

OSEEN'S CORRECTION TO STOKES DRAG ON AXIALLY  
SYMMETRIC ARBITRARY PARTICLE IN TRANSVERSE  
FLOW: A NEW APPROACH

Deepak Kumar Srivastava      Nirmal Srivastava

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## Oseen's correction to Stokes drag on axially symmetric arbitrary particle in transverse flow: a new approach

Deepak Kumar Srivastava \*      Nirmal Srivastava †

### Abstract

In this paper, Oseen's correction to Stokes drag experienced by axially symmetric particle placed in a uniform stream perpendicular to axis of symmetry (i.e. transverse flow) is obtained. For this, the linear relationship between axial and transverse Stokes drag is utilized to extend the Brenner's formula for axial flow to transverse flow. General expression of Oseen's correction to Stokes drag on axially symmetric particle placed in transverse flow is found to be new. This general expression is applied to some known axially symmetric bodies and obtained values of Oseen's drag, up to first order terms in Reynolds number 'R', are also claimed to be new and never exist in the literature. Numerical values of Oseen drag are also evaluated and their variations with respect to Reynolds number, eccentricity and deformation parameter are depicted in figures and compared with some known values. Some important applications are also highlighted.

**Keywords:** Stokes drag, Oseen's drag, axially symmetric body, Oseen's flow, transverse flow.

## 1 Introduction

The method and formulation for analysis of flow at a very low Reynolds number is important. The slow motion of small particles in a fluid is common in bio-engineering. Oseen's drag formulation can be used in connection with flow of fluids under various special conditions, such as: containing particles,

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\*Department of Mathematics B.S.N.V. Post Graduate College(University of Lucknow, Lucknow) Station Road, Charbagh, Lucknow (U.P.)-226001, India, e-mail: dks-flow@hotmail.com

†e-mail: nirmalsri.25@gmail.com

sedimentation of particles, centrifugation or ultracentrifugation of suspensions, colloids, and blood through isolation of tumors and antigens. The fluid does not even have to be a liquid, and the particles do not need to be solid. It can be used in a number of applications, such as smog formation and atomization of liquids. The problem of great importance in the hydrodynamics of low Reynolds number flows is the drag or resistance experienced by a particle moving uniformly through an infinite fluid. Since the appearance of Stokes's approximate solution for the flow of a viscous fluid past a sphere (Stokes, 1851), very well known as Stokes law, numerous attempts have been made, both to generalize the problem by changing the shape of the body, and to improve the calculation by including the effect of the inertia terms which were neglected in the original calculation. Oseen (1927) tackled this type of problem involving the correction to Stokes drag extensively. Oseen provided solutions for the flow past various bodies at small Reynolds number 'R' and calculated the force to the first order in R, one term more than would be given by the Stokes approximation. By the inclusion of the effect of the inertia terms, Oseen improved the flow picture far from the body where the Stokes approximation is inadequate, but near the body the difference between the two solutions is of an order of smallness which is outside the accuracy of either approximation. Oseen's calculation for the force thus requires some further justification, for flow past a sphere, by the work of Kaplun (1957), Kaplun and Lagerstrom (1957) and Proudman and Pearson (1957). Oseen failed to calculate correctly the velocity field, his result for the drag on the sphere, namely

$$D = D_0[1 + (3/8)R], \quad (1.1)$$

where  $D_0$  is the Stokes drag, is in fact valid because the correction to the velocity field makes no contribution to the total force on the sphere. Almost similar problem has been considered by Chang (1960) for the axially symmetric Stokes flow of a conducting fluid past a body of revolution in the presence of a uniform magnetic field. An equation identical to that cited above, except that the dimensionless Hartmann number, M, appears in place of the Reynolds number, 'R'. It is interesting to note here that Chang's (1960) result is restricted to axially symmetric flows because of the requirement that there be sufficient symmetry to preclude the existence of an electric field. While, on the other hand, Brenner's (1961) result is limited only by the requirement that the Stokes drag on the particle (and thus the Oseen drag) be parallel to its direction of motion. Krasovitskaya et al. (1970) proposed a formula based on Oseen's correction for calculating the settling of solid particles of powdered materials with enhanced accuracy in carrying out sedimentation analysis. Dyer and Ohkawa (1992) have

used the Oseen drag in acoustic levitation. These two works are the main practical applications of Oseen's correction which was not possible with the Stokes drag. In biology (Fung, 1997), blood flow in small vessels, such as capillaries, is characterized by small Reynolds numbers. A vessel of diameter of 10  $\mu\text{m}$  with a flow of 1 millimetre/second, viscosity of 0.02 poise for blood, density of 1 g/cm<sup>3</sup> and a heart rate of 2 Hz, will have a Reynolds number of 0.005. At these small Reynolds numbers, the viscous effects of the fluid become predominant. Oseen's method is better in understanding of the movement of these particles for drug delivery and studying metastasis movements of cancers.

Datta and Srivastava (1999) have proposed a new method to evaluate axial Stokes drag and transverse Stokes drag based on geometry of axially symmetric particle under no-slip boundary condition. From these two expressions of Stokes drag, the linear relationship between axial and transverse Stokes drag can be proved. With the help of this linear relationship, we can evaluate the Oseen's correction in Stokes drag experienced by axially symmetric particles placed in transverse flow followed by Brenner's formula (Brenner, 1961). Srivastava et al. (2012) solved the problem of steady Stokes flow past dumbbell shaped axially symmetric body of revolution by using newly developed analytic approach based on D-S conjecture (Datta and Srivastava, 1999). The method is described in section 2 and applied to various axially symmetric bodies in section 4.

## 2 Method

Let us consider the axially symmetric body of characteristic length  $L$  placed along its axis ( $x$ -axis, say) in a uniform stream  $U$  of viscous fluid of density  $\rho$  and kinematic viscosity  $\nu$ . When Reynolds number  $UL/\nu$  is small, the steady motion of incompressible fluid around this axially symmetric body is governed by Stokes equations [Happel and Brenner, 1964],

$$\mathbf{0} = - \left( \frac{1}{\rho} \right) \text{grad} p + \nu \nabla^2 \mathbf{u}, \quad \text{div } \mathbf{u} = 0 \quad (2.1)$$

subject to the no-slip boundary condition.

We have taken up the class of those axially symmetric bodies which possess continuously turning tangent, placed in a uniform stream  $U$  along the axis of symmetry (which is  $x$ -axis), as well as constant radius 'b' of maximum circular cross-section at the mid of the body. This axi-symmetric body is obtained by the revolution of meridional plane curve (depicted in figure 1) about

axis of symmetry which obeys the following limitations:

- i. Tangents at the points A, on the x-axis, must be vertical,
- ii. Tangents at the points B, on the y-axis, must be horizontal,
- iii. The semi-transverse axis length 'b' must be fixed.

The point P on the curve may be represented by the Cartesian coordinates  $(x, y)$  or polar coordinates  $(r, \theta)$  respectively; PN and PM are the length of tangent and normal at the point P. The symbol R stands for the intercepting length of normal between the point on the curve and point on axis of symmetry and symbol  $\alpha$  is the slope of normal PM which can vary from 0 to  $\pi$ .

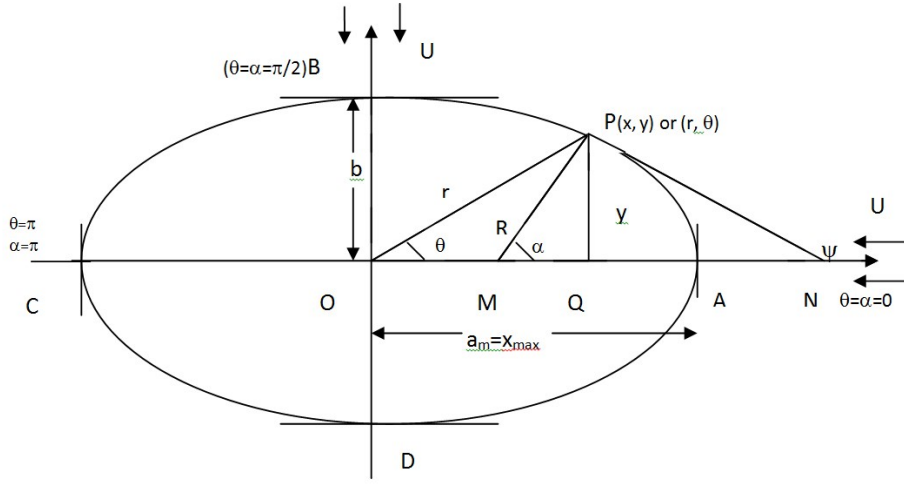


Figure 1: Geometry of axially symmetric body

## 2.1 Axial flow

The expression of Stokes drag on such type of axially symmetric bodies placed in axial flow (uniform flow parallel to the axis of symmetry) is given by [Datta and Srivastava, 1999]

$$F_x = \frac{1}{2} \frac{\lambda b^2}{h_x}, \quad \text{where} \quad \lambda = 6\pi\mu U \quad (2.2)$$

$$\text{and} \quad h_x = \left(\frac{3}{8}\right) \int_0^\pi R \sin^3 \alpha d\alpha. \quad (2.3)$$

## 2.2 Transverse flow

The expression of Stokes drag on axially symmetric bodies placed in transverse flow (uniform flow perpendicular to the axis of symmetry) is given by [Datta and Srivastava, 1999]

$$F_y = \left(\frac{1}{2}\right) \frac{\lambda b^2}{h_y}, \quad \text{where} \quad \lambda = 6\pi\mu U \quad (2.4)$$

$$\text{and} \quad h_y = \left(\frac{3}{16}\right) \int_0^\pi R (2\sin\alpha - \sin^3\alpha) d\alpha. \quad (2.5)$$

Here the suffix 'y' has been introduced to assert that the force is in the transverse direction,

$$\frac{F_x}{F_y} = \frac{h_y}{h_x} = \frac{1}{2} \frac{\int_0^\pi R (2\sin\alpha - \sin^3\alpha) d\alpha}{\int_0^\pi R \sin^3\alpha d\alpha} = K \quad (\text{say}). \quad (2.6)$$

Now, on dividing (2.2) and (2.4), we get

$$F_x = K F_y. \quad (2.7)$$

On applying Brenner's formula (Brenner, 1961), Oseen's correction to Stokes drag on a body placed in axial uniform flow, in general, may be written as

$$\frac{F}{F_x} = 1 + \frac{F_x}{16\pi\mu LU} R + O(R^2), \quad (2.8)$$

by using linear relationship between axial and transverse Stokes drag (2.7), equation (2.8) provides Oseen's correction to Stokes drag on a body placed in transverse uniform flow

$$\frac{F}{F_y} = K \frac{F}{F_x} = K \left[ 1 + \frac{F_x}{16\pi\mu LU} R + O(R^2) \right]. \quad (2.9)$$

where K is a real factor defined in (2.6) and  $R = \rho UL/\mu$ , the particles Reynolds number.

