encounters the magnetic field and moving charged plasma of the solar wind. The solar wind flows around the earth's magnetic field but distorts the field as it does so. All the magnetic fields are the result of moving electric charges. Several researchers have investigated natural convection boundary layer flow of an electrically conducting fluid in presence of magnetic field. Arora & Gupta [4] presented an exact solution for the magnetohydrodynamic flow between two rotating cylinders under radial magnetic field. Agarwal *et al.* [5] analyzed the effect of MHD free convection and mass transfer flow past a vibrating infinite vertical circular cylinder. Raptis & Agarwal [6] studied the effect of MHD free convection and mass transfer on the flow past an oscillating infinite coaxial vertical circular cylinder. Ganesan & Loganathan [7] presented an analysis of magnetic field effect on a semi-infinite moving vertical cylinder with constant heat flux by employing an implicit finite-difference scheme of Crank-Nicolson type. Elgazery & Hassan [8] presented a numerical study of radiation effect on MHD transient mixed-convection flow over a moving vertical cylinder with constant heat flux through a porous medium.

Recently, Reddy & Reddy [9] performed a numerical study of the interaction of radiation and mass transfer effects on unsteady MHD free convection flow of an incompressible viscous fluid past a semi-infinite moving vertical cylinder by finite-difference scheme of Crank-Nicolson type. Reddy & Reddy [10] also studied the interaction of free convection with thermal radiation of a viscous incompressible unsteady MHD flow past a semi-infinite vertical cylinder with variable surface temperature and concentration numerically.

However, no exact solution on unsteady free convective flow past vertical cylinder with magnetic field effect seems to have been reported and this motivates the present investigation. The problem of unsteady magnetohydrodynamic flow past vertical cylinders have important applications in the study of geological formations, in the exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. The objective of the present study is to make an analytic investigation of unsteady one-dimensional free convective flow of a viscous incompressible and electrically conducting fluid past an infinite vertical cylinder with constant temperature under the action of uniform magnetic field applied normal to the direction of flow. The governing boundary layer equations along with the initial and boundary conditions are first transformed into a dimensionless form and the exact solutions of the resulting system of equations are obtained by using Laplace transform technique. The behaviour of the velocity, temperature, skin-friction and Nusselt number are investigated for various set of physical parameters viz; Grashof number, magnetic field parameter, Prandtl number and time.

2 Mathematical analysis

An unsteady one-dimensional laminar free convective flow of a viscous incompressible fluid past an infinite vertical cylinder of radius r_0 is considered. The x-axis is being taken vertically upwards along the axis of the cylinder and the radial co-ordinate r is taken normal to the cylinder. Initially, it is assumed that the cylinder and fluid are at the same temperature T'_∞. It is also assumed that at $t' > 0$, the temperature of the cylinder raised to constant temperature T'_w. A uniform magnetic field is applied in the direction perpendicular to the cylinder. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity and hence the induced magnetic field is negligible in comparison to the applied magnetic field. The viscous dissipation is also assumed to be negligible in the energy equation as the motion is due to free convection only. It is also assumed that all the fluid properties are constant except for the density in the buoyancy term, which is given by the usual Boussinesq's approximation. Under these assumptions the governing boundary layer equations are:

$$\frac{\partial u}{\partial t'} = g\beta\left(T' - T'_\infty\right) + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\sigma B_0^2 u}{\rho} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right) \tag{2}$$

with initial and boundary conditions,

$$\left.\begin{array}{llll} t' \le 0: & u = 0, & T' = T'_\infty & \forall \;\; r \\ t' > 0: & u = 0, & T' = T'_w & at \;\; r = r_0 \\ & u \to 0, & T' \to T'_\infty & as \;\; r \to \infty \end{array}\right\} \tag{3}$$

