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SIMPLE AND ACCURATE APPROACH FOR SOLVING OF NONLINEAR HEAT CONVECTIVE-RADIATIVE EQUATION IN FIN BY USING THE COLLOCATION METHOD AND COMPARISON WITH HPM AND VIM

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According to: *Tib Journal Abbreviations (C) Mathematical Reviews*, the abbreviation TEOPM7 stands for TEORIJSKA I PRIMENJENA MEHANIKA.

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Simple and accurate approach for solving of nonlinear heat convective-radiative equation in fin by using the collocation method and comparison with HPM and VIM

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Abstract

Collocation Method (CM) such as analytical technique, which does not need small parameters is here used to evaluate the analytical approximate solutions of the nonlinear heat transfer equation. The obtained results from Collocation Method are compared with other analytical techniques such as Homotopy Perturbation Method (HPM) and Variation Iteration Method (VIM). Also, boundary value problem (BVP) is applied as a numerical method for validation. The results reveal that the Collocation Method is very effective, simple and more accurate than other techniques. Also, it is found that this method is a powerful mathematical tool and can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering especially at some heat transfer equations.

Keywords: Heat transfer, collocation method, homotopy perturbation method, variational iteration method.

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Nomenclature

A_c	Cross sectional area of the fin
X	spatial coordinate
h	heat transfer coefficient at convection
k_0	thermal conductivity at zero temperature
P	fin perimeter
CM	Collocation Method
VIM	Variation Iteration Method
HPM	Homotopy Perturbation Method
L	length of the fin
N	fin parameter or convection–conduction parameter(dimensionless)
T	local fin temperature
T_b	fin base temperature
T_S	sink temperature for radiation
T_∞	sink temperature for convection
x	dimensionless spatial coordinate

Greek symbols

θ_∞	dimensionless convection sink temperature
θ_s	dimensionless radiation sink temperature
θ	dimensionless temperature
ε'	fin surface emissivity (dimensionless)
ε	radiation–conduction number (dimensionless)
σ	Stefan–Boltzmann constant

Subscripts

a	surrounding fluid
c	tip end of the fin
b	conditions at the fin base

1 Introduction

The heat transfer rate enhancement in fins with reducing size and cost is the aim of many researchers in engineering applications. To achieve this goals, convective heat transfer coefficient, surface area available and temperature difference between surface and surrounding fluid are such as ways can be used. Most of problems and scientific phenomenon such as heat transfer are inherently of nonlinearity. We know that except a limited number of these problems, most of them do not have exact solution. Therefore, these nonlinear equations should be solved using other methods, Such as numerical techniques. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. Time consuming is another problem of numerical techniques. This is caused to led scientist to improve the traditional analytical method such as perturbation. In the analytical perturbation method, we should exert a small parameter in the equation. Therefore, finding this parameter and exerting it into the equation are difficulties of this method. Therefore, many different methods have recently introduced such as the δ-expansion method [1], Adomian's decomposition method [2], Homotopy Perturbation Method (HPM) [3–9], Variational Iteration Method (VIM) [10–18], Homotopy analysis method [19], Optimal Homotopy Asymptopic Method (OHAM)[20,21] and optimal Homotopy Perturbation Method (OHPM)[22].

One of the semi-exact methods is the collocation method. In this article, the basic idea of the CM is introduced and then its application in some heat transfer equations is studied. The nonlinear heat convective-radiattive equations of a fin in the steady state are solved through the three methods: Collocation Method, Homotopy Perturbation Method and the Variation Iteration Method, and compared with each other and also with the numerical solution. Results demonstrate that Collocation Method is simple and offers superior accuracy compared with the VIM and HPM.