## 3 Formulation of the problem

Let us consider the axially symmetric arbitrary body of characteristic length L placed along its axis (x-axis, say) in a uniform stream U of viscous fluid of density  $\rho$  and kinematic viscosity, perpendicular to axis of symmetry. When

particle Reynolds number  $UL/\nu$  is small, the steady motion of incompressible fluid around this axially symmetric body is governed by Stokes equations [Happel and Brenner, 1964],

$$\mathbf{0} = - \left( \frac{1}{\rho} \right) \text{grad} p + \nu \nabla^2 \mathbf{u}, \quad \text{div} \mathbf{u} = 0, \quad (3.1)$$

subject to the no-slip boundary condition. This equation is the reduced form of complete Navier–Stokes equations neglecting inertia term  $(\mathbf{u} \cdot \text{grad}) \mathbf{u}$  which is unimportant in the vicinity of body where viscous term dominates (Stokes approximation). Solution of this equation (3.1), called Stokes law, ‘ $6\pi\mu Ua$ ’, for a slowly moving sphere having radius ‘ $a$ ’, is valid only in the vicinity of the body which breaks down at distance far away from the body. This breaks down in Stokes solution at far distance from the body being known as Whitehead’s paradox. It was Oseen in 1910, who pointed out the origin of Whitehead’s paradox and suggested a scheme for its resolution (see Oseen, 1927). In order to rectify the difficulty, Oseen went on to make the following additional observations.

In the limit where the particle Reynolds number  $\rho Ua/\mu \rightarrow 0$ , Stokes approximation becomes invalid only when  $r/a \rightarrow \infty$ . But at such enormous distances, the local velocity  $\mathbf{v}$  differs only imperceptibly from a uniform stream of velocity  $\mathbf{U}$ . Thus, Oseen was inspired to suggest that the inertial term  $(\mathbf{U} \cdot \text{grad}) \mathbf{u}$  could be uniformly approximated by the term  $(\mathbf{u} \cdot \text{grad}) \mathbf{u}$ . By such arguments, Oseen proposed that uniformly valid solutions of the problem of steady streaming flow past a body at small particle Reynolds numbers could be obtained by solving the linear equations

$$(\mathbf{U} \cdot \text{grad}) \mathbf{u} = - \left( \frac{1}{\rho} \right) \text{grad} p + \nu \nabla^2 \mathbf{u}, \quad \text{div} \mathbf{u} = 0 \quad (3.2)$$

known as Oseen’s equation. Oseen obtained an approximated solution of his equations for flow past a sphere, from which he obtained the Stokes drag formula [Happel and Brenner, page 44, eq.(2-6.5), 1964]

$$F = 6\pi\mu a U \left[ 1 + \frac{3}{8} N_{Re} + O(N_{Re}^2) \right], \quad (3.3)$$

where  $N_{Re} = \rho Ua/\mu$  is bodies Reynolds number.

We find the solution of these equations (3.2) for various axially symmetric bodies like sphere, spheroid (prolate and oblate), deformed sphere, cycloidal body, cassini oval, hypocycloidal body, cylindrical capsule with semi spherical ends and complicated egg-shaped body consisting of semi spherical and semi

spheroidal ends under no-slip boundary conditions by use of D-S conjecture (2.2), (2.4) followed by linear relationship (2.7) and Brenner's formula (2.8) valid for axial flow and its extension (2.9) for transverse flow.

## 4 Solution

### 4.1 Flow past a sphere

Stokes drag on sphere having radius 'a' placed in uniform axial flow, with velocity U, parallel to axis of symmetry(x-axis) and very well known as Stokes law of resistance is given by (by utilizing DS conjecture 2.4 and 2.5, Datta and Srivastava, 1999)

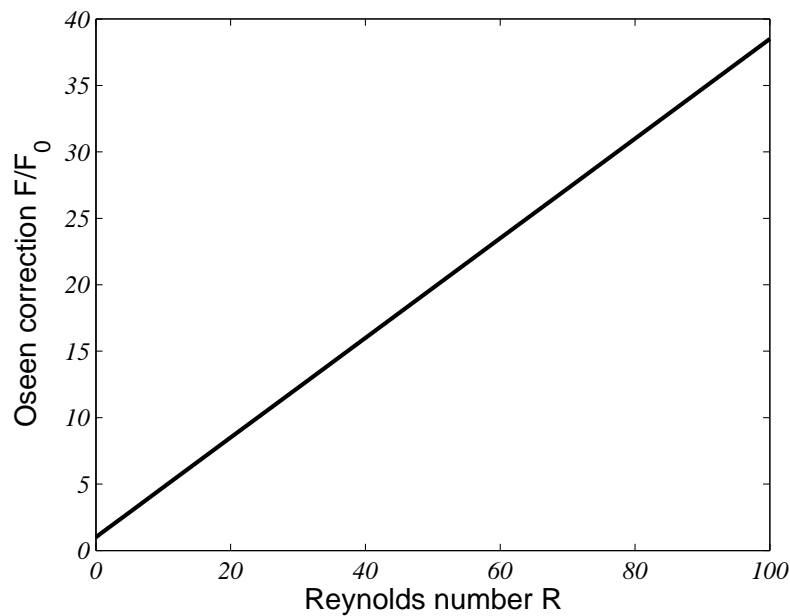


Figure 2: Variation of Oseen correction with respect to Reynolds number  $R = \rho U a / \mu$  for sphere

$$F_x = 6\pi\mu U a. \quad (4.1a)$$

$$F_y = 6\pi\mu U a. \quad (4.1b)$$



From relation (4.1a, b) and (2.7), the value of

$$K = 1. \quad (4.2)$$

Now, the Oseen's correction as well as the solution of Oseen's equation (3.2) may be obtained for same sphere by substituting the value of  $K = 1$  and Stokes drag (4.1a,b) in Brenner's formula (2.8) and (2.9) as

$$\frac{F}{F_y} = \frac{F}{F_x} = 1 + \frac{3}{8}R + O(R^2), \quad (4.3)$$

where  $R = \left(\frac{\rho U a}{\mu}\right)$  is particle Reynolds number. This Oseen's correction matches with that given by Oseen(1927) and Chester(1962).

## 4.2 Flow past a prolate spheroid

Stokes drag on prolate spheroid having semi-major axis length 'a' and semi-minor axis length 'b' placed in uniform velocity  $U$ , parallel to axis of symmetry(axial flow) and perpendicular to its axis of symmetry(transverse flow) is given as[by utilizing formulae 2.4 and 2.5, Datta and Srivastava, 1999]

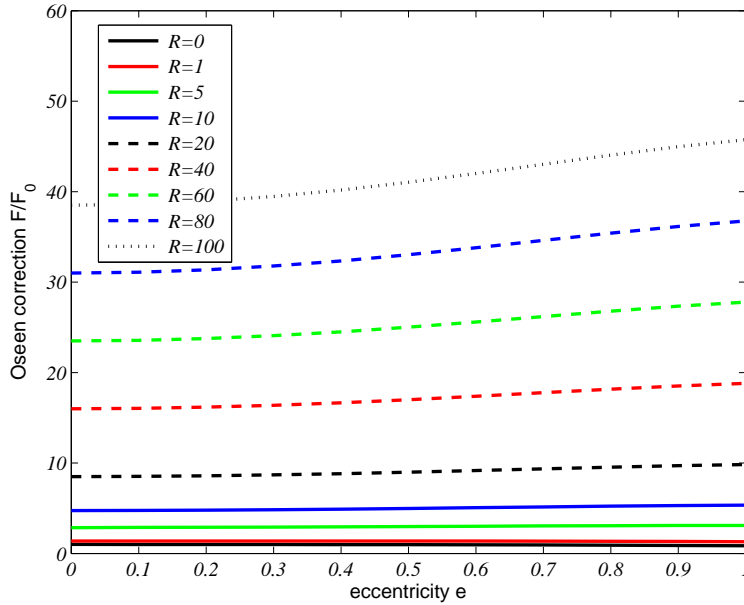


Figure 3: Variation of Oseen correction with respect to eccentricity 'e' for various values of Reynolds number  $R = \rho U a / \mu$  for prolate spheroid