In order to write the governing equations, initial and boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\left.\begin{array}{llll} R = \dfrac{r}{r_0}, & U = \dfrac{ur_0}{\nu}, & t = \dfrac{t'\nu}{r_0^2}, & T = \dfrac{T' - T'_\infty}{T'_w - T'_\infty} \\ Pr = \dfrac{\nu}{\alpha}, & Gr = g\beta r_0^3\dfrac{T'_w - T'_\infty}{\nu^2}, & M = \dfrac{\sigma B_0^2 r_0^2}{\nu\rho} & \end{array}\right\} \tag{4}$$

In view of the above, the governing Eqs. (1) and (2) reduce to the following non-dimensional form,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} - MU + GrT \tag{5}$$

$$Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} \tag{6}$$

with corresponding initial and boundary conditions as:

$$\left.\begin{array}{lllll} t \leq 0: & U = 0, & T = 0 & \forall & R \\ t > 0: & U = 0, & T = 1 & at & R = 1 \\ & U \to 0, & T \to 0 & as & R \to \infty \end{array}\right\} \tag{7}$$

Here, M is the magnetic field parameter, Gr is the Grashof number, and Pr is the Prandtl number.

3 Solution technique

In order to solve the governing unsteady non-dimensional Eqns.(5) and (6) subject to initial and boundary conditions (7), we apply usual Laplace transform technique.

Laplace transform of Eqns.(5) and (6) gives,

$$\frac{d^2\overline{U}}{dR^2} + \frac{1}{R}\frac{d\overline{U}}{dR} - (M + p)\,\overline{U} + Gr\overline{T} = 0 \tag{8}$$

and

$$\frac{d^2\overline{T}}{dR^2} + \frac{1}{R}\frac{d\overline{T}}{dR} - Prp\overline{T} = 0 \tag{9}$$

Where, p is the parameter of Laplace transform, defined by $Lf(t) = F(p)$, L being the Laplace operator, and $\overline{U}$, $\overline{T}$ are the Laplace transforms of U and T respectively.

Solution of (9), subject to transformed initial and boundary conditions (7) gives,

$$\overline{T} = \frac{K_0\left(R\sqrt{Prp}\right)}{pK_0\left(\sqrt{Prp}\right)} \tag{10}$$

Using equation (10), the solution of (8) subject to transformed initial and boundary conditions (7) gives,

$$\overline{U} = \frac{Gr}{p\,\{M + p\,(1 - Pr)\}}\left\{\frac{K_0\left(\sqrt{Prp}R\right)}{K_0\left(\sqrt{Prp}\right)} - \frac{K_0\left(\sqrt{M+p}R\right)}{K_0\left(\sqrt{M+p}\right)}\right\} \tag{11}$$

(The way of deriving $\overline{U}$ and $\overline{T}$ is presented in the Appendix)

Now, using the theorem of inverse Laplace transform in equation (10), we have,

$$T = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\frac{K_0\left(R\sqrt{pPr}\right)}{pK_0\left(\sqrt{pPr}\right)}dp \tag{12}$$

The integrand of (12) has a simple pole at $p = 0$ and a branch point at $p = 0$

Now $K_0(\sqrt{pPr})$ do not have zero at any point in the real and imaginary plane if the branch cut is made along the negative real axis. To obtain $T(t, R)$ from $\overline{T}(p, R)$, we use the adjoining contour Fig.1. Therefore the line integral in (12) may be replaced by the limit of the sum of the integrals over FE, ED, DC, CB, and BA as $S_1 \to \infty$ and $S_0 \to 0$.

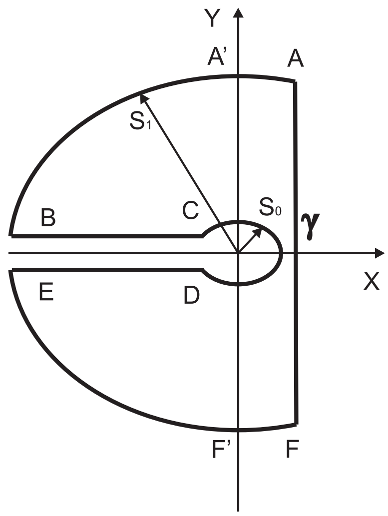

Figure 1: Path of contour integration for the inverse integral.

