Theoret.Appl.Mech. TEOPM7, Vol.41, No.3, pp. 233–245, Belgrade 2014 *

A LOW ORDER VISCOPLASTICITY OF TRANSVERSELY ISOTROPIC QUASI-RATE INDEPENDENT MATERIALS

Milan Mićunović **Ljudmila Kudrjavceva**

*doi:10.2298/TAM1403233M Math.Subj.Class.: 74-05,74C20,74C99,74D10,74E10,74L10.

According to: *Tib Journal Abbreviations (C) Mathematical Reviews*, the abbreviation TEOPM7 stands for TEORIJSKA I PRIMENJENA MEHANIKA.

Theoret. Appl. Mech., Vol.41, No.3, pp. 233–245, Belgrade 2014

A low order viscoplasticity of transversely isotropic quasi-rate independent materials

M. Mićunović * **Lj. Kudrjavceva** †

Abstract

As found by experiments *quasi rate independent materials* (*QRI*) describe very well behavior of steels in very wide range of strains and strain rates ([3],[4]). This property has been combined with tensor representation modeling using a generalized associative flow rule based not on the yield function but on a more general loading function. Seemingly rate independent QRI producing incremental evolution equations show rate sensitivity by means of variability of yield stress with stress rate. On the other hand transverse isotropy appears in metal forming issues like in rolled car body sheets [18]. Here an extension of tensor generators and invariants is needed to include the preferred anisotropy direction. Such a procedure has been made here. In addition we believe that the results of this paper are applicable to dynamic deformation of orthogneiss rocks treated recently in [5].

Keywords: Viscoplasticity, QRI materials, reactor stainless steels, orthogneiss rock

1 Introduction and preliminaries

Theoretical consideration of viscoplasticity has become an important issue for finite element codes which pretend to perform calculations of complex structures with a high precision. In a majority of them (like ABAQUS, ADINA, MARC, etc.) evolution equation for plastic strain rate is of associate type, i.e., it is perpendicular to yield surface in stress space. As another essential

*Mašinski fakultet, The University of Kragujevac, ul. Sestre Janjića 6, 34000 Kragujevac, Serbia, e-mail: mmicun@kg.ac.rs

†The State University of Novi Pazar, ul. Vuka Karadziá bb., 36000 Novi Pazar, Serbia e-mail: ljuta1@gmail.com

simplification yield function is firmly connected to the second invariant of the deviatoric stress, so called J_2 invariant. This leads to coaxiality of plastic stretching and stress deviator tensors. Such a surface most commonly is based on the above–mentioned unique flow curve. It should be noted that usually yield function is detected from tension tests and then applied to calculation during arbitrary stress–strain histories appearing in real structures. This procedure could produce significant errors destroying geometrical accuracy which FEM codes offer.

The best check for a theory is to compare it to experiments. We have at the disposal experiments performed in Dynamic testing laboratory of JRC-Ispra, Italy with specimen made of austenitic stainless steel AISI 316 in the range of small, medium and high strain rates from 0.001 s^{-1} to 1000 s^{-1}. The testing has been done mainly at room temperature. Calibration of the theory shortly sketched below is explained in detail in [4].

Let us devote few words about the notions used in this paper. For finite elasto-visco-plastic strains it is commonly accepted for a considered body that aside from undeformed configuration (χ_0) and deformed current configuration (χ_t) there exists a local reference configuration of natural state elements (ν_t)[4]. Then, Kröner's decomposition rule [6, 7] holds in the following form:

$$\mathbf{F}_P := \mathbf{F}_E^{-1}\mathbf{F}, \tag{1.1}$$

where $\mathbf{F}$ is the deformation gradient tensor, $\mathbf{F}_E$ the elastic distortion tensor and $\mathbf{F}_P$ the plastic distortion tensor, determined by the mappings $(\chi_0) \to (\chi_t)$, $\chi_n \to (\chi_t)$ and $(\chi_0) \to (\nu_t)$, respectively.