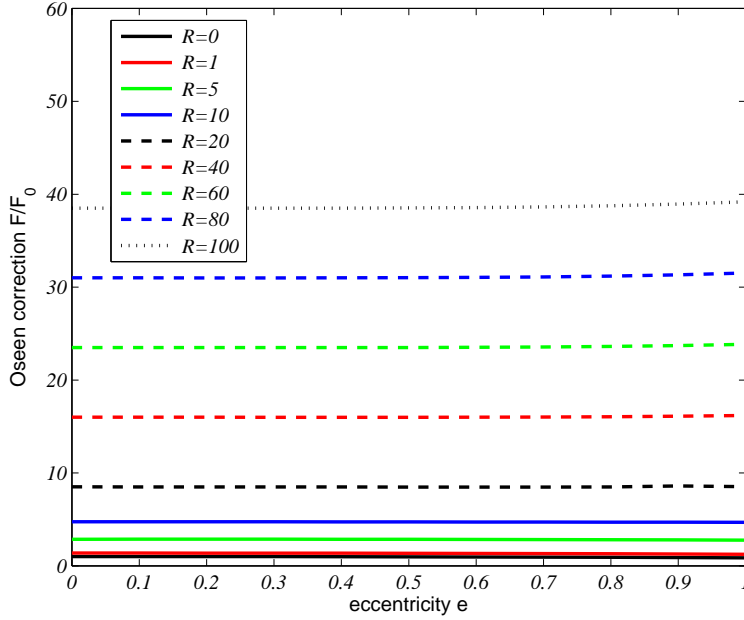


Figure 4: Variation of Oseen correction with respect to eccentricity ‘e’ for various values of Reynolds number  $R = \rho Ub/\mu$  for prolate spheroid

$$F_x = 16\pi\mu Uae^3 \left[ -2e + (1 + e^2) \ln \frac{1+e}{1-e} \right]^{-1}, \quad (4.3a)$$

$$F_y = 32\pi\mu Uae^3 \left[ 2e + (3e^2 - 1) \ln \frac{1+e}{1-e} \right]^{-1}. \quad (4.3b)$$

By using (4.3a,b), from (2.7), the value of real factor ‘K’ comes out to be

$$\begin{aligned} K &= \frac{1}{2} \left[ 2e + (3e^2 - 1) \ln \frac{1+e}{1-e} \right] \left[ -2e + (1 + e^2) \ln \frac{1+e}{1-e} \right]^{-1} \\ &= 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 \dots \end{aligned} \quad (4.4)$$

Now, from Brenner's formula (2.8) and (2.9), the Oseen's correction, with the use of real factor  $K$  (cf. (4.4)) may be written as

$$\begin{aligned}
 \frac{F}{F_y} &= K \frac{F}{F_x} \\
 &= \left[ 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 - \frac{2}{5}e^2 - \frac{17}{175}e^4 \dots \right\} R \right. \\
 &\quad \left. + O(R^2) \right] \\
 &= 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 + \frac{3}{8} \left\{ 1 + \frac{3}{10}e^2 - \frac{118}{175}e^4 \dots \right\} R + O(R^2),
 \end{aligned} \tag{4.5}$$

where  $R = \rho U a / \mu$ , is particle Reynolds number. The same solution may be re-written, when we take particle Reynolds number  $R = \rho U b / \mu$ , by using  $b/a = (1 - e^2)^{1/2}$ , as

$$\begin{aligned}
 \frac{F}{F_y} &= K \frac{F}{F_x} \\
 &= K \left[ 1 + \frac{e^3}{\sqrt{1-e^2} \left[ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right]} R + O(R^2) \right], \\
 &= \left[ 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 \dots \right] \left[ 1 + \frac{e^3}{\sqrt{1-e^2} \left[ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right]} R \right. \\
 &\quad \left. + O(R^2) \right] \\
 &= \left[ 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 + \frac{1}{10}e^2 + \frac{109}{1400}e^4 \dots \right\} R \right. \\
 &\quad \left. + O(R^2) \right] \\
 &= 1 - \frac{1}{10}e^2 - \frac{8}{175}e^4 + \frac{3}{8} \left\{ 1 + \frac{31}{1400}e^4 \right\} R + O(R^2).
 \end{aligned} \tag{4.6}$$

Equations (4.5) and (4.6) immediately reduce to the case of sphere (given in (4.3)) in the limiting case as  $e \rightarrow 0$ . On the other hand, the expressions (4.5) and (4.6) due to Oseen for prolate spheroid appear to be new as no such type of expressions are available in the literature for comparison.

### 4.3 Flow past oblate spheroid

Stokes drag on oblate spheroid having semi-major axis length 'b' and semi-minor axis length 'a' placed in uniform velocity U, parallel to axis of symmetry(axial flow) and perpendicular to its axis of symmetry(transverse flow) is given as [by utilizing DS conjecture 2.4 and 2.5, Datta and Srivastava, 1999]

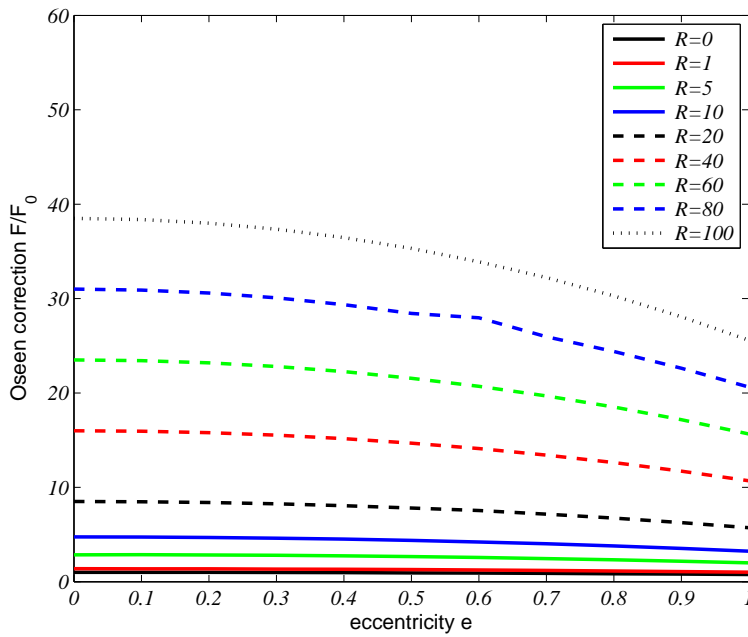


Figure 5: Variation of Oseen correction with respect to eccentricity 'e' for various values of Reynolds number  $R = \rho U a / \mu$  for oblate spheroid

$$F_x = 8\pi\mu U a e^3 \left[ e\sqrt{1-e^2} - (1-2e^2)\sin^{-1}e \right]^{-1}, \quad (4.7a)$$

$$F_y = 16\pi\mu U a e^3 \left[ -e\sqrt{1-e^2} + (1+2e^2)\sin^{-1}e \right]^{-1}. \quad (4.7b)$$

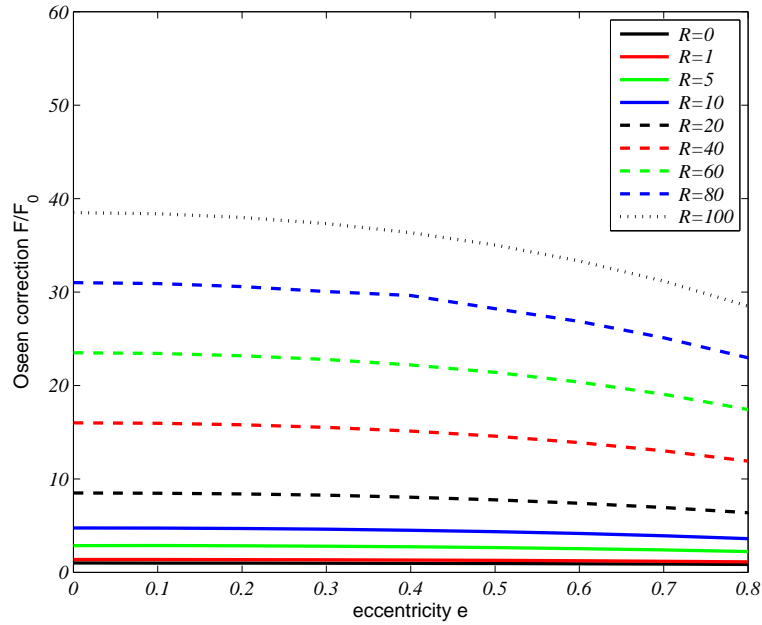


Figure 6: Variation of Oseen correction with respect to eccentricity ‘e’ for various values of Reynolds number  $R = \rho Ub/\mu$  for oblate spheroid

By using (4.7a,b) and (2.7), the value of real factor ‘K’ comes out to be

$$\begin{aligned}
 K &= \frac{F_x}{F_y} \\
 &= \frac{1}{2} \left[ -e\sqrt{1-e^2} + (1+2e^2)\sin^{-1}e \right] \left[ e\sqrt{1-e^2} \right. \\
 &\quad \left. - (1-2e^2)\sin^{-1}e \right]^{-1} = 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 \dots
 \end{aligned} \tag{4.8}$$