The particular form of the inversion integral, equation (12) has chosen because the value along the paths DC, BA and FE approaches zero as $S_1 \to \infty$ and $S_0 \to 0$.

Along the paths CB and ED we choose $p = e^{i\pi}V^2/Pr$ and $p = e^{-i\pi}V^2/Pr$, respectively.

On the path CB,

$$T_{CB} = \frac{1}{\pi i}\int_{0}^{\infty} e^{-\frac{V^2}{\mathrm{Pr}}t}\frac{J_0\left(RV\right) - iY_0\left(RV\right)}{V\left\{J_0\left(V\right) - iY_0\left(V\right)\right\}}dV \tag{13}$$

and on the path ED,

$$T_{ED} = \frac{1}{\pi i} \int_0^{\infty} e^{-\frac{V^2}{Pr}t} \frac{J_0(RV) + iY_0(RV)}{V\{J_0(V) + iY_0(V)\}} dV \tag{14}$$

The sum of the integrals along CB and ED gives,

$$T_{CB+ED} = \frac{2}{\pi} \int_0^{\infty} \frac{e^{-\frac{V^2}{Pr}t}}{V} \frac{J_0(RV)\,Y_0(V) - Y_0(RV)\,J_0(V)}{J_0^2(V) + Y_0^2(V)} dV \tag{15}$$

Also, the residue of the integrand of (12) at the point $p = 0$ is $= 1$
Thus from the theory of residues we have,

$$T = 1 + \frac{2}{\pi} \int_0^{\infty} e^{-\frac{V^2}{Pr}t} \Gamma(R, V) \frac{dV}{V} \tag{16}$$

where

$$\Gamma(R, V) = \frac{J_0(RV)\,Y_0(V) - Y_0(RV)\,J_0(V)}{J_0^2(V) + Y_0^2(V)} \tag{17}$$

Similarly, the inverse Laplace transform of equation (11) gives the expression of velocity profile as:

$$\begin{aligned} U \;=\; & \frac{Gr}{M}\left\{1 - \frac{K_0\left(R\sqrt{M}\right)}{K_0\left(\sqrt{M}\right)}\right\} + \\ & \frac{2Gr}{\pi}\int_0^{\infty} \frac{e^{-\frac{V^2}{\mathrm{Pr}}t}}{\left\{M - V^2\left(\frac{1}{Pr} - 1\right)\right\} V} \Gamma(R, V)\, dV \\ & + \frac{2Gr}{\pi}\int_0^{\infty} \frac{Ve^{-\left(V^2+M\right)t}}{(V^2+M)\{(V^2+M)(1-Pr) - M\}} \Gamma(R, V)\, dV \end{aligned}$$

(for $M \neq 0$) (18)

$$\begin{aligned} U \;=\; & \frac{2GrPr}{(Pr-1)\,\pi} \int_0^{\infty} \left(1 - e^{-\frac{V^2}{Pr}t}\right) \\ & \left\{\Gamma\left(R, \frac{V}{\sqrt{Pr}}\right) - \Gamma(R, V)\right\} \frac{dV}{V^3} \end{aligned}$$

(for $M = 0$) (19)

Knowing the velocity and temperature fields, it is necessary to study the skin-friction and Nusselt number. In non-dimensional form the skin friction and Nusselt number are defined respectively as follows:

$$\tau = -\left.\frac{\partial U}{\partial R}\right]_{R=1} \tag{20}$$

$$Nu = -\left.\frac{\partial T}{\partial R}\right]_{R=1} \tag{21}$$

3.1 Skin Friction

Expressions for the skin-friction τ obtained from Eqs. (18) and (19) as,

$$\begin{aligned}\tau &= -\frac{Gr}{\sqrt{M}}\frac{K_1\left(\sqrt{M}\right)}{K_0\left(\sqrt{M}\right)} + \\ &\frac{2Gr}{\pi}\int_0^\infty \frac{e^{-\frac{V^2}{Pr}t}}{\left\{M - V^2\left(\frac{1}{Pr}-1\right)\right\}}\Gamma_1(V)\,dV + \\ &\frac{2Gr}{\pi}\int_0^\infty \frac{V^2 e^{-\left(V^2+M\right)t}}{\left(V^2+M\right)\left\{\left(V^2+M\right)(1-Pr)-M\right\}}\Gamma_1(V)\,dV \\ (\text{for } M &\neq 0)\end{aligned} \tag{22}$$

$$\begin{aligned}\tau &= \frac{2GrPr}{(Pr-1)\pi}\int_0^\infty \left(1 - e^{-\frac{V^2}{Pr}t}\right) \\ &\left\{\frac{1}{\sqrt{Pr}}\Gamma_1\left(\frac{V}{\sqrt{Pr}}\right) - \Gamma_1(V)\right\}\frac{dV}{V^2} \\ (\text{for } M &= 0)\end{aligned} \tag{23}$$

3.2 Nusselt number

Expression for Nusselt number Nu obtained from Eq.(16) as:

$$Nu = \frac{2}{\pi}\int_0^\infty \left\{e^{-\frac{V^2}{Pr}t}\Gamma_1(V)\right\} dV \tag{24}$$

where

$$\Gamma_1(V) = \frac{J_1(V)\,Y_0(V) - Y_1(V)\,J_0(V)}{J_0^2(V) + Y_0^2(V)} \tag{25}$$

4 Results and discussions

In order to get an insight into the physics of the problem, the numerical computations of velocity, temperature, skin-friction and Nusselt number are made for different values of magnetic field parameter M, Grashof numbers Gr, Prandtl number Pr, time t and presented graphically in Figs. 2-10. Water, when mixed with salt; air when mixed with ionized gases, or salt water vapour, they are electrically conductive. Our observation is based on fluids which are slightly electrically conductive.

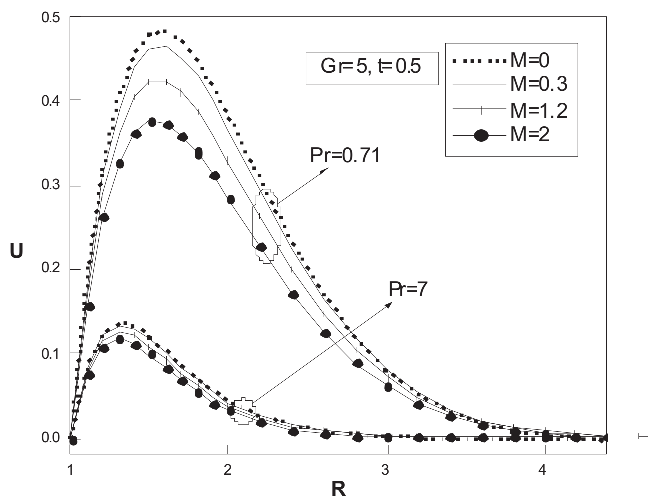

Figure 2: Effects of M and Pr on velocity profiles at Gr=5, t=0.5

The transient velocity profiles for different values of M and Pr at $Gr = 5$ and $t = 0.5$ are shown in Fig.2. It is observed from the figure that the presence of magnetic field leads to a decrease in the velocity field. It is due to the fact that the application of transverse magnetic field will result a resistive type of force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. The Prandtl number signifies the

