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REFLECTION AND REFRACTION OF PLANE WAVES AT THE INTERFACE BETWEEN MAGNETOELECTROELASTIC AND LIQUID MEDIA

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Reflection and refraction of plane waves at the interface between magnetoelectroelastic and liquid media

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Abstract

This paper analyzes the reflection and refraction of plane wave incidences at the interface between magnetoelectroelastic (MEE) and liquid media. The MEE medium is assumed to be transversely isotropic and the liquid medium to be nonviscous. Three cases, i.e., the coupled quasi-pressure wave incidence from the MEE medium, the coupled quasi-shear vertical wave incidence from the MEE medium, and the pressure wave incidence from the liquid medium, are discussed. The expressions of reflection and transmission coefficients varying with the incident angle are obtained. This investigation would be useful to the MEE acoustic device field.

Keywords: plane wave; reflection; refraction; magnetoelectroelastic; liquid

1 Introduction

Acoustic wave behavior at the interface between piezoelectric and liquid media is a significant topic in some fields such as acoustic device design and non-destructive evaluation. This problem has been studied for both viscous and nonviscous liquids, and some valuable solutions have been found. It is difficult to embrace all the related references and therefore just some of them are mentioned herein. Noorbehesht *et al* [1] studied the reflection and transmission of plane elastic waves at the boundary between piezoelectric

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material and water. Shana *et al* [2] theoretically investigated the reflection of obliquely incident shear horizontal bulk acoustic waves at the interface between a piezoelectrc crystal and a viscous conductive liquid. Nayfeh *et al* [3, 4] derived the analytical expressions for the reflection and transmission coefficients (RTCs) for the fluid-loaded piezoelectric plate and fluid-loaded piezoelectric half-space in order to study the influence of piezoelectricity on such waves. Recently, the reflection and transmission of plane waves from a fluid-porous piezoelectric solid interface was studied by Vashishth *et al* [5], and the variation of leaky wave velocity with the frequency was also studied.

Composites consisting of piezoelectric and piezomagnetic phases have attracted considerable interests because the composites have a remarkable magnetoelectric effect due to coupling between the two phases [6], and there also exist some literatures (or earlier works) on the reflection and refraction problem. For example, Pang *et al* [7] analyzed the reflection and refraction of a plane wave incidence at the interface between piezoelectric and piezomagnetic media, and later Pang *et al* [8] analyzed a case with an imperfectly bonded interface. Chen *et al* [9] investigated the RTCs of oblique incident plane waves to a multilayered system of piezomagnetic and/or piezoelectric materials. Besides the layered piezoelectric/piezomagnetic structures, the magnetoelectric effect can also be obtained by the use of homogeneous magnetoelectroelastic (MEE) solids which could avoid interface defects, and some studies on wave motion were also conducted. For instantce, Wu *et al* [10] investigated propagation of symmetric and antisymmetric Lamb waves in an infinite MEE plate. Feng *et al* [11] investigated the propagation properties of Stoneley waves between two MEE half planes, while 25 sets of magnetoelectric interface conditions were adopted. The 12 velocities of surface wave propagation in MEE materials were obtained in explicit forms by Melkumyan [12]. To the best knowledge of the author, however, reflection and transmission of incident waves at the interface between MEE and liquid media have not been investigated so far, and this could be encountered in the design of underwater acoustic devices. Therefore, the engineering-oriented problem motivates the present study.

In this paper, the reflection/refraction at a plane interface between MEE and liquid media is investigated for three different types of wave incidences. The RTCs are derived by solving a linear algebraic system of equations. This investigation is supposed to be helpful to the applications of MEE acoustic devices.

2 Basic equations and formulation

We shall discuss the reflection and refraction phenomena of plane waves at the interface between MEE and liquid half-spaces. Let the x-axis be taken along the interface and z-axis along the direction pointing vertically upward. The lower half-space is taken as the MEE medium ($z > 0$), while the elastic constants, piezoelectric coefficients, piezomagnetic coefficients, dielectric permittivities, magnetic permeabilities, magnetoelectric coefficients and density are denoted by c_{ijkl}, e_{ij}, f_{ij}, ε_{ij}, μ_{ij}, g_{ij}, ρ with i, j, k, l=1, 2, 3 respectively. The upper half-space is taken as the liquid medium (z¿0) with density ρ'. There exist three cases with this plane problem, viz., (1) incident quasi-pressure (QP) wave in the solid, (2) incident quasi-shear vertical (QSV) wave in the solid, (3) incident pressure (P) wave in the liquid. The complete geometry is shown in Fig.1.