In the next section for brief review of constitutive models we will need the plastic stretching tensor being equal to

$$\mathbf{D}_P = \mathrm{sym}(\mathbf{L}_P) \equiv \frac{1}{2}(\mathbf{L}_P + \mathbf{L}_P^T)$$

determined by the symmetric part of plastic "velocity gradient" tensor

$$\mathbf{L}_P = (D_t\mathbf{F}_P)\,\mathbf{F}_P^{-1}.$$

Here aside of the usual notation where the superimposed dot stands for material time derivative, we have applied $D_t\mathcal{A} \equiv \dot{\mathcal{A}}$ for arbitrary $\mathcal{A}$.

Concerning state variables we will use plastic strain and stress. As an invariant measure of plastic strain the Hill's logarithmic tensor

$$\boldsymbol{\varepsilon}_P = \ln \mathbf{V}_P = \ln\left(\mathbf{F}_P\mathbf{F}_P^T\right)\ /2 \tag{1.2}$$

is chosen here. Its principal advantage lies in the fact that it is a deviatoric tensor. In other words, its three principal invariants read

$$\pi_1 = \mathrm{tr}\boldsymbol{\varepsilon}_P = 0, \qquad \pi_2 = \mathrm{tr}\boldsymbol{\varepsilon}_P^2 \neq 0, \qquad \pi_3 = \mathrm{tr}\boldsymbol{\varepsilon}_P^3 \neq 0, \tag{1.3}$$

if plastic volume change is neglected. In the above definition, the polar decomposition theorem for the plastic distortion tensor has been applied by means of

$$\mathbf{F}_P = \mathbf{R}_P\mathbf{U}_P = \mathbf{V}_P\mathbf{R}_P,$$

where $\mathbf{R}_P^{-1} = \mathbf{R}_P^T$ holds for the plastic rotation tensor.

Let the second Piola-Kirchhoff stress, related to (ν_t)-configuration, be denoted by $\mathbf{S}$. It is connected with the Cauchy ("true") stress via the expression

$$\mathbf{S} = \mathrm{det}\mathbf{F}_E\mathbf{F}_E^{-T}\mathbf{T}\mathbf{F}_E^{-1}.$$

By means of its deviatoric part $\mathrm{dev}(\mathbf{S}) \equiv \mathbf{S}_d$ and Hill's logarithmic plastic strain the following set of invariants will be used throughout this paper [1]:

$$\gamma := \{s_1, s_2, s_3, \pi_2, \pi_3, \mu_1, \mu_2, \mu_3, \mu_4\}, \tag{1.4}$$

where

$$s_1 = \mathrm{tr}\ \mathbf{S}, \quad s_2 = \mathrm{tr}\mathbf{S}_d^2, \quad s_3 = \mathrm{tr}\mathbf{S}_d^3, \quad \mu_1 = \mathrm{tr}\{\mathbf{S}_d\boldsymbol{\varepsilon}_P\},$$
$$\mu_2 = \mathrm{tr}\{\mathbf{S}_d\boldsymbol{\varepsilon}_P^2\}, \quad \mu_3 = \mathrm{tr}\{\mathbf{S}_d^2\boldsymbol{\varepsilon}_P\}, \quad \mu_4 = \mathrm{tr}\{\mathbf{S}_d^2\boldsymbol{\varepsilon}_P^2\}.$$

Finally, let us mention that in papers oriented to experiments it is customary to use the following notations:

$$\sigma_{eq} \equiv \left(\frac{3}{2}s_2\right)^{1/2}, \quad D_t\varepsilon_{eq}^p \equiv \left(\frac{2}{3}\mathrm{tr}\{\mathbf{D}_P^2\}\right)^{1/2}, \tag{1.5}$$

and names *equivalent stress* and *equivalent plastic strain rate*, respectively. They are used throughout this paper.