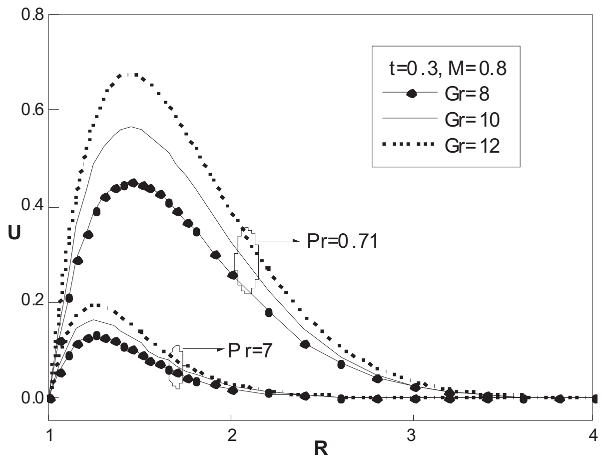

Figure 3: Effects of Gr and Pr on velocity profiles at t=0.3, M=0.8

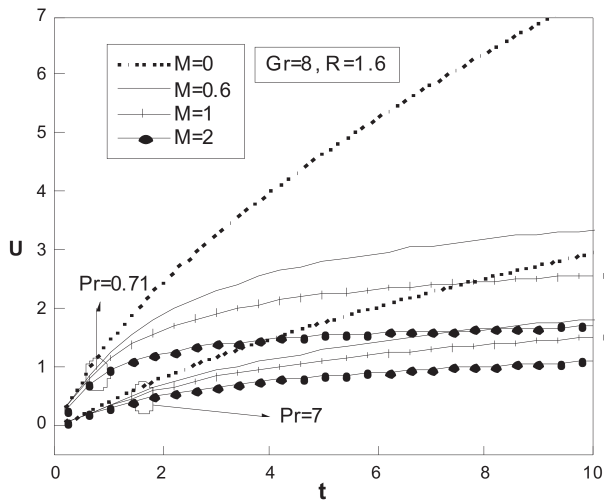

Figure 4: Effect of M with respect to time on velocity profiles at Gr=8, R=1.6

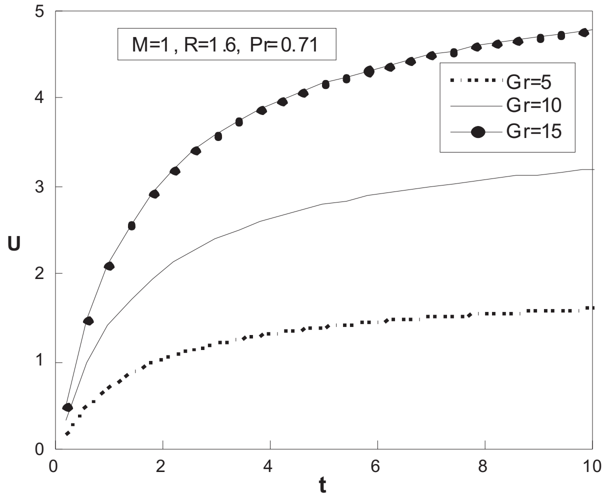

Figure 5: Effect of *Gr* with respect to time on velocity profiles at M=1, R=1.6, Pr=0.71

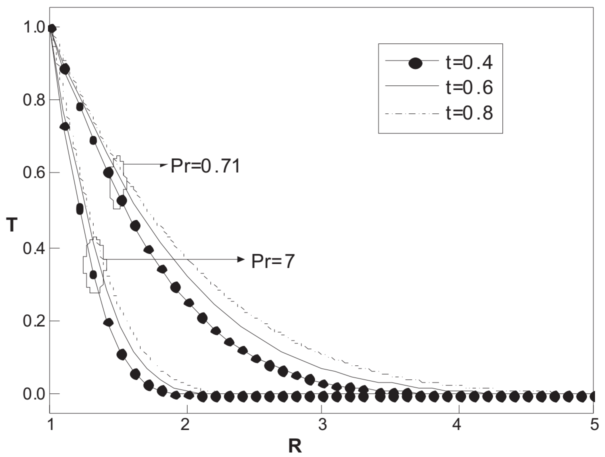

Figure 6: Effects of Prandtl number and time on temperature profiles.