According to the quasistatic approximation and linearity assumption, the governing equations with electricity and magnetism, in the absence of body force, are expressed as:

$$\begin{aligned} \sigma_{ij,j} &= \rho \ddot{u}_i \\ D_{i,i} &= 0 \\ B_{i,i} &= 0 \end{aligned} \tag{1}$$

where u_i, σ_{ij}, D_i and B_i are the displacement, stress, electric displacement and magnetic induction, respectively. The dot denotes time differentiation and the comma denotes space-coordinate differentiation; the repeated index in the subscript implies summation.

The constitutive equations of an MEE medium are:

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\gamma_{kl} - e_{kij}E_k - f_{kij}H_k \\ D_i &= e_{ikl}\gamma_{kl} + \varepsilon_{il}E_l + g_{il}H_l \\ B_i &= f_{ikl}\gamma_{kl} + g_{il}E_l + \mu_{il}H_l \end{aligned} \tag{2}$$

where $\gamma_{ij} = 0.5\,(u_{i,j} + u_{j,i})$, $E_i = -\varphi_{,i}$, $H_i = -\psi_{,i}$, φ and ψ are the electric potential and magnetic potential respectively.

Note that for the plane strain problem considered here all quantities are independent of the y coordinate. The MEE medium is assumed transversely isotropic and its electric and magnetic poling directions all parallel to the

z-axis. The liquid medium is assumed nonviscous. The constitutive equation of the MEE medium, i.e., Eq. (2), can be given in the two dimensional form:

$$\begin{aligned}
\sigma_{xx} &= c_{11}u_{x,x} + c_{13}u_{z,z} + e_{31}\varphi_{,z} + f_{31}\psi_{,z} \\
\sigma_{zz} &= c_{13}u_{x,x} + c_{33}u_{z,z} + e_{33}\varphi_{,z} + f_{33}\psi_{,z} \\
\sigma_{xz} &= c_{44}\left(u_{x,z} + u_{z,x}\right) + e_{15}\varphi_{,x} + f_{15}\psi_{,x} \\
D_x &= e_{15}\left(u_{x,z} + u_{z,x}\right) - \varepsilon_{11}\varphi_{,x} - g_{11}\psi_{,x} \\
D_z &= e_{31}u_{x,x} + e_{33}u_{z,z} - \varepsilon_{33}\varphi_{,z} - g_{33}\psi_{,z} \\
B_x &= f_{15}\left(u_{x,z} + u_{z,x}\right) - g_{11}\varphi_{,x} - \mu_{11}\psi_{,x} \\
B_z &= f_{31}u_{x,x} + f_{33}u_{z,z} - g_{33}\varphi_{,z} - \mu_{33}\psi_{,z}
\end{aligned} \tag{3}$$

Here, we shall allow the subscripts x, y and z to be synonymous with 1, 2 and 3, respectively. Substituting Eq. (3) into (1), the magneto-electro-mechanical coupling governing equations for the MEE medium in terms of mechanical displacements, electric potential and magnetic potential can be given as:

$$\begin{aligned}
&c_{11}u_{x,xx} + c_{44}u_{x,zz} + (c_{13} + c_{44})u_{z,xz} + \\
&\qquad (e_{31} + e_{15})\varphi_{,xz} + (f_{31} + f_{15})\psi_{,xz} = \rho\ddot{u}_x \\
&(c_{13} + c_{44})u_{x,xz} + c_{44}u_{z,xx} + c_{33}u_{z,zz} + \\
&\qquad e_{15}\varphi_{,xx} + e_{33}\varphi_{,zz} + f_{15}\psi_{,xx} + f_{33}\psi_{,zz} = \rho\ddot{u}_z \\
&(e_{31} + e_{15})u_{x,xz} + e_{15}u_{z,xx} + e_{33}u_{z,zz} - \\
&\qquad \varepsilon_{11}\varphi_{,xx} - \varepsilon_{33}\varphi_{,zz} - g_{11}\psi_{,xx} - g_{33}\psi_{,zz} = 0 \\
&(f_{31} + f_{15})u_{x,xz} + f_{15}u_{z,xx} + f_{33}u_{z,zz} - g_{11}\varphi_{,xx} - \\
&\qquad g_{33}\varphi_{,zz} - \mu_{11}\psi_{,xx} - \mu_{33}\psi_{,zz} = 0
\end{aligned} \tag{4}$$