Now, from Brenner’s formula (2.8) and (2.9), the Oseen’s correction, with the

use of real factor  $K$ (cf. (4.8)), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &= \left[ 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 - \frac{1}{10}e^2 - \frac{31}{1400}e^4 \dots \right\} R \right. \\ &\quad \left. + O(R^2) \right] \\ &= 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 + \frac{3}{8} \left\{ 1 - \frac{1}{3}e^2 - \frac{53}{11200}e^4 \dots \right\} R + O(R^2), \end{aligned} \tag{4.9}$$

where  $R = \left( \frac{\rho U a}{\mu} \right)$  is particle Reynolds number. The same solution may be re-written, when we take particle Reynolds number  $R = \left( \frac{\rho U b}{\mu} \right)$ , by using  $b/a = (1 - e^2)^{1/2}$ , as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &= K \left[ 1 + \frac{e^3}{2\sqrt{1-e^2} \left[ e\sqrt{1-e^2} - (1-2e^2)\sin^{-1}e \right]} R + O(R^2) \right] \\ &= \left[ 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 \dots \right] \left[ 1 \right. \\ &\quad \left. + \frac{e^3}{2\sqrt{1-e^2} \left[ e\sqrt{1-e^2} - (1-2e^2)\sin^{-1}e \right]} R + O(R^2) \right] \\ &= \left[ 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 \right. \right. \\ &\quad \left. \left. + \frac{2}{5}e^2 + \frac{61}{200}e^4 \dots \right\} R + O(R^2) \right] \\ &= 1 - \frac{7}{30}e^2 - \frac{199}{33600}e^4 + \frac{3}{8} \left\{ 1 - \frac{1}{3}e^2 - \frac{4079}{33600}e^4 \dots \right\} R \\ &\quad + O(R^2). \end{aligned} \tag{4.10}$$

Equations (4.9) and (4.10) immediately reduces to the case of sphere(given in eq. 4.3) in the limiting case as  $e \rightarrow 0$ . On the other hand, the expressions (4.9) and (4.10) due to Oseen for oblate spheroid appears to be new as no such type of expressions are available in the literature for comparison.

#### 4.4 Flow past deformed sphere

We consider the polar equation of deformed sphere as

$$r = a \left[ 1 + \varepsilon \sum_{k=0}^{\infty} d_k P_k(\mu) \right], \quad \mu = \cos\theta \quad (4.11)$$

where ‘ $\varepsilon$ ’ is deformation parameter and  $(r, \theta)$  are polar coordinates. The Stokes drag experienced by this deformed sphere placed in axial and transverse uniform stream is given by use of by D-S formulae 2.4 and 2.5( Datta and Srivastava, 1999) only up to first order of ‘ $\varepsilon$ ’ as

$$F_x = 6\pi\mu Ua \left[ 1 + \varepsilon \left( d_0 - \frac{1}{5}d_2 \right) \right], \quad (4.12a)$$

$$F_y = 6\pi\mu Ua \left[ 1 + \varepsilon \left( d_0 - \frac{1}{10}d_2 \right) \right] \quad (4.12b)$$

By using (4.12a,b) and (2.7), the value of real factor ‘K’ comes out to be

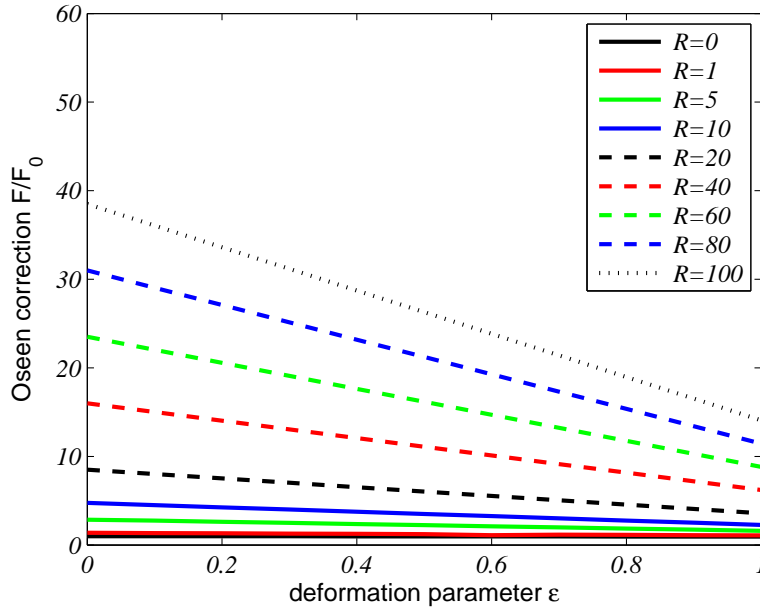


Figure 7: Variation of Oseen correction with respect to deformation parameter ‘ $\varepsilon$ ’ for various values of Reynolds number  $R = \rho Ua/\mu$  for deformed sphere

$$\begin{aligned}
 K &= \frac{F_x}{F_y} \\
 &= \left[ 1 + \varepsilon \left( d_0 - \frac{1}{5}d_2 \right) \right] \left[ 1 + \varepsilon \left( d_0 - \frac{1}{10}d_2 \right) \right]^{-1} \\
 &= \left[ 1 + \varepsilon \left( d_0 - \frac{1}{5}d_2 \right) \right] \left[ 1 - \varepsilon \left( d_0 - \frac{1}{10}d_2 \right) \right] \\
 &= 1 - \frac{1}{10}d_2\varepsilon + O(\varepsilon^2).
 \end{aligned} \tag{4.13}$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K (eq. 4.13), may be written as

$$\begin{aligned}
 \frac{F}{F_y} &= K \frac{F}{F_x} \\
 &= \left[ 1 - \frac{1}{10}d_2\varepsilon + O(\varepsilon^2) \right] \left[ 1 + \frac{3}{8} \left\{ 1 + \varepsilon \left( d_0 - \frac{1}{5}d_2 \right) + O(\varepsilon^2) \right\} R + O(R^2) \right] \\
 &= 1 - \frac{d_2}{10}\varepsilon + \frac{3}{8} \left\{ 1 - \left( d_0 + \frac{3}{10}d_2 \right) \varepsilon \right\} R + O(R^2),
 \end{aligned} \tag{4.14}$$

where  $R = \left( \frac{\rho U a}{\mu} \right)$  is particle Reynolds number. This expression (4.14) immediately reduces to the case of sphere (given in (4.2)) in the limiting case as  $\varepsilon \rightarrow 0$ . This further reduces to 1 for  $R = 0$  (case of Stokes drag in transverse flow).

#### 4.5 Cycloidal body of revolution

**Case 1.** We consider the equation of cycloidal body of revolution as

$$x = a(t + \sin t), y = a(1 + \cos t), \quad -\pi \leq t \leq \pi \tag{4.15}$$

The Stokes drag experienced by this cycloidal body of revolution placed in axial and transverse uniform stream is given by use of DS conjecture 2.4 and 2.5, [Datta and Srivastava, 1999]

$$F_x = \frac{128}{3} \mu U a, \tag{4.16a}$$

$$F_y = \frac{256}{5} \mu U a. \tag{4.16b}$$



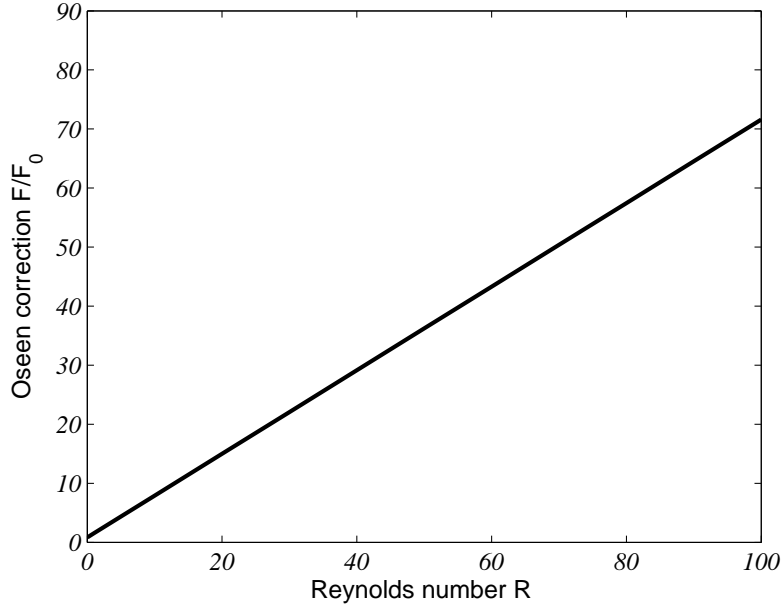


Figure 8: Variation of Oseen correction with respect to Reynolds number  $R = \rho U a \pi / \mu$  for cycloidal body

By using (4.16a,b) and (2.7), the value of real factor 'K' comes out to be

$$K = \frac{F_x}{F_y} = \frac{5}{6} \cong 0.83. \quad (4.17)$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K(eq. 4.18), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &= \frac{5}{6} \left[ 1 + \frac{8}{3\pi} R + O(R^2) \right] = \left[ \frac{5}{6} + \frac{20}{9\pi} R + O(R^2) \right] \\ &\cong 0.8333 + 0.7077 \times R + O(R^2), \end{aligned} \quad (4.18)$$

where  $R = \rho U a / \mu$ , is particle Reynolds number. This expression reduces to 0.8333 as  $R \rightarrow 0$ , the case of Stokes drag i.e.,  $F = 0.8333 F_y$  which should be equal to  $F_y$ . The reason behind this discrepancy is due to the error of

16.67 % which persists in the operation of D-S conjecture on cycloidal body of revolution (4.16).