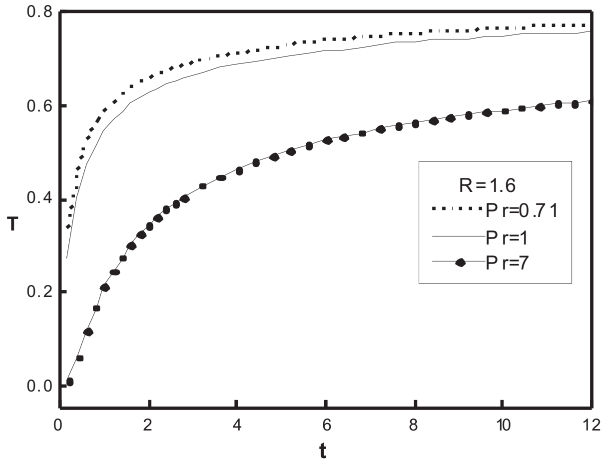

Figure 7: Temperature profiles with respect to time for different values of Pr

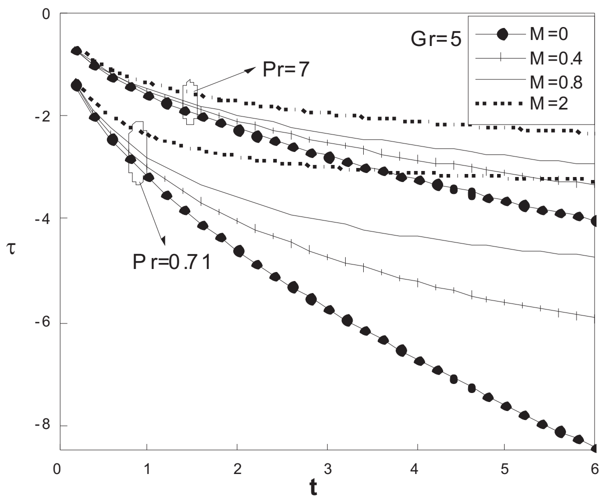

Figure 8: Effects of M and Pr on skin friction at Gr=5

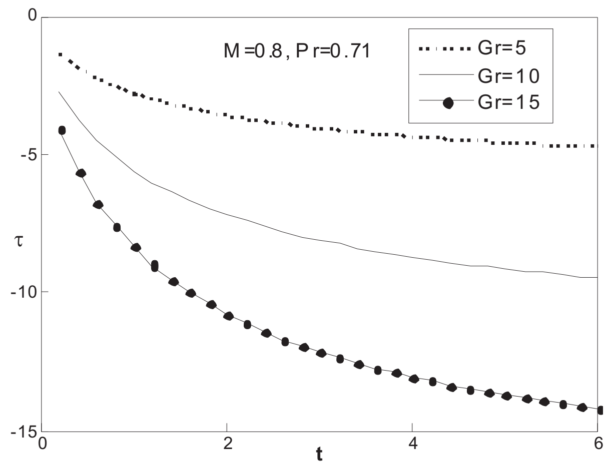

Figure 9: Effect of Gr on skin friction at M=0.8, Pr=0.71

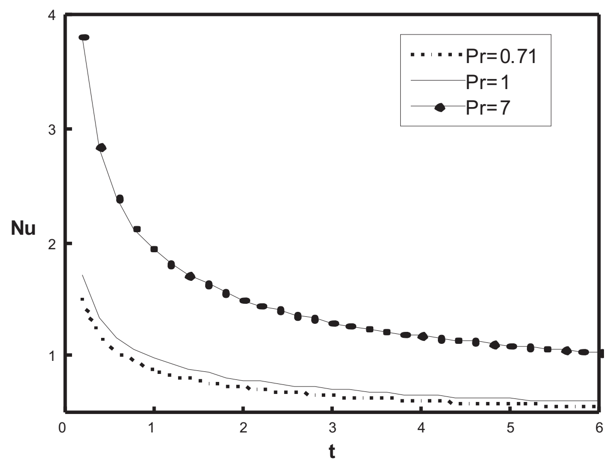

Figure 10: Nusselt number for different values of Pr

relative effects of the momentum and heat transport by diffusivity process. It physically relates the relative thickness of the hydrodynamic boundary layer and thermal boundary layer. It is observed here that transient velocity decreases with increase in Prandtl number.