2 QRI viscoplasticity

2.1 Quasi rate independence

It is known fact that initial yield stress under dynamic loading depends on strain rate or stress rate: at higher stress rates the initial stress yield is

[1] The choice of the deviatoric stress is motivated by the traditional approach in the existing plasticity papers. Of course, everything derived in this chapter holds if instead of $\mathrm{dev}\mathbf{S} \equiv \mathbf{S}_d$ we use the invariants formed by means of $\mathbf{S}$.

larger. On the other hand, the phenomenon of delayed yielding inherent to some metals and alloys is observed [4]: stress under dynamic loading exceeds its static value and plasticity starts after a certain time called delay time. Let plastic deformation commence at time t^*. Denote by Y the initial equivalent dynamic yield stress, i.e. $Y = Y\big(D_t\sigma_{eq}(t^*)\big)$. Its static counterpart, the initial equivalent static yield stress, is denoted here by $Y_0 \equiv (0)$.[2]

Then, the accumulated plastic strain is governed by corresponding constitutive equation having the following form [8]:

$$\varepsilon^p_{eq}(t) = \int_0^t J(t-\tau)\, D_t\sigma_{eq}(\tau)\, d\tau \quad \text{and} \quad D_t\varepsilon^p_{eq}(t) = J(0)\, D_t\sigma_{eq}(t), \qquad (2.1)$$

where $J(t-\tau) = \{0 \text{ if } \tau < t^*, \text{ and } \exp(-\mathcal{M}) \text{ if } \tau \geqslant t^*\}$. Here $\mathcal{M}$ is an "universal" constant introduced and determined in [8]. For the material considered it covers precisely diverse multiaxial strain histories at strain rates within the extremely wide range from $10^{-3}[s^{-1}]$ to $10[s^{-1}]$. This holds for reactor steels with large percentage of Nickel and Chromium as well as for ferritic steels. A calibration of experiments dealt with orthogneiss rocks as reported in [5] is in progress. It is expected that a similar behavior will be met. The evolution equations which originate from (2.1b) used here are referred as MAM model [8]. Below, we use a simplified version of this model. The evolution equation for the plastic stretching (plastic "strain rate") is shortly described by the following tensor representation:

$$\mathbf{D}_{\mathrm{P}} = \Lambda \sum \Gamma_\alpha\, \mathbf{H}_\alpha, \qquad (2.2)$$

For the time being, scalars $\Gamma_1, \Gamma_2, \ldots$ and tensors $\mathbf{H}_1, \mathbf{H}_2, \ldots$ are not specified but the experimental evidence acquired at the Dynamic testing laboratory of JRC, Ispra (cf. [8]) strongly suggests that the scalar coefficient Λ takes the form:

$$\Lambda = \eta\,(\sigma_{eq} - Y)\big(\frac{\sigma_{eq}}{Y_0} - 1\big)^{\lambda}\, D_t\sigma_{eq}\, \exp(-\mathcal{M}). \qquad (2.3)$$

Inserting of (2.3) into (2.2) leads to an evolution equation seemingly characteristic for rate independent materials. However, the rate dependence appears in stress rate dependent value of the initial yield stress Y, which has a triggering role for inelasticity onset. A model based on (2.3) could be named as "quasi rate independent". It is worth noting that the obtained evolution equations are endochronic permitting scaling of plastic strain rate, replacing

[2] In fact, instead of zero stress rate here a very small equivalent stress rate is taken - a minimal value of stress rate achievable at available experiment.

time as independent variable by the von Mises equivalent stress. This is useful for calibration in very wide strain rate range from low to almost impact strain rates.

Note about the universal constant: Regardless of the form of a constitutive model for the stainless steel AISI 316H, the exponent $\lambda = 0.554$ whereas the "universal" material constant for this material was found to have the value: $\mathcal{M} = 7.26$

2.2 MAM model for isotropic materials

According to ([9]) the increment of plastic strain tensor is perpendicular to a loading surface Ω = const where Ω depends on stress, temperature and *Pattern of Internal Rearrangement* (PIR). Translating this statement into the language of the previous section an evolution equation for plastic stretching should hold in the following form ([9]):

$$\mathbf{D}_P = \partial_{\mathbf{S}}\Omega(\mathbf{S}, T, PIR). \tag{2.4}$$

Here PIR is described by anholonomic internal variables representing crystal slips over active slip systems. The related geometric issues are reported in [4] in detail.