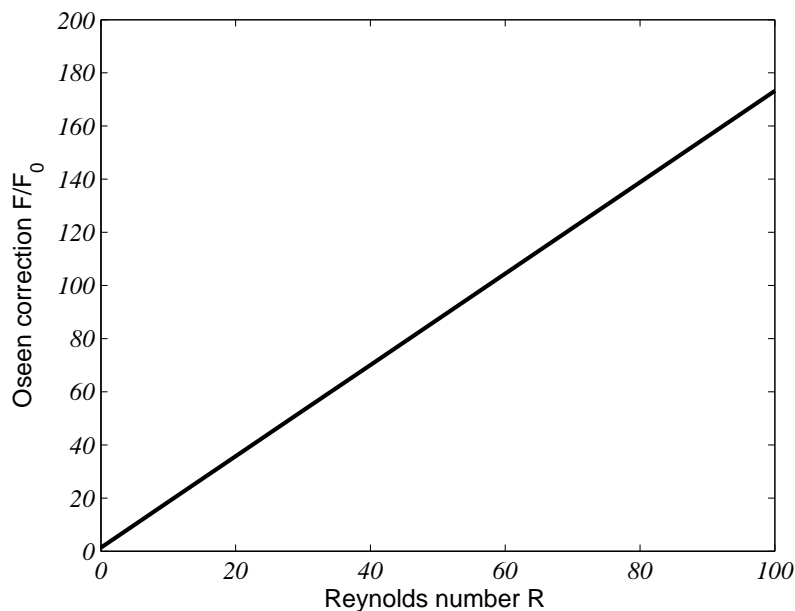


Figure 9: Variation of Oseen correction with respect to Reynolds number  $R = 2a\rho U/\mu$  for cycloidal body

**Case 2.** Consider the body generated by the rotation about x-axis of the curve composed of arcs of two cycloidal parts represented parametrically by

$$x = a(1 + \cos t), y = a(t + \sin t), 0 \leq t \leq \pi \quad (4.19a)$$

$$x = -a(1 + \cos t), y = a(t + \sin t), 0 \leq t \leq \pi. \quad (4.19b)$$

The Stokes drag experienced by this cycloidal body of revolution placed in axial and transverse uniform stream is given by use of by DS conjecture 2.4 and 2.5, [Datta and Srivastava, 1999]

$$F_x = \frac{96\pi^3}{3\pi^2 + 16} \mu U a, \quad (4.20a)$$

$$F_y = \frac{192\pi^3}{9\pi^2 + 32} \mu U a. \quad (4.20b)$$

By using (4.20 a,b) and (2.7), the value of real factor 'K' comes out to be

$$K = \frac{F_x}{F_y} = \frac{1}{2} \left( \frac{9\pi^2 + 32}{3\pi^2 + 16} \right) \cong 1.3244. \quad (4.21)$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K (eq. 4.21), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &= \frac{1}{2} \left( \frac{9\pi^2 + 32}{3\pi^2 + 16} \right) \left[ 1 + \frac{6\pi^2}{(3\pi^2 + 16)} R + O(R^2) \right] \\ &= 1.3244 + 1.718964199 \times R + O(R^2), \end{aligned} \quad (4.22)$$

where  $R = \rho U a / \mu$ , is particle Reynolds number. This expression reduces to 1.3244 as  $R \rightarrow 0$ , the case of Stokes drag i.e.,  $F = 1.3244 F_y$  which should be equal to  $F_y$ . The reason behind this discrepancy is due to the error of 32.44 % which persists in the operation of D-S conjecture on cycloidal body of revolution (4.20a,b).

#### 4.6 Egg-shaped body

We consider the egg-shaped body of revolution in which right portion is in the shape of semi spheroid obtained from revolution of ellipse

$$x = a \cos t, y = b \sin t, 0 \leq t \leq \pi/2 \quad (4.23a)$$

and left portion is in the shape of semi-sphere obtained from revolution of circle

$$x = b \cos t, y = b \sin t, \pi/2 \leq t \leq \pi, \quad (4.23b)$$

about the axis of symmetry(x-axis). The Stokes drag experienced by this axially symmetric egg-shaped body in axial and transverse uniform stream is given by use of DS conjecture 2.4 and 2.5(see Datta and Srivastava, 1999)

$$F_x = 8\pi\mu U a \left[ \frac{2}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1}, \quad (4.24a)$$

$$F_y = 16\pi\mu U a \sqrt{1-e^2} \left[ \frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ 2e + (3e^2-1) \ln \frac{1+e}{1-e} \right\} \right]^{-1}. \quad (4.24b)$$

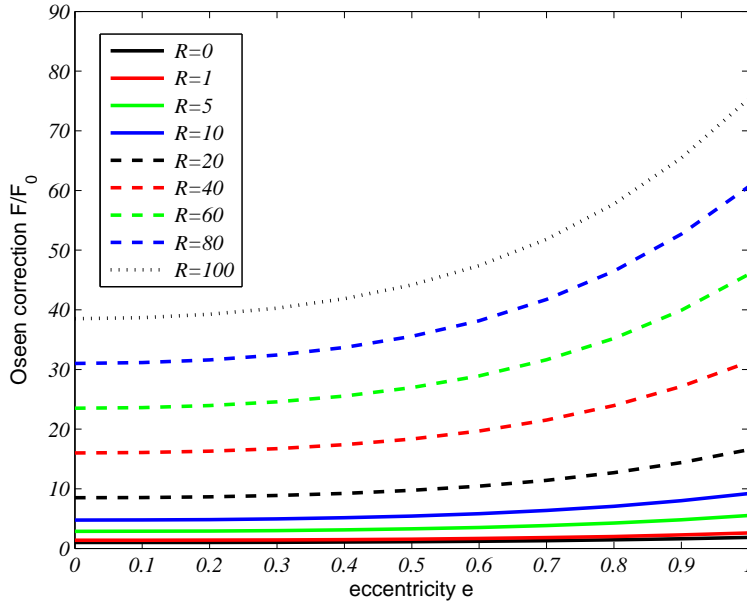


Figure 10: Variation of Oseen correction with respect to eccentricity ‘e’ for various values of Reynolds number  $R = \rho U a / \mu$  for egg-shaped body

By using (4.23a,b) and (2.7), the value of real factor ‘K’ comes out to be

$$\begin{aligned}
 K &= \frac{F_x}{F_y} \\
 &= \frac{1}{2\sqrt{1-e^2}} \left[ \frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ 2e + (3e^2 - 1) \ln \frac{1+e}{1-e} \right\} \right] \left[ \frac{2}{3} \right. \quad (4.25a)
 \end{aligned}$$

$$\begin{aligned}
 &+ \left. \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1} \\
 &= 1 + \frac{5}{12} e^2 + \frac{733}{1680} e^4 + \dots \quad (4.25b)
 \end{aligned}$$

Now, from Brenner’s formula (2.8) for axial flow and revised Brenner’s formula (2.9) for transverse flow, the Oseen’s correction, with the use of real factor

K(eq. 4.25), may be written as

$$\begin{aligned}
\frac{F}{F_y} &= K \frac{F}{F_x} \\
&= \frac{1}{2\sqrt{1-e^2}} \left[ \frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ 2e + (3e^2 - 1) \ln \frac{1+e}{1-e} \right\} \right] \left[ \frac{2}{3} \right. \\
&\quad \left. + \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1} \\
&\quad \times \left( 1 + \frac{1}{2} \left[ \frac{2}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1} R + O(R^2) \right) \\
&= \left[ 1 + \frac{5}{12}e^2 + \frac{733}{1680}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 + \frac{1}{20}e^2 + \frac{51}{1400}e^4 \dots \right\} R + O(R^2) \right] \\
&= \left( 1 + \frac{5}{12}e^2 + \frac{733}{1680}e^4 \dots \right) + \frac{3}{8} \left\{ 1 + \frac{7}{15}e^2 + \frac{14511}{29400}e^4 + \dots \right\} R + O(R^2),
\end{aligned} \tag{4.26}$$

where  $R = \rho U a / \mu$ , is particle Reynolds number. The same solution may be written, when we take particle Reynolds number  $R = \rho U b / \mu$ , by using  $b/a = (1 - e_2)^{1/2}$ , as

$$\begin{aligned}
\frac{F}{F_y} &= K \frac{F}{F_x} \\
&= \frac{1}{2\sqrt{1-e^2}} \left[ \frac{4}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ 2e + (3e^2 - 1) \ln \frac{1+e}{1-e} \right\} \right] \left[ \frac{2}{3} \right. \\
&\quad \left. + \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1} \\
&\quad \times \left( 1 + \frac{1}{2\sqrt{1-e^2}} \left[ \frac{2}{3} + \frac{\sqrt{1-e^2}}{4e^3} \left\{ -2e + (1+e^2) \ln \frac{1+e}{1-e} \right\} \right]^{-1} R + O(R^2) \right) \\
&= \left[ 1 + \frac{5}{12}e^2 + \frac{733}{1680}e^4 \dots \right] \left[ 1 + \frac{3}{8} \left\{ 1 + \frac{11}{20}e^2 + \frac{611}{1400}e^4 \dots \right\} R + O(R^2) \right] \\
&= \left( 1 + \frac{5}{12}e^2 + \frac{733}{1680}e^4 + \dots \right) + \frac{3}{8} \left\{ 1 + \frac{29}{30}e^2 + \frac{26271}{29400}e^4 + \dots \right\} R + O(R^2).
\end{aligned} \tag{4.27}$$

These expressions (4.27) and (4.28) reduce to  $1 + (3/8)R$  as  $e \rightarrow 0$ , the case of

Oseen's correction in transverse flow for sphere having radius 'a' which further reduces to 1 as  $R \rightarrow 0$ , the case of transverse Stokes drag over egg-shaped axially symmetric body.