The effect of Grashof number Gr on the transient velocity profiles for $M = 0.8$, $t = 0.3$ and $Pr = 0.71$ and 7 are shown in Fig.3. The Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force. The positive values of Gr correspond to cooling of the cylinder by free convection. Heat is therefore condvected away from the vertical cylinder into the fluid, which increases temperature and thereby enhances the buoyancy force. As expected, it is found that an increase in the Grashof number leads to an increase in the velocity due to enhancement in the buoyancy force.

Effects of magnetic field parameter and Prandtl number on velocity profile against time are presented in Fig.4. Also, the effect of Grashof number on velocity profiles against time is presented in Fig.5. It is observed that initially velocity increases with time but for larger time, it approaches steady state. Time required to reach steady state increases with Grashof number but decreases with magnetic field parameter or Prandtl number.

The transient temperature profiles for different t and Pr are plotted in Fig.6. It is observed that the temperature increases with decreased values of Pr or increased values of time t. Also, Fig.7 depicts the temperature profile against time shows that for larger time it tends to a steady state.

Fig.8 depicts the effects of magnetic field parameter and Prandtl number and Fig.9 depicts the effect of Grashof number on skin friction. It is found from the figures that the skin friction increases in presence of magnetic field. Also, skin friction increases with Pr but decreases with Gr. Also, it is found that skin friction tends to steady state for larger time and time required to reach steady state decreases with magnetic field parameter but increases with Grashof number.

Fig.10 shows the effects of Pr on the rate of heat transfer i.e. Nusselt number. Here, it is observed that Nusselt number increases with Pr and time required to reach steady state increase with Pr.

5 Conclusions

On the basis of the results obtained from the above discussions, the conclusions of this study are as follows:

i Time required to reach steady state increases with Grashof number but decreases with magnetic field effect or Prandtl number.

ii The transient velocity increases with Grashof number but decreases with magnetic field parameter or Prandtl number.

iii Skin friction increases with magnetic field effect or Prandtl number but decreases with Grashof number.

iv Nusselt number increases with Prandtl number.

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Appendix

Solutions of the coupled equations

$$\frac{d^2\overline{U}}{dR^2} + \frac{1}{R}\frac{d\overline{U}}{dR} - (M+p)\,\overline{U} + Gr\overline{T} = 0 \tag{I}$$

and

$$\frac{d^2\overline{T}}{dR^2} + \frac{1}{R}\frac{d\overline{T}}{dR} - \Pr p\overline{T} = 0 \tag{II}$$

Subject to the with boundary conditions,

$$\left.\begin{array}{llll} U=0, & T=1 & at & R=1 \\ U\to 0, & T\to 0 & as & R\to\infty \end{array}\right\} \tag{III}$$

The equation (II) is modified Bessel's equation in R with order zero. The solution of this equation is given by,

$$\overline{T} = C_1 I_0\left(R\sqrt{\Pr p}\right) + C_2 K_0\left(R\sqrt{\Pr p}\right) \tag{IV}$$

where C_1 and C_2 are two arbitrary constants and $I_0\left(R\sqrt{\Pr p}\right)$ and $K_0\left(R\sqrt{\Pr p}\right)$ are modified Bessel's functions of first kind and second kind respectively of order zero.