2 Governing equations

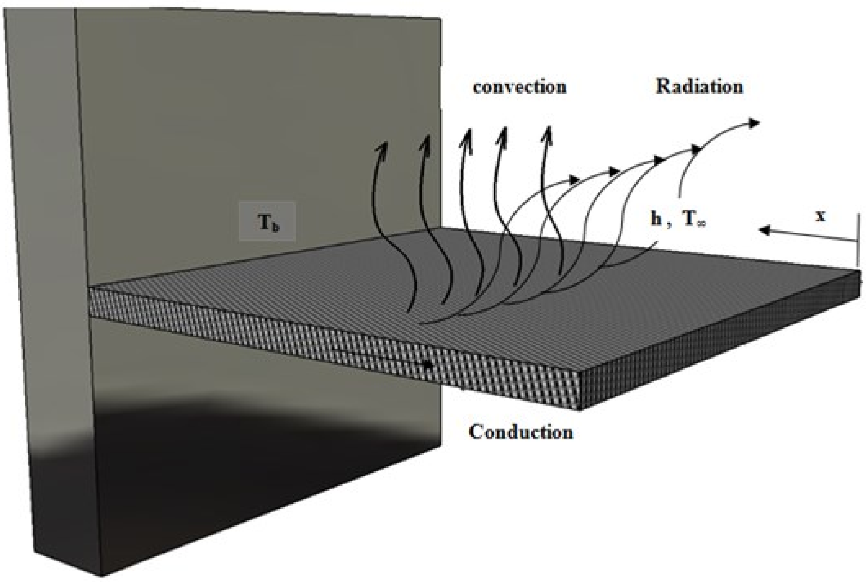

Figure 1: Schematic diagram of fin profile under consideration

The example to be studied is the one-dimensional heat transfer in a longitudinal fin of rectangular profile area A_c, length L, constant thermal conductivity k_0 and surface emissivity ε. The fin is attached to a primary surface at fixed temperature T_b and loses heat by simultaneous convection and radiation to the surrounding medium. The sink temperatures for convection and radiation are T_∞ and T_s, respectively. The convective heat transfer coefficient h is assumed to be a constant. The heat loss from the tip of the fin compared with the top and bottom surfaces of the fin is taken to be negligible. Since the transverse Biot number should be small for the fin to be truly effective, the temperature variation in the transverse direction can be neglected. Thus heat conduction occurs only in the longitudinal direction. For the problem just described, the appropriate differential equation and the boundary conditions may be written as

$$\frac{d^2T}{dx^2} - \frac{h\,p}{K_0A}\left(T - T_\infty\right) - \frac{\varepsilon'\sigma p}{K_0A}\left(T^4 - T_S^4\right) = 0 \tag{1}$$

$$x = 0, \quad \frac{dT}{dx} = 0 \qquad \text{as well as} \qquad x = L, \quad T = T_b, \tag{2}$$

where x is measured from the tip of the fin. For simplicity, the case of $T_\infty = T_S = 0$ is treated. With the introduction of following dimensionless quantities,

$$\theta = \frac{T}{T_b}, \quad X = \frac{x}{L}, \quad N^2 = \frac{hpL^2}{K_0A}, \quad \varepsilon = \frac{\varepsilon'\sigma pL^2T_b^3}{K_0A} \tag{3}$$

eqs. (1)–(3) take the form

$$\frac{d^2\theta}{dX^2} - N^2\,\theta - \varepsilon\,\theta^4 = 0 \tag{4}$$

$$X = 0, \quad \frac{d\theta}{dX} = 0 \qquad \text{as well as} \qquad X = 1, \quad \theta = 1. \tag{5}$$

3 Application of collocation method

In collocation method a trial family of approximate solution T containing a finite number of undetermined coefficient $C_1, C_2, \ldots$ and C_n can be constructed by the superposition of some basis functions such as polynomials, trigonometric functions and a trial solution so selected to satisfy the essential boundary conditions for the problem [23]. But when it is introduced into the differential equation,this one is not satisfied and leads to a residual R, because it is not the exact solution. For the true solution the residual vanishes identically, therefore the problem of constructing an approximate solution becomes one of determining the unknown coefficients $C_1, C_2, \ldots$ and C_n. So the residual stays close to zero throughout the domain of the solution. Depending on the number of terms taken for the trial solution, the type of base functions used and the way the unknown coefficients are determined, several different approximate solutions are possible for a given problem. We wish to obtain an approximate solution for this problem in the interval $0 < X < 1$. To construct a trial solution $\theta \equiv t/T_b$, we choose the basic function as a polynomial in X. The trial solution contains two undetermined coefficients and satisfies the condition for all values of C as follows:

$$\begin{aligned} \theta\left(X\right) = 1 &+ C_1\left(1 - X^2\right) + C_2\left(1 - X^3\right) \\ &+ C_3\left(1 - X^4\right) + C_4\left(1 - X^5\right) + C_5\left(1 - X^6\right), \end{aligned} \tag{6}$$

whereas the trial solution satisfies the boundary conditions of (5). When θ is introduced into differential equation (4) it yields residual $R\left(X\right)$ as follows:

$$\begin{aligned} R\left(X\right) = {} & 24\varepsilon C_1 C_2 C_3 X^4 C_4 - 24\varepsilon C_1 C_2 C_3 X^{10} C_5 + 24\varepsilon C_1 C_2 C_3 X^4 C_5 \\ & + 24\varepsilon C_1 C_2 C_4 C_5 X^6 - 24\varepsilon C_1 C_2 X^8 C_3 C_4 + 24\varepsilon C_1 C_2 X^3 C_3 C_4 \\ & + 24\varepsilon C_1 C_2 X^3 C_3 C_5 + 24\varepsilon C_1 C_2 X^3 C_4 C_5 + 24\varepsilon C_1 C_2 X^{13} C_3 C_5 \\ & + \cdots - 48\varepsilon C_1 C_2 X^7 C_3 C_4 - 4\varepsilon C_4^3 X^2 1 C_5 - 12\varepsilon C_4 X^{11} C_5^3 \\ & - 4\varepsilon C_4 X^{23} C_5^3 + 12\varepsilon C_4 X^{17} C_5^3 = 0. \end{aligned} \tag{7}$$

This residual vanishes only with the exact solution for the problem. Now the problem of finding approximate solution of the problem in the interval $0 < X < 1$ becomes one adjusting the values of C_1, C_2, C_3, C_4, C_5 so that residual stays close to zero throughout the interval $0 < X < 1$. The basic assumption is that the residual does not deviate much from zero between collocation locations:

$$R\left(\frac{1}{6}\right) = 0,\ R\left(\frac{2}{6}\right) = 0,\ R\left(\frac{3}{6}\right) = 0,\ R\left(\frac{4}{6}\right) = 0,\ R\left(\frac{5}{6}\right) = 0. \tag{8}$$

This produces the following set of algebraic equations:

$$
\begin{aligned}
R\left(\frac{1}{6}\right) &= -\varepsilon - N^2 - 0.333333\,C_3 - 0.0231148\,C_5 \\
&- 23.206891464\varepsilon\,C_1\,C_2C_3\,C_5 - 0.999978566\,N^2\,C_5 \\
&+ \cdots - C_2 - 2C_1 - 23.20440\varepsilon C_1\,C_2\,C_3\,C_4 - 0.09258\;C_4 \\
&- 0.9953703N^2\,C_2 - 0.999223N^2\,C_3 - 0.9998713N^2\,C_4 \\
&- 11.98868\varepsilon\,C_3\,C_4\,C_5^2 = 0,
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
R\left(\frac{2}{6}\right) &= -\varepsilon - N^2 - 1.333333\,C_3 - 2C_2 - 2C_1 - 0.7407407\,C_4 \\
&- 20.261757\varepsilon\,C_1\,C_2C_3\,C_5 - 0.99862825\,N^2\,C_5 + \cdots \\
&- 20.206093\varepsilon C_1\,C_2\,C_3\,C_4 - 0.3703703\,C_5 - 1.333333\,C_3 \\
&- 0.9629629N^2\,C_2 - 0.9876543N^2\,C_3 - 0.995884N^2\,C_4 \\
&- 11.64137\varepsilon\,C_3^2\,C_4\,C_5 = 0,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
R\left(\frac{3}{6}\right) &= -\varepsilon - N^2 - 3\,C_3 - 3\,C_2 - 2\;C_1 - 2.5\;C_4 - 1.875\,C_5 \\
&- 0.75\,N^2\,C_1 - 0.96875N^2\,C_4 - 0.984375N^2\,C_5 \\
&+ \cdots - 14.3041992\varepsilon\,C_1\,C_2\,C_3\,C_4 - 10.05764007\varepsilon\,C_3^2\,C_4\,C_5 \\
&- 0.9375N^2\,C_3 - 14.53491\varepsilon\,C_1\,C_2\,C_3\,C_5 = 0,
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
R\left(\frac{4}{6}\right) &= -\varepsilon - N^2 - 5.3\,3333C_3 - 4\,C_2 - 2\;C_1 - 5.9259255\,C_4 \\
&- 5.9259259C_5 - 0.86831275N^2\,C_4 + \cdots - 0.91220850N^2\,C_5 \\
&- \;6.53782200\varepsilon\,C_1\,C_2\,C_3\,C_4 - 0.703703703N^2\,C_2 \\
&- 0.80246913N^2\,C_3 - 6.12080044\varepsilon\,C_3^2\,C_4\,C_5 = 0,
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
R\left(\frac{5}{6}\right) &= -\varepsilon - N^2 - 8.3\,3333C_3 - 5\,C_2 - 2\;C_1 - 11.5740740\,C_4 \\
&- 14.467592C_5 - 0.59812242N^2\,C_4 + \cdots - 0.66510202N^2\,C_5 \\
&- 0.95674604\varepsilon\,C_1\,C_2\,C_3\,C_4 - 0.51774691N^2\,C_3 \\
&- 1.279660132\varepsilon C_3^2\,C_4\,C_5 - 1.063885414\varepsilon\,C_1\,C_2\,C_3\,C_5 = 0.
\end{aligned} \tag{13}
$$

Thus we can obtain coefficients $C_1, \ldots, C_5$ for different values of ε and N as displayed in the table 1.

Table 1: Values of coefficients $C_1, \ldots, C_5$ obtained for different ε and N

ε	N	C_1	C_2	C_3	C_4	C_5
0.8	1	−0.3522584539	0.02885438874	−0.1134243827	0.07539448499	−0.04601045594
0.6	1	−0.3464428016	0.01950052041	−0.08788115417	0.05125548469	−0.03240810318
0.8	0.5	−0.2199369690	0.01089417921	−0.05348342586	0.02889652790	−0.01999639036
0.6	0.5	−0.2033419045	0.005979100628	−0.03634567139	0.01598823992	−0.01189368851
0.8	0.25	−0.1818050885	0.007559259348	−0.0418417101	0.02015195596	−0.01457829786
0.6	0.25	−0.1608119863	0.003696128908	−0.02682120282	0.009941296203	−0.007882485075

4 Application of Variational Iteration Method

First we construct a correction functional which reads

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda \left\{ \theta_n''(x) - N^2 \theta_n(x) + \varepsilon \theta^4(x) \right\} d\tau \tag{14}$$

where λ is general Lagrange multiplier.

Making the above correction functional stationary, we can obtain following stationary conditions

$$\lambda''(t) - M^2 \lambda(t) = 0, \quad 1 - \lambda'(t)|_{t=x} = 0, \quad \lambda(t)|_{t=x} = 0. \tag{15}$$

The Lagrange multiplier, therefore, can be identified as

$$\lambda = -\frac{\sinh[N(x-\tau)]}{N}. \tag{16}$$

As a result, we obtain the following iteration formula

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda \left\{ \theta_n''(x) - N^2 \theta_n(x) + \varepsilon \theta^4(x) \right\} d\tau. \tag{17}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition

$$\theta_0(x) = \frac{e^{-Nx}}{e^{-N}+e^{N}} + \frac{e^{Nx}}{e^{-N}+e^{N}}. \tag{18}$$