3 Solution and boundary conditions

Let us assume the harmonic solution as [13]:

$$\left(\vec{u}^{(n)}, \varphi^{(n)}, \psi^{(n)}\right) = \left(A^{(n)}, B^{(n)}, C^{(n)}\right)\vec{d}^{(n)}\exp\left(i\eta^{(n)}\right) \tag{5}$$

where different values of the index n serve to label the various types of waves that occur when an incident wave is reflected and refracted, $\vec{d}^{(n)}$ is the unit vector of motion, $\eta^{(n)}$ is defined as:

$$\eta^{(n)} = k^{(n)}\left(\vec{x}\cdot\vec{p}_{(n)} - c^{(n)}t\right) \tag{6}$$

where $\vec{p}_(n)$ is the unit propagation vector and $\vec{x}$ is the coordinate vector, $A^{(n)}, B^{(n)}, C^{(n)}$ the unknown amplitudes, $c^{(n)}$ the phase velocity and $k^{(n)}$ the wave number. Substituting Eq.(5) into Eq.(4) and eliminating electric potential and magnetic potential, one can obtain the Christoffel's equation of the MEE medium; then with a specified direction, the phase velocities could be numerically solved from the secular equation [9].

To demonstrate the solution procedure, we consider the case of an incident QSV wave in the solid as an example. By assigning n=0, there is an incident QSV wave propagating with velocity ${c_T}^{(0)}$ in the MEE medium and it makes an angle $\theta^{(0)}$ with the z-axis. For the two dimensional problem, the corresponding displacement components, electric potential and magnetic potential could be expressed from Eq.(5) as

$$\begin{aligned}
u_x^{(0)} &= -A^{(0)} \cos\theta^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_T^{(0)} t\right)\right] \\
u_z^{(0)} &= A^{(0)} \sin\theta^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_T^{(0)} t\right)\right] \\
\varphi^{(0)} &= B^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_T^{(0)} t\right)\right] \\
\psi^{(0)} &= C^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_T^{(0)} t\right)\right]
\end{aligned} \tag{7}$$

Then, we postulate the following reflected and refracted waves to satisfy the problem as:

(1) Reflected wave. There is a QP wave propagating with velocity c_L^1 in the MEE medium and it makes an angle $\theta^(1)$ with the z-axis. The displacement components, electric potential and magnetic potential could be expressed as:

$$\begin{aligned}
u_x^{(1)} &= A^{(1)} \sin\theta^{(1)} \exp\left[ik^{(1)}\left(x\sin\theta^{(1)} - z\cos\theta^{(1)} - c_L^{(1)} t\right)\right] \\
u_z^{(1)} &= -A^{(1)} \cos\theta^{(1)} \exp\left[ik^{(1)}\left(x\sin\theta^{(1)} - z\cos\theta^{(1)} - c_L^{(1)} t\right)\right] \\
\varphi^{(1)} &= B^{(1)} \exp\left[ik^{(1)}\left(x\sin\theta^{(1)} - z\cos\theta^{(1)} - c_L^{(1)} t\right)\right] \\
\psi^{(1)} &= C^{(1)} \exp\left[ik^{(1)}\left(x\sin\theta^{(1)} - z\cos\theta^{(1)} - c_L^{(1)} t\right)\right]
\end{aligned} \tag{8}$$

(2) Reflected wave. There is a QSV wave propagating with velocity ${c_T}^{(2)}$ in the MEE medium and it makes an angle $\theta^{(2)}$ with the z-axis. The displacement components, electric potential and magnetic potential could be

expressed as:

$$\begin{aligned}
u_x^{(2)} &= A^{(2)}\cos\theta^{(2)}\exp\left[ik^{(2)}\left(x\sin\theta^{(2)} - z\cos\theta^{(2)} - c_T{}^{(2)}t\right)\right]\\
u_z^{(2)} &= A^{(2)}\sin\theta^{(2)}\exp\left[ik^{(2)}\left(x\sin\theta^{(2)} - z\cos\theta^{(2)} - c_T{}^{(2)}t\right)\right]\\
\varphi^{(2)} &= B^{(2)}\exp\left[ik^{(2)}\left(x\sin\theta^{(2)} - z\cos\theta^{(2)} - c_T{}^{(2)}t\right)\right]\\
\psi^{(2)} &= C^{(2)}\exp\left[ik^{(2)}\left(x\sin\theta^{(2)} - z\cos\theta^{(2)} - c_T{}^{(2)}t\right)\right]
\end{aligned} \tag{9}$$