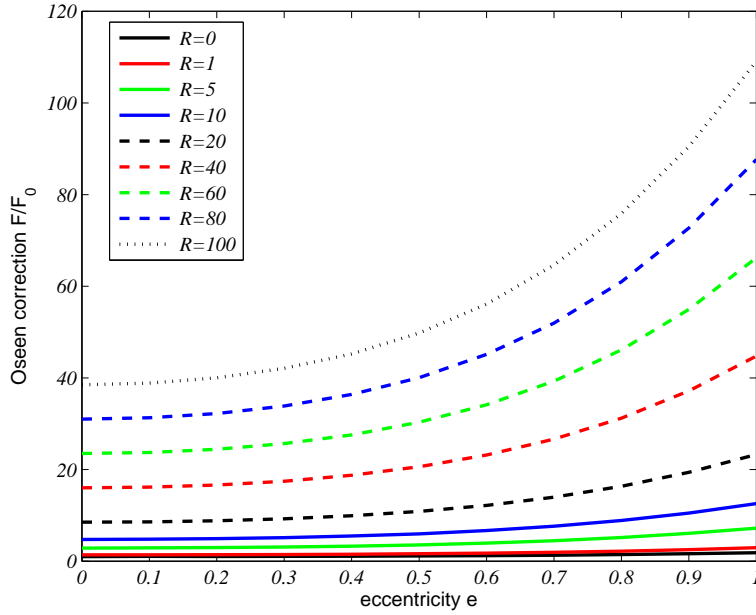


Figure 11: Variation of Oseen correction with respect to eccentricity 'e' for various values of Reynolds number  $R = \rho U b / \mu$  for egg-shaped body

#### 4.7 Cassini body of revolution

We consider the cassini body of revolution obtained by revolving the curve

$$y^2 = \frac{2}{3}(1 + 3x^2)^{1/2} - x^2 - \frac{1}{3}, 0 \leq x \leq 1, \tag{4.28}$$

about x-axis. The Stokes drag experienced by this axially symmetric cassini body of revolution placed in axial and transverse uniform stream is given by DS conjecture 2.4 and 2.5 (see Srivastava, 2001), on taking  $a = 1$ ,  $b = 0.577$ , is

$$F_x \cong 0.8\pi\mu U, \tag{4.29a}$$

$$F_y \cong 0.82\pi\mu U. \tag{4.29b}$$

By using (4.30 a,b) and (2.7), the value of real factor 'K' comes out to be

$$K = \frac{F_x}{F_y} \cong 0.9756098. \quad (4.30)$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K (eq. 4.30), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &\cong 0.9756098 [1 + 0.3R + O(R^2)] \\ &= 0.97561 + 0.2926829 \times R + O(R^2) \\ &\cong 1 + 0.3R + O(R^2), \end{aligned} \quad (4.31)$$

where  $R = \rho U a / \mu$  is particle Reynolds number. This expression reduces to 1 as  $R \rightarrow 0$ , the case of transverse Stokes drag on cassini body of revolution (Srivastava, 2001).

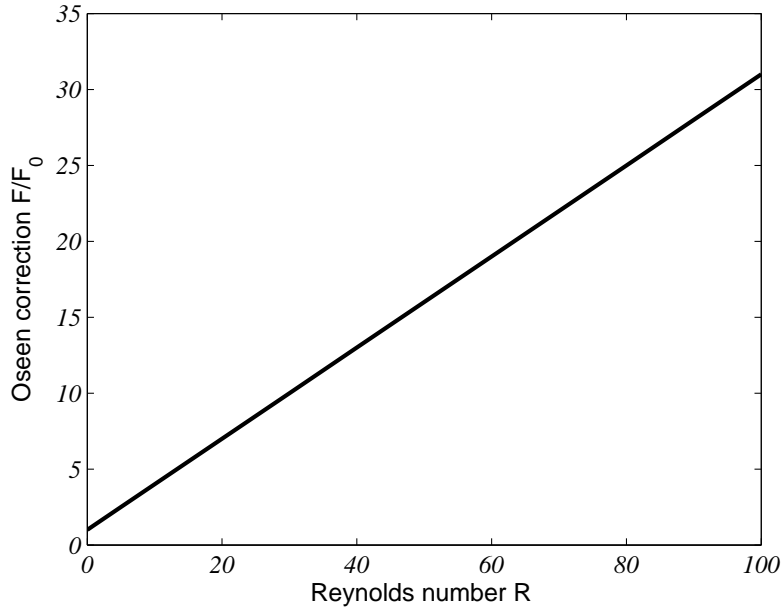


Figure 12: Variation of Oseen correction with respect to Reynolds number  $R = \rho U / \mu$  for cassini body

#### 4.8 Hypocycloidal body of revolution

We consider the hypocycloidal body of revolution obtained by revolving the curve

$$y^2 = -3x^2 + \sqrt{(1 + 8x^4)}, 0 \leq x \leq 1, \quad (4.32)$$

about x-axis. The Stokes drag experienced by this axially symmetric hypocycloidal body of revolution placed in axial and transverse uniform stream is given by DS conjecture 2.4 and 2.5 (see Srivastava, 2001), on taking  $a = 1$ , is

$$F_x \cong 6.264\pi\mu U, \quad (4.33a)$$

$$F_y \cong 7.92\pi\mu U. \quad (4.33b)$$

By using (4.34 a,b) and (2.7), the value of real factor 'K' comes out to be

$$K = \frac{F_x}{F_y} \cong 0.7909091. \quad (4.34)$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K (eq. 4.34), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &\cong 0.8 [1 + 0.3915R + O(R^2)] \\ &= 0.8 + 0.31 \times R + O(R^2), \end{aligned} \quad (4.35)$$

where  $R = \rho U a / \mu$  is particle Reynolds number. This expression reduces to 0.8 as  $R \rightarrow 0$ , which should be 1, the case of transverse Stokes drag on hypocycloidal body of revolution (Srivastava, 2001). The reason behind this discrepancy is due to the error of 20% which persists in the operation of D-S formulae on hypocycloidal body of revolution (4.32).



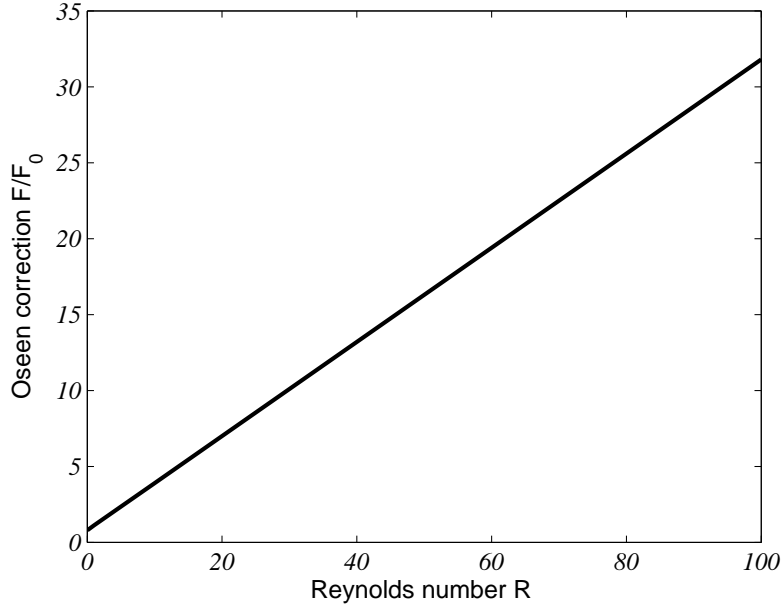


Figure 13: Variation of Oseen correction with respect to Reynolds number  $R = \rho U / \mu$  for hypocycloidal body

#### 4.9 Cylindrical capsule or shell

We consider the cylindrical capsule having semi-spherical caps on both ends having same radius 'b' obtained by revolving the curves( PA, the circular segment, AA' , the line segment, A'P' , again circular segment)

$$PA, x = b \cos t, y = b \sin t, \quad 0 \leq t \leq \pi/2, \quad (4.36a)$$

$$AA', y = b, \theta = \pi/2, \quad (4.36b)$$

$$A'P', x = b \cos t, y = b \sin t, \quad \pi/2 \leq t \leq \pi. \quad (4.36c)$$

The Stokes drag experienced by this axially symmetric cylindrical body of revolution placed in axial and transverse uniform stream is given by DS conjecture 2.4 and 2.5 (see Srivastava, 2001), is

$$F_x = 6\pi\mu Ua, \quad (4.37a)$$

$$F_y = 6\pi\mu Ua. \quad (4.37b)$$

By using (4.38a,b) and (2.7), the value of real factor 'K' comes out to be

$$K = \frac{F_x}{F_y} = 1. \quad (4.38)$$

Now, from Brenner's formula (2.8) for axial flow and revised Brenner's formula (2.9) for transverse flow, the Oseen's correction, with the use of real factor K (eq. 4.34), may be written as

$$\begin{aligned} \frac{F}{F_y} &= K \frac{F}{F_x} \\ &= 1 \left[ 1 + \frac{3}{16}R + O(R^2) \right] \\ &= 1 + 0.1875R + O(R^2), \end{aligned} \quad (4.39)$$

where  $R = \rho U(2a)/\mu$  is particle Reynolds number. This expression reduces to 1 as  $R \rightarrow 0$ , the case of transverse Stokes drag on cylindrical shell of revolution (Srivastava, 2001).