The plastic deformation "gradient" (i.e. distortion) tensor is incompatible, represents also slips and may reflect transformation of anholonomic coordinates. Thus, taking into account that plastic rotation tensor may be either fixed or taken to be unity, it was assumed in [8] that in the above equation PIR may be represented by the plastic strain tensor. Moreover, we extend the above evolution equation inserting in it a scalar function Λ which must account for the linear connection between rates of Mises equivalent stress and equivalent plastic strain rate. The structure of Λ is shown in (2.3). Therefore,

$$\mathbf{D}_P(D_t\mathbf{S}, \mathbf{S}, \boldsymbol{\varepsilon}_P, T) \;=\; \Lambda\, \partial_{\mathbf{S}}\Omega(\mathbf{S}, \boldsymbol{\varepsilon}_P, T), \tag{2.5}$$

where Rice's loading function depends on temperature and the above given invariants, i.e.,

$$\Omega \;=\; \Omega(\gamma, T) \;\equiv\; \Omega(s_1, s_2, s_3, \pi_2, \pi_3, \mu_1, \mu_2, \mu_3, \mu_4, T).$$

Suppose that damage is negligible until localization onset and that this function is approximated by a fourth order polynomial with respect to $\mathbf{S}$ and first order in $\boldsymbol{\varepsilon}_P$. With such an approximation we would have [4]:

$$\begin{aligned} 2\,\Omega \;&= a_1 s_2 + (a_2 + a_4\mu_1)(s_1 s_2 - s_3) \\ &\quad + \frac{1}{2}a_3 s_2^2 + \frac{1}{3}a_5(3\mu_3 s_2 - 2\mu_1 s_3). \end{aligned} \tag{2.6}$$

Therefore, applying tensor representation to the evolution equation for plastic stretching (2.2) we have the following four tensor generators:

$$\begin{aligned} Y_0\,\mathbf{H}_1 &= \mathbf{S} - \frac{1}{3}\mathbf{1}\,\mathrm{tr}\mathbf{S} \equiv \mathrm{dev}\mathbf{S} \equiv \mathbf{S}_d, \\ Y_0^2\,\mathbf{H}_2 &= \mathrm{dev}(\mathbf{S}_d^2), \\ \mathbf{H}_3 &= \boldsymbol{\varepsilon}_P, \\ Y_0\mathbf{H}_4 &= \mathrm{dev}(\mathbf{S}_d\boldsymbol{\varepsilon}_P + \boldsymbol{\varepsilon}_P\mathbf{S}_d) \end{aligned} \tag{2.7}$$

whereas corresponding scalar coefficients depend on the listed invariants in the following manner:

$$\begin{aligned} \Gamma_1 &= a_1 + a_2 s_1 + a_3 s_2 + a_4\mu_1 + a_5\mu_3, \\ \Gamma_2 &= -\frac{3}{2}(a_2 + a_4\mu_1) - 2a_5\mu_1, \\ \Gamma_3 &= \frac{1}{2}a_4(s_1 s_2 - s_3) - \frac{2}{3}a_5 s_3, \\ \Gamma_4 &= a_5 s_2. \end{aligned} \tag{2.8}$$

The coefficient Λ has the form (2.3) given above.

Calibration of the MAM-model was done in [4] for a AISI 316H stainless steel. The material constants of this model were found to be: $\mathcal{A} = \{a_1, a_2, a_3, a_4, a_5\} = \{0.925,\ -0.065,\ -0.039,\ 0.017,\ -0.134\}$ with $\lambda = 0.554$ and correlation constant $\eta = 0.985$.