Since $T \to 0$ and so $\overline{T} \to 0$ as $R \to \infty$, so we must choose $C_1 = 0$

Also, $T = 1$ and so $\overline{T} = \frac{1}{p}$ at $R = 1$, so equation (IV) gives,

$$C_2 = \frac{1}{pK_0\left(\sqrt{\Pr p}\right)} \tag{V}$$

Therefore equation (IV) reduces to,

$$\overline{T} = \frac{K_0\left(R\sqrt{\Pr p}\right)}{pK_0\left(\sqrt{\Pr p}\right)} \tag{VI}$$

Substituting the expression for $\overline{T}$ from (VI) in equation (I), we have,

$$\frac{d^2\overline{U}}{dR^2} + \frac{1}{R}\frac{d\overline{U}}{dR} - (M+p)\,\overline{U} = -Gr\frac{K_0\left(R\sqrt{\Pr p}\right)}{pK_0\left(\sqrt{\Pr p}\right)} \tag{VII}$$

Now, consider the equation

$$\frac{d^2\overline{U}}{dR^2} + \frac{1}{R}\frac{d\overline{U}}{dR} - (M+p)\,\overline{U} = 0 \qquad \text{(VIII)}$$

which is modified Bessel's equation of order zero. The solution of this equation gives

$$\overline{U} = C_3 I_0\left(R\sqrt{M+p}\right) + C_4 K_0\left(R\sqrt{M+p}\right) \qquad \text{(IX)}$$

where C_3 and C_4 are arbitrary constants. Considering solution (IX) as complimentary function (CF) of equation (VII), the general solution of equation (VII) i.e. $\overline{U} = CF + Particularintegral$, is obtained by variation of parameter technique. Following, variation of parameter method, and using the following identities,

$$\int RI_0\left(aR\right)I_0\left(bR\right)dR = \frac{R}{a^2-b^2}\left[aI_0\left(bR\right)I_1\left(aR\right)\, - bI_0\left(aR\right)I_1\left(bR\right)\right]$$

$$\int RI_0\left(aR\right)I_0\left(bR\right)dR = \frac{R}{a^2-b^2}\left[aI_0\left(bR\right)I_1\left(aR\right)\, - bI_0\left(aR\right)I_1\left(bR\right)\right]$$

$$\int RI_0\left(aR\right)K_0\left(bR\right)dR = \frac{R}{a^2-b^2}\left[aI_1\left(aR\right)\,K_0\left(bR\right) + bI_0\left(aR\right)\,K_1\left(bR\right)\right]$$

and properties of modified Bessel's functions [*cf.* Carslaw and Jaeger [11,12]]

$$\frac{d}{dR}\left\{I_0(aR)\right\} = aI_1(aR), \qquad \frac{d}{dR}\left\{K_0(aR)\right\} = -aK_1(aR),$$

$$I_0\left(aR\right)K_1\left(aR\right) + I_1\left(aR\right)K_0\left(aR\right) = \frac{1}{aR}$$

R being the dummy variable; finally using boundary conditions (III), the expression for $\overline{U}$ is obtained as

$$\overline{U} = \frac{Gr}{p\left\{M + p\left(1-\Pr\right)\right\}}\left\{\frac{K_0\left(\sqrt{\Pr p}R\right)}{K_0\left(\sqrt{\Pr p}\right)} - \frac{K_0\left(\sqrt{M+p}R\right)}{K_0\left(\sqrt{M+p}\right)}\right\} \qquad \text{(X)}$$

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Prolazna slobodna konvekcija MHD tečenja duž beskonačnog vertikalnog cilindra

Ovaj rad predstavlja analiticki rastvor nestabilan jednodimenzionalni prirodni tok konvektivnog viskozna nestiljiva i elektricno sprovodenje tecnosti pored beskonacne vertikalne cilindra sa konstantnim temperature i magnetno polje, primenjeno normalno na pravac toka. Egzaktna reenja bezdimenziono nestacionarnog linearnih jednacina su vladajucih dobijeni koricenjem Laplasove transformacije tehnike. Numericki proracuni za prelazni brzina, temperatura, koe-trenje, Nusselt broj su obracunat i prikazan u grafikonima za razne fizicke skup parametarskih Vrednosti naime, Grashofov broj, Prandtl broj, magnetni parametar i vreme.

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