Using the above iteration formula (18), after some simplifications, we get

$$\begin{aligned}
\theta_1(x) = {} & \frac{1}{15}\frac{1}{(e^N+e^{-N})(e^{8N}+4e^{6N}+6e^{4N}+4e^{2N}+1)}\big[15N^2e^{Nx} \\
& + 15N^2e^{N(x+8)} + 60N^2e^{N(x+6)} + 60N^2e^{N(x+2)} + 15N^2e^{-Nx} \\
& + 15N^2e^{-N(x-8)} + 60N^2e^{-N(x-6)} + 90N^2e^{-N(x-4)} \\
& + 24\varepsilon e^{N(x+4)} + 24\varepsilon e^{N(x+3)} + 24\varepsilon e^{-N(x-5)} \\
& + 24\varepsilon e^{-N(x-3)} + \varepsilon e^{N(4x+5)} - 90\varepsilon e^{3N} + \varepsilon e^{N(4x+3)} \\
& + 60N^2e^{-N(x-2)} + 20\varepsilon e^{-N(2x-5)} + 20\varepsilon e^{-N(2x-3)} \\
& + \varepsilon e^{-N(4x-5)} + \varepsilon e^{-N(4x-3)} + 20\varepsilon e^{N(2x+5)} + 90N^2e^{N(x+4)} \\
& - 90\varepsilon e^{5N} + 20\varepsilon e^{N(2x+3)}\big],
\end{aligned} \tag{19}$$

where $C_0 = \frac{1}{A}$, that is

$$\begin{aligned}A = &\frac{1}{15}\frac{1}{(e^{N}+e^{-N})(e^{8N}+4e^{6N}+6e^{4N}+4e^{2N}+1)}\big[15N^2e^{9N}\\&+60N^2e^{7N}+90N^2e^{5N}+60N^2e^{3N}+15N^2e^{N}+15N^2e^{7N}\\&+60N^2e^{5N}+90N^2e^{3N}-90\varepsilon e^{5N}+\varepsilon e^{N}+15N^2e^{-N}\\&+24\varepsilon e^{5N}+24\varepsilon e^{4N}+24\varepsilon e^{2N}+\varepsilon e^{9N}+\varepsilon e^{-N}-90\varepsilon e^{3N}\\&+\varepsilon e^{7N}+60N^2e^{N}+20\varepsilon e^{3N}+20\varepsilon e^{7N}+20\varepsilon e^{5N}\big].\end{aligned}$$

5 Application of Homotopy Perturbation Method

We define He's general form of eq. (4) as:

$$A(\theta) - U(x) = 0, \qquad x \in \Omega\,. \tag{20}$$

With the boundary condition of:

$$B\left(\theta\,,\,\frac{\partial\theta}{\partial n}\right) = 0\,, \qquad x \in \Gamma. \tag{21}$$

where A is a general differential operator, B a boundary operator, $U(x)$ a known analytical function and Γ is the boundary of the domain Ω.

So according to eq. (4) we will have:

$$A(\theta) = \left[\theta''(X) - N^2\,\theta(X) - \varepsilon\,\theta^4(X)\right], \qquad U(x) = 0. \tag{22}$$

Here A can be divided into two parts, L and N, where L is linear and N is nonlinear:

$$L(\theta) + N(\theta) = 0\,, \qquad x \in \Omega\,. \tag{23}$$

The Homotopy Perturbation structure is shown as follows:

$$H(\theta,\,p) = (1-p)\left[L(\theta) - L(\theta_0)\right] + p\left[A(\theta) - U(x)\right] = 0 \tag{24}$$

where, $p \in [0\,,\,1]$ is an embedding parameter and θ_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of eq.(4) can be written as a power series in p, as following:

$$\theta(X) = \theta_0(X) + p\,\theta_1(X) + p^2\,\theta_2(X) + \cdots = \sum_{i=0}^{n} p^i\,\theta_i(X). \tag{25}$$

Considering eqs. (22) and (24), we will have:

$$(1-p)\,\left[\theta''(X) - N^2\,\theta(X)\right] + p\,\left[\theta''(X) - N^2\,\theta(X) - \varepsilon\,\theta^4(X)\right] = 0, \qquad (26)$$