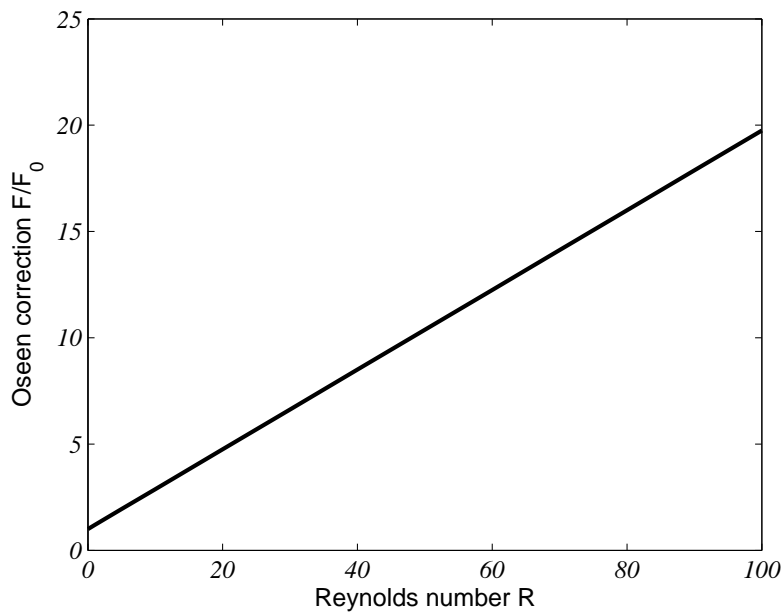


Figure 14: Variation of Oseen correction with respect to Reynolds number  $R = 2a\rho U/\mu$  for cylindrical capsule

## 5 Numerical discussion

The numerical values of Oseen correction for various axi-symmetric bodies are calculated for finite Reynolds number and presented in tables 1 to 13. For

Table 1: Numerical values of  $F/F_y$  with respect to Reynolds number ( $R = \rho U a / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for sphere [calculated from eq.(4.3) and depicted in figure 2]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
1	1.3750	2.8750	4.7500	8.5000	16.000	23.500	31.000	38.500

Table 2: Numerical values of  $F/F_y$  with respect to eccentricity 'e' of prolate spheroid ( $0 \leq e \leq 1$ ) for various values of Reynolds number ( $R = \rho U a / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) [calculated from eq.(4.5) and depicted in figure 3]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	0.9989	1.3751	2.8796	4.7602	8.5214	16.0438	23.5663	31.0887	38.6111
0.2	0.9959	1.3753	2.8931	4.7903	8.5847	16.1735	23.7622	31.3510	38.9398
0.3	0.9906	1.3754	2.9147	4.8388	8.6869	16.3831	24.0794	31.7756	39.4719
0.4	0.9828	1.3748	2.9429	4.9029	8.8231	16.6633	24.5036	32.3438	40.1841
0.5	0.9721	1.3729	2.9757	4.9793	8.9864	17.0007	25.0150	33.0293	41.0436
0.6	0.9581	1.3686	3.0106	5.0631	9.1681	17.3781	25.5881	33.7982	42.0082
0.7	0.9400	1.3609	3.0443	5.1487	9.3573	17.7746	26.1919	34.6091	43.0264
0.8	0.9173	1.3485	3.0733	5.2293	9.5413	18.1653	26.7893	35.4134	44.0374
0.9	0.8890	1.3298	3.0931	5.2972	9.7054	18.5217	27.3381	36.1545	44.9708
1.0	0.8543	1.3032	3.0989	5.3436	9.8329	18.8114	27.7900	36.7686	45.7471

sphere, from table 1, it is clear that non-dimensional drag value  $F/F_y$  increases from 1 to 38.50 with respect to increment in Reynolds number R from 0 (case of Stokes drag) to 100. This variation is depicted by straight line in figure 2. Similar behaviour of  $F/F_y$  persists for cycloidal body (both cases), cassini body, hypocycloidal body and cylindrical capsule whose values are given in the related tables (11,12,13) while their variations are depicted in the figures 12,13,14. For prolate spheroid, in both situations when  $R = \rho U a / \mu$  and  $\rho U b / \mu$ , non-dimensional drag value  $F/F_y$  decreases slowly for low Reynolds number

Table 3: Numerical values of  $F/F_y$  with respect to eccentricity 'e' of prolate spheroid ( $0 \leq e \leq 1$ ) for various values of Reynolds number ( $R = \rho U b / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) [calculated from eq.(4.6) and depicted in figure 4]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	0.9990	1.3740	2.8740	4.7490	8.4990	15.9990	23.4990	30.9991	38.4991
0.2	0.9959	1.3709	2.8709	4.7460	8.4961	15.9965	23.4967	30.9970	38.4973
0.3	0.9906	1.3657	2.8660	4.7413	8.4919	15.9933	23.4947	30.9960	38.4974
0.4	0.9828	1.3580	2.8589	4.7349	8.4870	15.9913	23.4956	30.9998	38.5041
0.5	0.9721	1.3477	2.8497	4.7273	8.4825	15.9929	23.5033	31.0137	38.5240
0.6	0.9581	1.3342	2.8385	4.7188	8.4796	16.0011	23.5226	31.0442	38.5657
0.7	0.9400	1.3170	2.8250	4.7099	8.4799	16.0198	23.5596	31.0995	38.6394
0.8	0.9173	1.2957	2.8093	4.7012	8.4853	16.0533	23.6213	31.1894	38.7574
0.9	0.8890	1.2695	2.7913	4.6935	8.5979	16.1069	23.7159	31.3248	38.9338
1.0	0.8543	1.2376	2.7708	4.6873	8.5203	16.1864	23.8525	31.5186	39.1846

Table 4: Numerical values of  $F/F_y$  with respect to eccentricity ‘e’ of oblate spheroid ( $0 \leq e \leq 1$ ) for various values of Reynolds number ( $R = \rho U a / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100,$ ) [calculated from eq.(4.9) and depicted in figure 5]

e	Oseen correction ‘ $F/F_y$ ’								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	0.9976	1.3714	2.8664	4.7351	8.4726	15.9477	23.4227	30.8977	38.3726
0.2	0.99026	1.3606	2.84064	4.6906	8.3906	15.7905	23.1905	30.5904	37.9904
0.3	0.9789	1.3426	2.7976	4.6163	8.2536	15.5284	22.8031	30.0778	37.3525
0.4	0.9625	1.3174	2.7372	4.5120	8.0616	15.1607	22.2598	29.3589	36.4580
0.5	0.9412	1.2849	2.6594	4.3776	7.8140	14.6869	21.5596	28.4324	35.3052
0.6	0.9152	1.2450	2.5640	4.2129	7.5406	14.1060	20.7014	27.968	33.8922
0.7	0.8842	1.1975	2.4508	4.0174	7.1507	13.4172	19.6837	25.9502	32.2166
0.8	0.8482	1.1425	2.3196	3.7909	6.7337	12.6192	18.5046	24.3901	30.2756
0.9	0.8071	1.0797	2.1700	3.5329	6.2588	11.7105	17.1623	22.6140	28.0657
1.0	0.7607	1.0089	2.0018	3.2430	5.7252	10.6898	15.6543	20.6188	25.5833

( $0 \leq R \leq 1$ ) and increases slowly with respect to eccentricity ‘e’ (from 0 to 1.0) for Reynolds number beyond 5 whose values are given in tables 2-3 and corresponding variations are depicted in figures 3-4. Contrary to this fact, for oblate spheroid, in both situations when  $R = \rho U a / \mu$  and  $\rho U b / \mu$ , non-dimensional drag value  $F/F_y$  decreases slowly with respect to eccentricity ‘e’ (from 0 to 1.0) for various specific values of finite Reynolds number 0 (case of Stokes drag) to 100 whose values are given in tables 4-5 and corresponding variations are depicted in figures 5-6. It is interesting to note that for specific value of eccentricity ‘e’, non-dimensional drag value  $F/F_y$  increases with respect to finite increment in R from 0 (Stokes drag) to 100. For design factors  $d_0 = 0.5$ ,

Table 5: Numerical values of  $F/F_y$  with respect to eccentricity 'e' of oblate spheroid(0 e 1) for various values of Reynolds number( $R = \rho Ub/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100,$ ) [calculated from eq.(4.10) and depicted in figure 6]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	0.9976	1.3714	2.8663	4.7351	8.4725	15.9475	23.4224	30.8973	38.3722
0.2	0.9906	1.3605	2.8402	4.6899	8.3892	15.7877	23.1863	30.5848	37.9834
0.3	0.9789	1.3423	2.7958	4.6127	8.265	15.5142	22.7818	30.0495	37.3171
0.4	0.9625	1.3163	2.7316	4.5008	8.0392	15.1159	22.1926	29.6293	36.3460
0.5	0.9412	1.2822	2.6458	4.3503	7.7593	14.5775	21.3956	28.2137	35.0318
0.6	0.9152	1.2393	2.5357	4.1562	7.3972	13.8792	20.3612	26.8432	33.3252
0.7	0.8842	1.1870	2.3983	3.9124	6.9406	12.997	19.0534	25.1098	31.1662
0.8	0.8482	1.1245	2.2300	3.6117	6.3753	11.9024	17.4294	22.9565	28.4836
0.9	0.8071	1.0510	2.0265	3.2459	58.6847	10.5624	15.4400	20.3176	25.1952
1.0	0.7607	0.9652	1.7831	2.8055	4.8502	8.9397	3.0293	17.1188	21.2083

$d_2 = 0.5$ , from table 7, it is clear that non-dimensional drag value  $F/F_y$  for deformed sphere decreases with respect to deformation parameter ' $\varepsilon$ ' for various values of increment in Reynolds number R from 0 (case of Stokes drag) to 100. This variation is depicted by straight lines in figure 7. For egg-shaped body, in both situations when  $R = \rho Ua/\mu$  and  $\rho Ub/\mu$ , non-dimensional drag value  $F/F_y$  increases with respect to eccentricity 'e'(from 0 to 1.0) for various specific values of finite Reynolds number 0 (case of Stokes drag) to 100 whose values are given in tables 7-8 and corresponding variations are depicted in figures 11-10. It is interesting to note that for specific value of eccentricity 'e', non-dimensional drag value slowly increases with respect to finite increment in R from 0 (case of Stokes drag) to 100.