(3) Refracted wave. There is a P wave propagating with velocity $c_L^{(3)}$ in the liquid medium and it makes an angle $\theta^{(3)}$ with the z-axis. The electric potential and magnetic potential are neglected, and the displacement components could be expressed as:

$$\begin{aligned}
u_x^{(3)} &= A^{(3)}\sin\theta^{(3)}\exp\left[ik^{(3)}\left(x\sin\theta^{(3)} + z\cos\theta^{(3)} - c_L^{(3)}t\right)\right]\\
u_z^{(3)} &= A^{(3)}\cos\theta^{(3)}\exp\left[ik^{(3)}\left(x\sin\theta^{(3)} + z\cos\theta^{(3)} - c_L^{(3)}t\right)\right]
\end{aligned} \tag{10}$$

Owing to the coupled mechanical, electric and magnetic fields, we should find the relations between the amplitudes. The magnetoelectric boundary condition falls into four types:

(1) electrically open and magnetically shorted $D_z^{(n)} = B_z^{(n)} = 0$;

(2) electrically open and magnetically open $D_z^{(n)} = \psi_z^{(n)} = 0$;

(3) electrically shorted and magnetically open

$$\varphi_z^{(n)} = \psi_z^{(n)} = 0; \tag{11}$$

(4) electrically shorted and magnetically shorted $\varphi_z^{(n)} = B_z^{(n)} = 0$.

Herein, the first type is taken to illustrate the analysis procedure.

Substituting Eqs.(7), (8), (9) and (10) into (11), one may get the relations:

$$\begin{aligned}
&B^{(0)} = \xi^{(0)}A^{(0)}, \quad B^{(1)} = \xi^{(1)}A^{(1)}, \quad B^{(2)} = \xi^{(2)}A^{(2)},\\
&C^{(0)} = \zeta^{(0)}A^{(0)}, \quad C^{(1)} = \zeta^{(1)}A^{(1)}, \quad C^{(2)} = \zeta^{(2)}A^{(2)}
\end{aligned} \tag{12}$$

where the definitions of $\xi^{(n)}$ and $\zeta^{(n)}$ are given in Appendix A.

Apparently, the phase velocities and wave numbers satisfy the relations:

$$c_T^{(0)}k^{(0)} = c_L^{(1)}k^{(1)} = c_T^{(2)}k^{(2)} = c_L^{(3)}k^{(3)} = \omega \tag{13}$$

where ω represents the circular frequency.

The directions of the propagation vectors are given by the Snell's law, so that

$$\theta^{(0)} = \theta^{(2)}, \quad \frac{\sin\theta^{(0)}}{c_T^{(0)}} = \frac{\sin\theta^{(1)}}{c_L^{(1)}} = \frac{\sin\theta^{(3)}}{c_L^{(3)}} \tag{14}$$

The appropriate mechanical boundary conditions at the interface between the two half-spaces (z=0) can be described as: (1) the normal component of displacement is continuous; (2) the tangential stresses are zero because the liquid is nonviscous; (3) the normal stress is equal and opposite to the acoustic overpressure δp in the liquid. Mathematically, these boundary conditions could be written as:

$$\begin{aligned} &u_z^{(0)} + u_z^{(1)} + u_z^{(2)} = u_z^{(3)} \\ &\sigma_{xz}^{(0)} + \sigma_{xz}^{(1)} + \sigma_{xz}^{(2)} = 0 \\ &\sigma_{zz}^{(0)} + \sigma_{zz}^{(1)} + \sigma_{zz}^{(2)} = -\delta p = -Z\dot{u}_z^{(3)} = -i\omega Z u_z^{(3)} \end{aligned} \tag{15}$$

where $Z = \rho' c_L^{(3)}$ is the acoustic impedance of the liquid.