Two special cases of the loading function leading to reduced forms of the evolution equation (2.2) have remarkable simplicity.

- If $a_4 = 0$ and $a_5 = 0$, then the plastic stretching is of third–order power of stress. The loading function becomes

$$2\,\Omega \;=\; a_1 s_2 + a_2(s_1 s_2 - s_3) + \frac{1}{2}a_3 s_2^2. \tag{2.9}$$

- For the second–order stress–dependent plastic stretching the loading function is specified with only two material constants

$$2\,\Omega \;=\; a_1 s_2 + a_2(s_1 s_2 - s_3). \tag{2.10}$$

2.3 Transversely isotropic materials

When the material body possesses a single preferred anisotropy direction, say $\vec{N}$, then the arguments of the evolution equation (2.2) have to be extended to include the dyadic $\mathbf{N} \equiv \vec{N} \otimes \vec{N}$. If $\vec{N}$ is unit vector then $\vec{N} \cdot \vec{N} = tr\mathbf{N} = 1$. Therefore,

$$\mathbf{D}_{\mathrm{P}} = \Lambda\, \partial_{\mathbf{S}} \Omega\left(\mathbf{S}, \boldsymbol{\varepsilon}_{\mathbf{p}}, \mathbf{N}, T\right), \qquad \text{where} \qquad \Omega = \Omega\left(\mathbf{S}, \boldsymbol{\varepsilon}_{\mathbf{p}}, \mathbf{N}, T\right). \tag{2.11}$$

Accordingly the set of invariants to be used as the source of tensor generators reads:

$$\begin{aligned}
s_1 &= tr\mathbf{S}, \quad s_2 = tr\mathbf{S}_d^2, \quad s_3 = tr\mathbf{S}_d^3, \\
\pi_1 &= \mathrm{tr}\boldsymbol{\varepsilon}_{\mathbf{p}} = 0, \quad \pi_2 = \mathrm{tr}{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2, \quad \pi_3 = \mathrm{tr}\boldsymbol{\varepsilon}_P^3, \quad \pi_4 = \mathrm{tr}\mathbf{N}\boldsymbol{\varepsilon}_{\mathbf{p}}, \quad \pi_5 = \mathrm{tr}\mathbf{N}{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2, \\
\mu_1 &= tr\mathbf{S}_d\boldsymbol{\varepsilon}_{\mathbf{p}}, \quad \mu_2 = tr\mathbf{S}_d^2\boldsymbol{\varepsilon}_{\mathbf{p}}, \quad \mu_3 = tr\mathbf{S}_d{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2, \quad \mu_4 = tr\mathbf{S}_d^2{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2, \\
\varkappa_1 &= tr\mathbf{N}\mathbf{S}_d, \quad \varkappa_2 = tr\mathbf{N}\mathbf{S}_d^2, \\
\lambda_1 &= tr\mathbf{N}\mathbf{S}_d\boldsymbol{\varepsilon}_{\mathbf{p}}, \lambda_2 = tr\mathbf{N}\mathbf{S}_d^2\boldsymbol{\varepsilon}_{\mathbf{p}}, \lambda_3 = tr\mathbf{N}\mathbf{S}_d{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2, \ \lambda_4 = tr\mathbf{N}\mathbf{S}_d^2{\boldsymbol{\varepsilon}_{\mathbf{p}}}^2.
\end{aligned}$$