By substituting $\theta\,(X)$ from eq.(25) into eq.(26) and after some simplifications and rearrangements based on powers of p-terms, we have:

$$\begin{aligned} p^0: \qquad & -N^2\,\theta_0\,(X) \;+\theta_0''\,(X) = 0, \\ & \theta_0\,(1) = 1\;\;,\quad \theta_0'\,(0) = 0, \end{aligned} \qquad (27)$$

$$\begin{aligned} p^1: \qquad & \theta_1''\,(X) - \theta_1(X) - \varepsilon\;\theta_0^4\,(X) = 0, \\ & \theta_1\,(1) = 0\;\;,\quad \theta_1'\,(0) = 0. \end{aligned} \qquad (28)$$

Solving Eqs.(23)-(24) with boundary conditions, we have:

$$\theta_0(x) = \frac{e^{NX}}{e^N + e^{-N}} + \frac{e^{-NX}}{e^N + e^{-N}} \qquad (29)$$

$$\begin{aligned} \theta_1(X) = & -\frac{e^{NX}}{15\,N^2}\varepsilon\big(e^{8\,N} + 20\;e^{6\,N} - 90\;e^{4\,N} + 20\;e^{2\,N} + 1\big)\big(e^{-N}e^{8\,N} \\ & + 4\;e^{-N}e^{6\,N} + 6\;e^{-N}e^{6\,N} + 4\;e^{-N}e^{2\,N} + e^{-N} + 6\;e^{N}e^{4\,N} + e^{N} \\ & + 4\;e^{N}e^{2\,N} + e^{N}e^{8\,N} + 4\,e^{N}e^{6\,N}\big)^{-1} - \frac{e^{-NX}}{15\,N^2}\varepsilon\big(e^{8N} +\;20\,e^{6N} \\ & - 90\,e^{4N} + 20\,e^{2N} + 1\big)\big(e^{-N}e^{8\,N} + 4\;e^{-N}e^{6\,N} + 6\;e^{-N}e^{6\,N} + e^{-N} \\ & + 4\;e^{-N}e^{2\,N} + 6\;e^{N}e^{4\,N} + 4\,e^{N}e^{2\,N} + e^{N} + e^{N}e^{8\,N} + 4\,e^{N}e^{6\,N}\big)^{-1} \\ & + \frac{e^{-4\,N(-1+X)}}{15\,N^2}\;\frac{\varepsilon(e^{8\,N\,X} + 20\,e^{6\,N\,X} - 90\;e^{4N\,X} + 20\;e^{2\,N\,X} + 1)}{e^{8\,N} + 4\,e^{6\,N} + 6\,e^{4\,N} + 4\,e^{2\,N} + 1}. \end{aligned} \qquad (30)$$

The solution of this equation, when $p \to 1$, will be as follows:

$$\theta\,(X) = \theta_0\,(X) + \theta_{\,1}\,(X) \qquad (31)$$

6 Results and discussion

In this manuscript the Collocation Method such as analytical technique is employed to find an analytical solution of the nonlinear fin problem. The results are compared with other analytical methods such as VIM and HPM, for validation all these results are compared with the BVP. The main goal of this article is to show the simplicity, stability and power of Collocation Method rather than the other mentioned techniques. Figures 2.-6. show the temperature distribution with the axial distance in a convective-radiative fin with constant thermal conductivity by three methods. Comparing these figures gives closer results to numerical solution. It is observed that the Collocation Method is very effective, simple, stable and more accurate than other methods.