Using Eq.(12) and substituting Eqs.(7), (8), (9), (10) into Eq.(15), and after some algebraic manipulation, one may get

$$\begin{aligned} a_{11}\tfrac{A^{(1)}}{A^{(0)}} + a_{12}\tfrac{A^{(2)}}{A^{(0)}} + a_{13}\tfrac{A^{(3)}}{A^{(0)}} = m_1 \\ a_{21}\tfrac{A^{(1)}}{A^{(0)}} + a_{22}\tfrac{A^{(2)}}{A^{(0)}} + a_{23}\tfrac{A^{(3)}}{A^{(0)}} = m_2 \\ a_{31}\tfrac{A^{(1)}}{A^{(0)}} + a_{32}\tfrac{A^{(2)}}{A^{(0)}} + a_{33}\tfrac{A^{(3)}}{A^{(0)}} = m_3 \end{aligned} \tag{16}$$

where a_{ij} and m_i $(i,\, j = 1,2,3)$ are defined in Appendix B.

From Eq.(16), the RTCs of displacement, viz. $\frac{A^{(1)}}{A^{(0)}}, \frac{A^{(2)}}{A^{(0)}}, \frac{A^{(3)}}{A^{(0)}}$, can be readily solved by the use of Cramer's rule. For brevity they are not given here.

Keeping Eq.(12) in mind, then one may get the RTCs of electric potential and magnetic potential as:

$$\frac{B^{(1)}}{B^{(0)}} = \frac{A^{(1)}}{A^{(0)}} \cdot \frac{\xi^{(1)}}{\xi^{(0)}}, \frac{B^{(2)}}{B^{(0)}} = \frac{A^{(2)}}{A^{(0)}} \cdot \frac{\xi^{(2)}}{\xi^{(0)}}, \frac{C^{(1)}}{C^{(0)}} = \frac{A^{(1)}}{A^{(0)}} \cdot \frac{\zeta^{(1)}}{\zeta^{(0)}}, \frac{C^{(2)}}{C^{(0)}} = \frac{A^{(2)}}{A^{(0)}} \cdot \frac{\zeta^{(2)}}{\zeta^{(0)}} \tag{17}$$

Now we have the solutions for all incident angles smaller than the critical angle θ_{cr} which is implicitly expressed as

$$\sin\theta_{cr} = \frac{{c_T}^{(0)}}{c_L^{(1)}} \tag{18}$$

The critical angle could be numerically solved with Christoffel's equation of the MEE medium.

If $\theta^{(0)} > \theta_{cr}$, the reflected QP wave is a wave propagating along the x-axis, and the amplitude decays with the depth into the MEE medium (decreasing z). For this surface-type wave, the component $\cos\theta^{(1)}$ becomes $i(c_L^{(1)}/c_T^{(0)})\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}$ [13], and the displacement components, electric potential and magnetic potential are rewritten as:

$$
\begin{aligned}
u_x^{(1)} &= A^{(1)} \exp\left(k^{(0)} z\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}\right) \exp\left[ik^{(0)}\left(\sin\theta^{(0)} x - c_T^{(0)} t\right)\right] \\
u_z^{(1)} &= 0 \\
\varphi^{(1)} &= B^{(1)} \exp\left(k^{(0)} z\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}\right) \exp\left[ik^{(0)}\left(\sin\theta^{(0)} x - c_T^{(0)} t\right)\right] \\
\psi^{(1)} &= C^{(1)} \exp\left(k^{(0)} z\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}\right) \exp\left[ik^{(0)}\left(\sin\theta^{(0)} x - c_T^{(0)} t\right)\right]
\end{aligned} \tag{19}
$$

To obtain the RTCs when $\theta^{(0)} > \theta_{cr}$, the modified a_{ij}, $\xi^{(n)}$ and $\zeta^{(n)}$ are defined in Appendices A and B.

Since some of the RTCs become complex numbers beyond the critical angle [14], we merely show the magnitude ratio of the RTCs and the phase changes are not discussed, so we employ the following expressions:

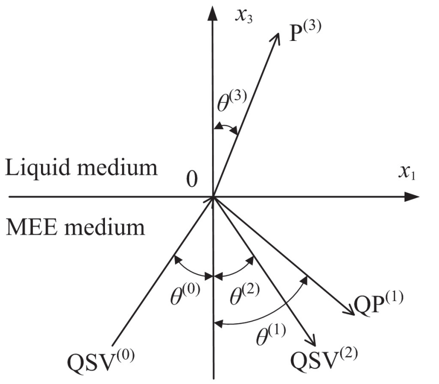
$$
\begin{aligned}
&Z_1 = \left|\frac{A^{(1)}}{A^{(0)}}\right|, Z_2 = \left|\frac{A^{(2)}}{A^{(0)}}\right|, Z_3 = \left|\frac{A^{(3)}}{A^{(0)}}\right|, \\
&Z_4 = \left|\frac{B^{(1)}}{B^{(0)}}\right|, Z_5 = \left|\frac{B^{(2)}}{B^{(0)}}\right|, Z_6 = \left|\frac{C^{(1)}}{C^{(0)}}\right|, Z_7 = \left|\frac{C^{(2)}}{C^{(0)}}\right|;
\end{aligned} \tag{20}
$$