Suppose now that Ω is a polynominal of third order in $\mathbf{S}$ and linear in $\boldsymbol{\varepsilon}$. Then the loading function has the following form:

$$\begin{aligned}
2\,\Omega \;=\; & a_0 s_1^2 + a_1 s_2 + a_2 s_3 + a_3 s_1 s_2 + a_4 s_1^3 + a_5 s_1\mu_1 + \\
& a_6 s_1\mu_2 + a_7 s_2\mu_1 + a_8\mu_2 + \\
& b_1\varkappa_1^2 + b_2\varkappa_2 + b_3\varkappa_1^3 + b_4\varkappa_1\varkappa_2 + b_5\varkappa_1 s_1 + \\
& b_6\varkappa_1 s_2 + b_7\varkappa_2 s_1 + b_8\mu_1\varkappa_1 + b_9\varkappa_1\mu_2 + \\
& b_{10}\varkappa_1 s_1\mu_1 + b_{11}\varkappa_2 s_1 + b_{12}\varkappa_2\mu_1 + \\
& c_1\lambda_1 s_1 + c_2\lambda_1 s_1^2 + c_3\lambda_1 s_1 + c_4\lambda_1\varkappa_1 + c_5\lambda_1\varkappa_1^2 + \\
& c_6\lambda_1\varkappa_2 + c_7\lambda_2\varkappa_1.
\end{aligned} \tag{2.12}$$

For convenience, we introduce notations:

$$\begin{aligned}
&\bar{a}_1 \equiv a_1, \quad \bar{a}_2 \equiv a_2, \quad \bar{a}_3 \equiv a_5, \quad \bar{a}_4 \equiv a_7, \\
&\bar{b}_1 \equiv b_1, \quad \bar{b}_2 \equiv b_2 \quad \bar{b}_3 \equiv b_6, \\
&\bar{b}_4 \equiv b_8, \quad \bar{b}_5 \equiv b_9, \quad \bar{b}_6 \equiv b_7, \\
&\bar{c}_1 \equiv c_1 \quad \bar{c}_2 \equiv c_2 \quad \bar{c}_3 \equiv c_5.
\end{aligned}$$

If damage is neglected, then $tr\mathbf{D}_{\mathrm{P}} = 0$ holds. On the other hand, the plastic stretching vanishes when stress is absent i.e. $\mathbf{D}_{\mathrm{P}/\mathbf{S}=0}$. With these two restrictions number of relevant material constants is much smaller so that the loading function (2.12) is reduced into

$$\begin{aligned} 2\,\Omega \;=\;& \bar{a}_1 s_2 + \bar{a}_2\,(s_3 - s_1 s_2) + \bar{a}_3\left(s_1\mu_1 - \frac{3}{2}\mu_2\right) + \bar{a}_4\mu_1 s_2 + \\ & \bar{b}_1\left(\frac{1}{9}s_1^2 + \varkappa_1^2 - \frac{2}{3}s_1\varkappa_1\right) + \bar{b}_2\left(\frac{1}{9}s_1^2 + \varkappa_2 - \frac{2}{3}s_1\varkappa_1\right) + \\ & \bar{b}_3\left(-\frac{1}{3}s_1 s_2 + \varkappa_1 s_2\right) + \bar{b}_4\left(\mu_1\varkappa_1 - \frac{1}{2}\mu_2\right) + \\ & \bar{b}_5\left(-\frac{1}{3}s_1\mu_2 + \varkappa_1\mu_2 + \varkappa_1 s_1\mu_1 - \mu_1\varkappa_2\right) + \\ & \bar{c}_1\,(s_1\lambda_1 - 3\lambda_1\varkappa_1 + \pi_4 s_1\varkappa_1) + \\ & \bar{c}_2\left(\lambda_1 s_1^2 - 3s_1\lambda_2 - 9\lambda_1\varkappa_2 + 9\varkappa_1\lambda_2 - \frac{1}{9}\pi_4 s_1^3 - 6\pi_4\varkappa_1^3 + 6\pi_4\varkappa_1\varkappa_2\right) + \\ & \bar{c}_3\left(\lambda_1\varkappa_1^2 - \lambda_1\varkappa_2 - \pi_4\varkappa_1^3 + \pi_4\varkappa_1\varkappa_2\right). \end{aligned} \tag{2.13}$$