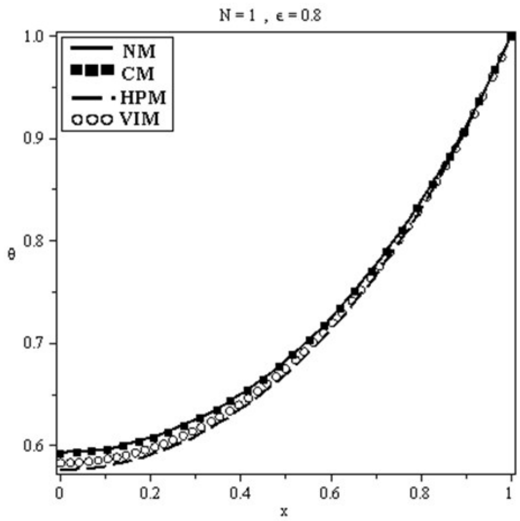

Figure 2: Comparison of temperature distributions with different approximate methods: $N = 1,\ \varepsilon = 0.8$

Moreover, In order to show the effectiveness of Collocation Method, numerical comparison between CM with other different approximate solutions are tabulated in Table 2. It is interesting to note that Collocation Method is very close to the numerical results and the results of HPM, VIM are significantly in error.

Table 2: The result of different method for $N = 0.25\,,\ \ \varepsilon = 0.8$.

X	CM	VIM	HPM	NUM	Error of VIM	Error of HPM	Error of CM
0	0.78948611	0.71141895	0.61776338	0.78959579	0.07817683	0.17183240	0.00010967
0.1	0.79130060	0.71423546	0.62149187	0.79139981	0.07716435	0.16990794	0.00009921
0.2	0.79675927	0.72269325	0.63268853	0.79684168	0.07414843	0.16415315	0.00008241
0.3	0.80594505	0.73681712	0.65138694	0.80601244	0.06919532	0.15462550	0.00006739
0.4	0.81901564	0.75664855	0.67764330	0.81906965	0.06242110	0.14142635	0.00005400
0.5	0.83620562	0.78224594	0.71153672	0.83624684	0.05400090	0.12471012	0.00004122
0.6	0.85783898	0.81368494	0.75316967	0.85786822	0.04418327	0.10469855	0.00002923
0.7	0.88435216	0.85105887	0.80266856	0.88437047	0.03331160	0.08170191	0.00001831
0.8	0.91632761	0.89447921	0.86018444	0.91633474	0.02185552	0.05615029	0.00000712
0.9	0.95453786	0.94407629	0.92589390	0.95453344	0.01045714	0.02863954	0.00000442

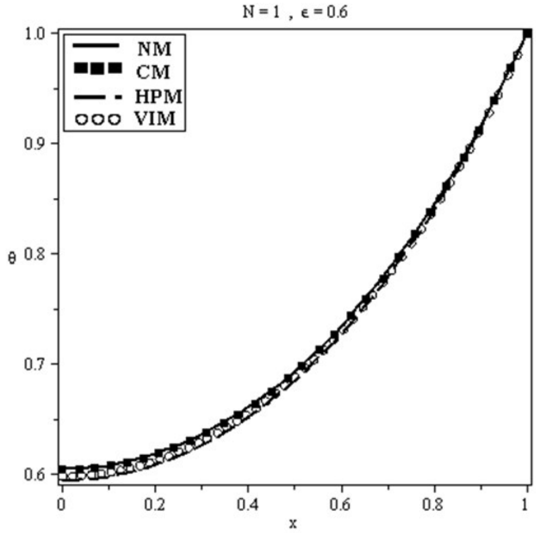

Figure 3: Comparison of temperature distributions with different approximate methods: $N = 1$, $\varepsilon = 0.6$

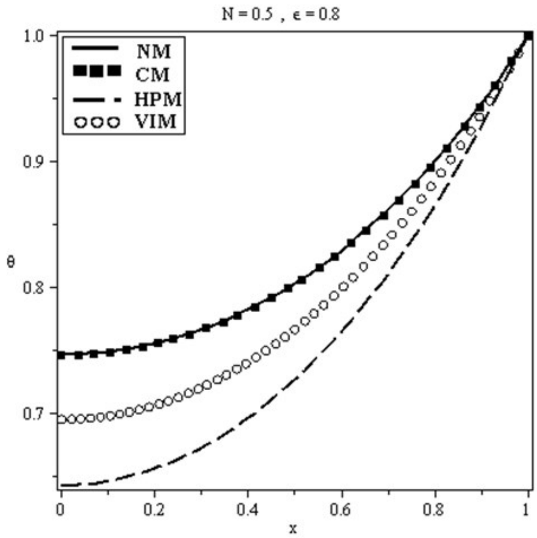

Figure 4: Comparison of temperature distributions with different approximate methods: $N = 0.5$, $\varepsilon = 0.8$