On the other hand, if an incident QP wave originates from the solid, or an incident P wave originates from the liquid medium, the corresponding displacement components are

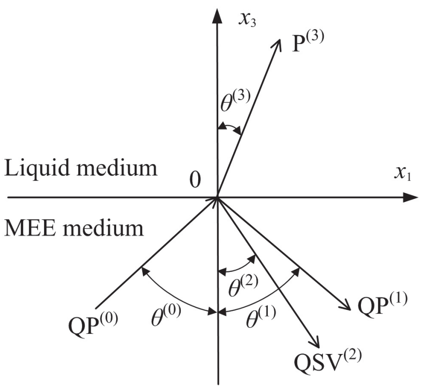
$$
\begin{aligned}
u_x^{(0)} &= A^{(0)} \sin\theta^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_L^{(0)} t\right)\right] \\
u_z^{(0)} &= A^{(0)} \cos\theta^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_L^{(0)} t\right)\right]
\end{aligned} \tag{21}
$$

In the former case the electric potential and magnetic potential could be expressed as

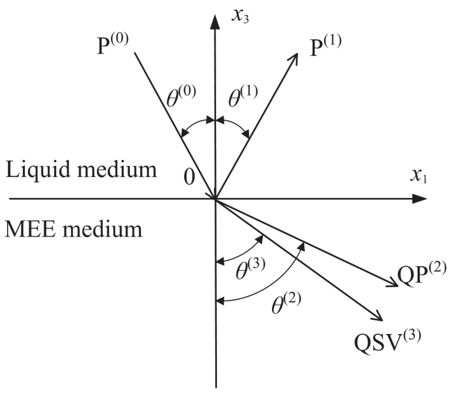
$$
\begin{aligned}
\varphi^{(0)} &= B^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_L^{(0)} t\right)\right] \\
\psi^{(0)} &= C^{(0)} \exp\left[ik^{(0)}\left(x\sin\theta^{(0)} + z\cos\theta^{(0)} - c_L^{(0)} t\right)\right]
\end{aligned} \tag{22}
$$



(a) QSV wave incidence in the solid



(b) QP wave incidence in the solid.



(c) P wave incidence in the liquid

Figure 1: Reflected and refracted waves at the boundary between MEE and liquid media.

While in the latter case the electric potential and magnetic potential are neglected. The RTCs for the two cases can be evidently obtained by following the above procedure.

4 Conclusion

Exact solutions are obtained for the reflection and refraction of plane waves at the interface between linear MEE material and liquid. The solutions implicitly contain the cases for elastic material/liquid [14], piezoelectric material/liquid [1] and piezomagnetic material/liquid.

Appendix A

$\xi^{(n)}$ and $\zeta^{(n)}$ in Eq. (12) are given as:

$$\xi^{(0)} = \xi^{(2)} = \frac{\sin\theta^{(0)}\left[(e_{31}-e_{33})\,\mu_{33} + (f_{33}-f_{31})\,g_{33}\right]}{g_{33}^2 - \varepsilon_{33}\mu_{33}},$$

$$\xi^{(1)} = \frac{\cos\theta^{(1)}\,(e_{33}\mu_{33} - f_{33}g_{33}) + \sin\theta^{(1)}\tan\theta^{(1)}\,(e_{31}\mu_{33} - f_{31}g_{33})}{g_{33}^2 - \varepsilon_{33}\mu_{33}}, \quad \text{(A1)}$$

$$\zeta^{(0)} = \zeta^{(2)} = \frac{\sin\theta^{(0)}\left[(f_{31}-f_{33})\,\varepsilon_{33} + (e_{33}-e_{31})\,g_{33}\right]}{g_{33}^2 - \varepsilon_{33}\mu_{33}},$$

$$\zeta^{(1)} = \frac{\cos\theta^{(1)}\,(f_{33}\varepsilon_{33} - e_{33}g_{33}) + \sin\theta^{(1)}\tan\theta^{(1)}\,(f_{31}\varepsilon_{33} - e_{31}g_{33})}{g_{33}^2 - \varepsilon_{33}\mu_{33}}.$$