Table 6: Numerical values of  $F/F_y$  with respect to deformation parameter ' $\varepsilon$ ' of deformed sphere ( $0 \leq \varepsilon \leq 1$ ) for various values of Reynolds number ( $R = \rho U a / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) [calculated from eq.(4.15) for  $d_0=d_2=1$  and depicted in figure7]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	0.9950	1.34563	2.7481	4.5012	8.0075	15.0200	22.0325	29.0450	36.0575
0.2	0.9900	1.31625	2.6212	4.2525	7.5150	14.0400	20.5650	27.0900	33.6150
0.3	0.9850	1.2868	2.4943	4.0037	7.0225	13.0600	19.0975	25.1350	31.1725
0.4	0.9800	1.2575	2.3675	3.7550	6.5300	12.0800	17.6300	23.1800	28.7300
0.5	0.9750	1.2281	2.2406	3.5062	6.0375	11.1000	16.1625	21.2250	26.2875
0.6	0.9700	1.0987	2.1137	3.2575	5.5450	10.1200	14.6950	19.2700	23.8450
0.7	0.9650	1.1693	1.9868	3.0087	5.0525	9.1400	13.2275	17.3150	21.4025
0.8	0.9600	1.1400	1.8600	2.7600	4.5600	8.1600	11.7600	15.3600	18.9600
0.9	0.9550	1.1106	1.7331	2.5112	4.0675	7.1800	10.2925	13.4050	16.5175
1.0	0.9500	1.0812	1.6062	2.2625	3.5750	6.2000	8.8250	11.4500	14.0750

## 6 Conclusion

In the present problem, we have extended the Brenner's formulae (Brenner, 1961), which were valid only for axial flow to transverse flow. This general expression is used to correct the Stokes drag, called Oseen's correction or Oseen's drag, up to the first order of Reynolds number 'R'. This proposed solution is the solution of Oseen's equation. The numerical values of non-dimensional drag  $F/F_y$  with respect to various parameters like eccentricity 'e', deformation parameter ' $\varepsilon$ ' and Reynolds number 'R' related to axially symmetric bodies are calculated and presented in tables 1-13. The respective variations between these quantities are shown in figures 2-14. It has been observed that for low

Table 7: Numerical values of  $F/F_y$  with respect to eccentricity 'e' of egg-shaped body ( $0 \leq e \leq 1$ ) for various values of Reynolds number ( $R = \rho U a / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) [calculated from eq.(4.26) and depicted in figure 10]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	1.0042	1.3809	2.8880	4.7719	8.5395	16.0750	23.6103	31.1457	38.6811
0.2	1.01736	1.3996	2.9288	4.8403	8.6632	16.3092	23.9551	31.6011	39.2470
0.3	1.04103	1.4332	3.0022	4.9635	8.8860	16.7310	24.5760	32.4210	40.2660
0.4	1.0778	1.4855	3.1165	5.1552	9.2326	17.3874	25.5421	33.6969	41.8517
0.5	1.1314	1.5617	3.2830	5.4346	9.7378	18.3442	26.9505	35.5569	44.1632
0.6	1.2065	1.6685	3.5164	5.8264	10.4463	19.6860	28.9258	38.1656	47.4053
0.7	1.3089	1.8141	3.8348	6.3608	11.4127	21.5165	31.6203	41.7241	51.8279
0.8	1.4453	2.0082	4.2594	7.0735	12.7016	23.9579	35.2141	46.4704	57.7266
0.9	1.6237	2.2619	4.8147	8.0056	14.3875	27.1512	39.9150	52.6787	65.4425
1.0	1.8529	2.5880	5.5284	9.2038	16.5548	31.2565	45.9583	60.6601	75.3619

particle Reynolds number (between 0 and 1), non-dimensional drag value decreases and increases for other values of Reynolds number. The increment in drag value is related to the increment in Reynolds number. The proposed analysis may provide a strong platform to study the optimal profiles in Oseen's flow which may appear in author's future work. Other important applications of Oseen's correction are in calculation of the settling of solid particles of powdered materials with enhanced accuracy in carrying out sedimentation analysis and in acoustic levitation. These two works are the main practical applications of Oseen's correction which was not possible with the Stokes drag.



Table 8: Numerical values of  $F/F_y$  with respect to eccentricity 'e' of egg-shaped body ( $0 \leq e \leq 1$ ) for various values of Reynolds number ( $R = \rho Ub/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) [calculated from eq.(4.27) and depicted in figure 11]

e	Oseen correction ' $F/F_y$ '								
	R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.0	1.0	1.3750	2.8570	4.7500	8.5000	16.0000	23.5000	31.0000	38.5000
0.1	1.0420	1.3828	2.8975	4.7908	8.5773	16.1506	23.7237	31.2969	38.8701
0.2	1.0173	1.4074	2.9675	4.9177	8.8180	16.6188	24.4195	32.2203	40.0210
0.3	1.0410	1.4513	3.0927	5.1444	9.2478	17.4546	25.6614	33.8682	42.075
0.4	1.0778	1.5194	3.2857	5.4936	9.9094	18.7410	27.56725	36.4041	45.2357
0.5	1.1314	1.6180	3.5642	5.971	10.8628	20.5942	30.3255	40.0569	49.7882
0.6	1.2065	1.7554	3.9511	6.6958	12.1851	23.1636	34.1422	45.1208	56.0993
0.7	1.3089	1.9420	4.4743	7.6397	13.9705	26.6321	39.2937	51.9553	64.6169
0.8	1.4453	2.1896	5.1666	8.8879	16.3304	31.2155	46.1005	60.9856	75.8706
0.9	1.6237	2.5122	6.0661	10.5085	19.3933	37.1628	54.9324	72.7019	90.4715
1.0	1.8529	2.9255	7.2159	12.5789	23.3048	44.7565	66.2083	87.6601	109.112

Table 9: Numerical values of  $F/F_y$  with respect to various values of Reynolds number ( $R = \rho U a \pi / \mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for cycloidal body (case 1) [calculated from eq.(4.19) and depicted in figure 8]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.8333	1.5410	4.3718	7.9103	14.9873	29.1413	43.2953	57.4493	71.6033

Table 10: Numerical values of  $F/F_y$  with respect to various values of Reynolds number ( $R = 2\rho Ua/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for cycloidal body (case 2) [calculated from eq.(4.23) and depicted in figure 9]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
1.3244	3.04336	9.9192	18.514	35.7037	70.083	104.462	138.842	173.221

Table 11: Numerical values of  $F/F_y$  with respect to various values of Reynolds number ( $R = \rho U/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for cassini body of revolution [calculated from eq.(4.31) and depicted in figure 12]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
1.0	1.3	2.5	4.0	7.0	13.0	19.0	25.0	31.0

Table 12: Numerical values of  $F/F_y$  with respect to various values of Reynolds number ( $R = \rho U/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for hypocycloidal body of revolution [calculated from eq.(4.35) and depicted in figure 13]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
0.80	1.11	2.35	3.90	7.00	13.20	19.40	25.60	31.80

Table 13: Numerical values of  $F/F_y$  with respect to various values of Reynolds number ( $R = 2\rho Ua/\mu = 0, 1, 5, 10, 20, 40, 60, 80, 100$ ) for cylindrical capsule [calculated from eq.(4.39) and depicted in figure 14]

Oseen correction ' $F/F_y$ '								
R=0	R=1.0	R=5.0	R=10.0	R=20	R=40	R=60	R=80	R=100
1.0	1.1875	1.9375	2.875	4.75	8.50	12.25	16.0	19.75

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### Oseen-ova ispravka Stokes-ovog otpora na osno simetričnoj proizvoljnoj čestici u poprečnom toku: novi pristup

Dobijena je Oseen-ova korekcija Stokes-ovog otpora uslovljenog aksijalno simetričnom česticom postavljenom u uniformnu struju upravnu na osu simetrije (znazči poprečno tečenje). U tom cilju je iskorišćen linearni odnos između uzdužnog i poprečnog Stokes-og otpora za proširenje Brenner-ove formule sa uzdužnog na poprečno tečenje. Za opšti izraz Oseen-ova korekcija Stokes-ovog otpora na uzdužno simetričnu česticu postavljenu u poprečno tečenje nadjeno je da je nova. Ovaj opšti izraz primenjuje se na neka poznata uzdužno simetrična tela pa su dobijene vrednosti Oseen-ovog otpora, zaključno sa prvim članovima reda po Reynolds-ovom broju, takodje nove i ne postoje u literaturi. Numeričke vrednosti Oseen-ovog otpora su takodje određene i njihove promene u odnosu na Reynolds-ov broj, ekscentricitet i deformacioni parametar su prikazane i uporedjene sa nekim poznatim vrednostima. Neke važne primene su takodje istaknute.