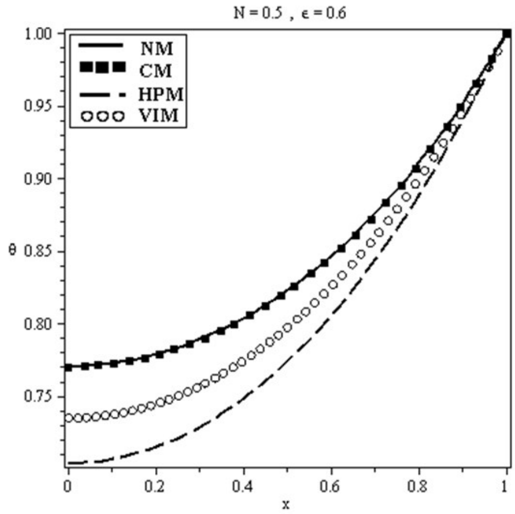

Figure 5: Comparison of temperature distributions with different approximate methods: $N = 0.5,\ \varepsilon = 0.6$

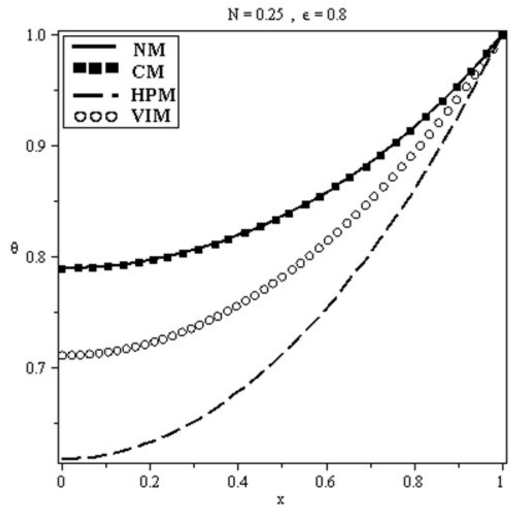

Figure 6: Comparison of temperature distributions with different approximate methods: $N = 0.25,\ \varepsilon = 0.8$

7 Conclusion

In this letter, the basic idea of the Collocation Method is introduced and then we have applied to solve the governing non-linear ordinary differential equations. Furthermore, the obtained solutions by Collocation Method are compared with VIM , HPM and numerical solutions. The results demonstrates that Collocation Method is very effective, simple and offers superior accuracy in comparison with the Variation Iteration Method and Homotopy Perturbation Method. Also, it is found that these methods are powerful mathematical tools and that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering specially some heat transfer equations.

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Prost i tačan pristup rešavanju nelinearne jednačine pri konvekciji i radijaciji u peraju korišćenjem kolokacionog metoda i uporedjenje sa HPM i VPM

Koristi se kolokaciona metoda (CM), kao analitička metoda kojoj nije potreban mali parametar, za analitička približna rešenja nelinearne jednačine prenosa toplote. Rezultati dobijeni kolokacionom metodom su uporedjeni sa drugim analitičkim tehnikama kao što su: perturbaciona metoda homotopijom (HPM) i varijaciona iterativna metoda (VIM). Takodje, za overu se koristi problem granične vrednosti (BVP) kao numerička metoda. Rezultati pokazuju da je kolokaciona metoda vrlo efektivna, prosta i tačnija od drugih tehnika. Takodje, nadjeno je da je ova metoda snažna matematička alatka i može se primeniti na veliku klasu linearnih i nelinearnih problema koji se pojavljuju u raznim naučnim i tehničkim poljima - posebno jednačinama prenosa toplote.

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