Special case of negligible plastic strain provides vanishing of following invariants $\mu_1 \approx 0$, $\mu_2 \approx 0$, $\lambda_1 \approx 0$, $\lambda_2 \approx 0$, $\pi_4 \approx 0$ leading to further simplification

$$\begin{aligned} 2\,\Omega \;\approx\;& \bar{a}_1 s_2 + \bar{a}_2\,(s_3 - s_1 s_2) + \bar{b}_1\left(\frac{1}{9}s_1^2 + \varkappa_1^2 - \frac{2}{3}s_1\varkappa_1\right) + \\ & \bar{b}_2\left(\frac{1}{9}s_1^2 + \varkappa_2 - \frac{2}{3}s_1\varkappa_1\right) + \bar{b}_3\left(-\frac{1}{3}s_1 s_2 + \varkappa_1 s_2\right). \end{aligned} \tag{2.14}$$

For further analysis only the simplest evolution equation following from the above reduced loading function is shown:

$$\begin{aligned} \mathbf{D}_{\mathrm{P}} \;=\;& \bar{a}_1\mathbf{S}_d + \bar{a}_2\left(\frac{3}{2}\mathbf{S}_d^2 - s_1\mathbf{S}_d - \frac{1}{2}\,s_2\mathbf{1}\right) + \\ & \bar{b}_1\left(\frac{1}{9}s_1\mathbf{1} + \varkappa_1\mathbf{N} - \frac{1}{3}s_1\mathbf{N} - \frac{1}{3}\varkappa_1\mathbf{1}\right) + \\ & \bar{b}_2\left(\frac{1}{9}s_1\mathbf{1} + \frac{1}{2}\mathbf{N}\mathbf{S}_d + \frac{1}{2}\mathbf{S}_d\mathbf{N} - \frac{1}{3}s_1\mathbf{N} - \frac{1}{3}\varkappa_1\mathbf{1}\right) + \\ & \bar{b}_3\left(-\frac{1}{3}s_1\mathbf{S}_d - \frac{1}{6}s_2\mathbf{1} + \frac{1}{2}s_2\mathbf{N} + \varkappa_1\mathbf{S}_d\right). \end{aligned} \tag{2.15}$$

2.4 Classical theory of transversely isotropic materials

In classical theory of plasticity of transversely isotropic materials the evolution equation is based on the equivalent stress:

$$\sigma_{eq}^2 = \frac{3}{2}\left(s_2 + \frac{1}{2}\bar{R}\varkappa_1^2\right) \tag{2.16}$$

and the corresponding yield function

$$f = \frac{1}{3}\frac{\sigma_{eq}^2}{h(\varepsilon_{Peq})} - 1$$

Then

$$\mathbf{D}_P = \partial_{\mathbf{S}} f = \frac{1}{2h(\varepsilon_{peq})}\partial_{\mathbf{S}}\sigma_{eq}^2 = \frac{1}{2h(\varepsilon_{peq})}\left(\mathbf{S}_d + \bar{R}\varkappa_1\mathbf{N}\right). \tag{2.17}$$

Comparing these expressions (2.16 and 2.17) with the simplest loading function in MAM theory we see that anisotropy coefficient $\bar{R}$ is proportional to $\bar{b}_1$. However this classical theory does not contain either coefficients $\bar{b}_2$ and $\bar{b}_3$ or new invariant expressions present in (2.15). Indeed analyzing data in [12] we conclude that evolution (2.17) is not able to cover properly deformation behavior of car body sheet. This will be demonstrated bellow.

2.5 Some loading histories

In order to illustrate significance of coefficients in (2.10), let us consider some special cases of loading calculating plastic stretching from the simplest evolution equation when direction of anisotropy is along the x_1 axis i.e.

$$\mathbf{N} = \left\{\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right\}.$$

The following special histories enlighten meaning of the material constants appearing in (2.15).