$\xi^{(n)}$ and $\zeta^{(n)}$ in Eq. (12) modified in terms of Eq. (19) are given as:

$$\left.\begin{aligned} \xi^{(1)} &= \frac{i\sin\theta^{(0)}\,(f_{31}g_{33} - e_{31}\mu_{33})}{\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}\,\left(g_{33}^2 - \varepsilon_{33}\mu_{33}\right)} \\ \zeta^{(1)} &= \frac{i\sin\theta^{(0)}\,(e_{31}g_{33} - f_{31}\varepsilon_{33})}{\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}\,\left(g_{33}^2 - \varepsilon_{33}\mu_{33}\right)} \end{aligned}\right\}, \quad \text{(A2)}$$

Appendix B

a_{ij} and m_i in Eq. (16) are given as:

$$
\begin{aligned}
a_{11} &= -\cos\theta^{(1)}, \\
a_{12} &= \sin\theta^{(2)}, \\
a_{13} &= -\cos\theta^{(3)}, \\
a_{21} &= 2\cos\theta^{(1)} c_{44} - e_{15}\xi^{(1)} - f_{15}\zeta^{(1)}, \\
a_{22} &= \frac{\cos 2\theta^{(2)} c_{44} - \sin\theta^{(2)}\left(e_{15}\xi^{(2)} + f_{15}\zeta^{(2)}\right)}{\sin\theta^{(2)}}, \\
a_{23} &= 0,
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
a_{31} &= \sin\theta^{(1)} c_{13} + \frac{\cos\theta^{(1)} c_{33} - e_{33}\xi^{(1)} - f_{33}\zeta^{(1)}}{\tan\theta^{(1)}}, \\
a_{32} &= \frac{\sin\theta^{(2)}\left(c_{13} - c_{33}\right) - e_{33}\xi^{(2)} - f_{33}\zeta^{(2)}}{\tan\theta^{(2)}}, \\
a_{33} &= \frac{Zc_L^{(3)}}{\sin\theta^{(3)}}.
\end{aligned}
$$

$$
\begin{aligned}
m_1 &= -\sin\theta^{(0)}, \\
m_2 &= -\frac{\cos 2\theta^{(0)} c_{44} - \sin\theta^{(0)}\left(e_{15}\xi^{(0)} + f_{15}\zeta^{(0)}\right)}{\sin\theta^{(0)}}, \\
m_3 &= \frac{\sin\theta^{(0)}\left(c_{13} - c_{33}\right) - e_{33}\xi^{(0)} - f_{33}\zeta^{(0)}}{\tan\theta^{(0)}}.
\end{aligned}
\tag{B2}
$$

a_{ij} and m_i in Eq.(16) modified in terms of Eq.(19) are given as:

$$
\begin{aligned}
a_{11} &= 0, \\
a_{21} &= -\sin\theta^{(0)}\left(e_{15}\xi^{(1)} + f_{15}\zeta^{(1)}\right) + i\csc\theta^{(0)} c_{44}\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}, \\
a_{31} &= c_{13} - i\csc\theta^{(0)}\left(e_{33}\xi^{(1)} + f_{33}\zeta^{(1)}\right)\sqrt{\sin^2\theta^{(0)} - (c_T^{(0)}/c_L^{(1)})^2}.
\end{aligned}
\tag{B3}
$$

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Odbijanje i prelamanje ravanskog talasa na granici izmedju magnetoelektroelastične i tečne sredine

Analizira se prelamanje ravanskog upadnog talasa na granici izmedu magnetoelektroelastične (MEE) i tečne sredine. Za MEE sredinu pretpostavlja se da je poprečno izotropna dok je tečna sredina po pretpostavci neviskozna. Tri slučaja, tj., (1) spregnuti skoro - pritiskujući talas dolazeći od MEE sredine, (2) spregnuti skoro - smičući vertikalni talas dolazeći od MEE sredine, kao i (3) pritiskujući talas dolazeći od tečne sredine, se razmatraju. Izrazi za koeficijente refleksije i prenosa se variraju sa upadnim uglom. Ovo istraživanje bi moglo biti korisno za MEE uredjaje sa akustičnim poljem.

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