(a) longitudinal uniaxial tension:

$$\mathbf{S} = \sigma\left\{\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right\} \Longrightarrow$$

$$\mathbf{D}_P = \left(\frac{a_1}{3}\sigma - \frac{a_2}{6}\sigma^2 + \frac{b_1+b_2}{9}\sigma + \frac{2b_3}{6}\sigma^2\right)\left\{\begin{array}{ccc} 2 & 0 & 0 \\ & -1 & 0 \\ & & -1 \end{array}\right\},$$

$$\varkappa = \frac{D_{P22}}{D_{P11}} = -\frac{1}{2}.$$

(b) transverse uniaxial tension

$$\mathbf{S} = \sigma\left\{\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right\} \Longrightarrow D_{P\alpha\beta} = 0, \alpha \neq \beta,$$

$$D_{P11} = -\frac{a_1}{3}\sigma + \frac{a_2}{6}\sigma^2 - \frac{4(b_1+b_2)}{9}\sigma + \frac{4b_3}{6}\sigma^2,$$

$$D_{P22} = \frac{2a_1}{3}\sigma - \frac{a_2}{3}\sigma^2 + \frac{2(b_1+b_2)}{9}\sigma - \frac{5b_3}{9}\sigma^2,$$

$$D_{P33} = -\frac{a_1}{3}\sigma + \frac{a_2}{6}\sigma^2 + \frac{2(b_1+b_2)}{9}\sigma + \frac{b_3}{9}\sigma^2$$

$$\varkappa = 6\frac{2(b_1+b_2) - b_3\sigma}{6a_1 - 3a_2\sigma^2 + 8(b_1+b_2) - 8b_3\sigma}.$$

(c) longitudinal-transverse shear

$$\mathbf{S} = \tau\left\{\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right\} \Longrightarrow$$

$$\mathbf{D}_P = \left\{\begin{array}{ccc} a_2\tau^2/2 + 2b_3\tau^2/3 & a_1\tau + b_2\tau/2 & 0 \\ & a_2\tau^2/2 - 2b_3\tau^2/3 & 0 \\ Sym & & -a_2\tau^2 - b_3\tau^2 \end{array}\right\},$$

$$\varkappa = 1 - \frac{6b_3}{3a_2 + 4b_3}.$$

(d) transverse shear

$$\mathbf{S} = \tau\left\{\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right\} \Longrightarrow \mathbf{D}_P = \left\{\begin{array}{ccc} -2\delta & 0 & 0 \\ & \delta & a_1\tau \\ Sym & & \delta \end{array}\right\},$$

where

$$\delta \equiv \frac{1}{2}a_2\tau^2 - \frac{1}{3}b_3\tau^2 \qquad \text{and} \qquad \varkappa = -\frac{1}{2}.$$

3 Some concluding remarks

In the above loading cases to the direction of plastic strain a special attention is given. Meaning of coefficients b_2 and b_3 will be analyzed especially with relation to plastic strain induced anisotropy and transition form isotropy to transverse isotropy by previous loading. However, an immediate conclusion could be made now. Namely the coefficient b_3 is indispensable either for longitudinal-transverse shear or for the case of pure transverse shear.

Acknowledgement. This paper is made within the research project 174004 funded by the Serbian Ministry of Science.

References

[1] Hill R (1952) On Discontinuous Plastic States with Special Reference to Localized Necking in Thin Sheets, *Journal of Mechanics and Physics of Solids*, **1** , pp. 19-30.

[2] Hill R, Extremal Paths of Plastic Work and Deformation (1986) *Journal of Mechanics and Physics of Solids*, **34/5** , pp. 511-523.

[3] Mićunović M (2005) Self-consistent method applied to quasi-rate independent polycrystals, *Philosophical Magazine*, **85 (33-35)**, pp. 4031–4054

[4] Mićunović M (2009) *Thermomechanics of Viscoplasticity: Fundamentals and Applications* Springer, 2009.

[5] Cadoni E (2010) Dynamic characterization of orthogneiss rock subjected to intermediate and high strain rates in tension *Rock Mechanics and Rock Engineering*, **43-6**, pp. 667-67

[6] Kröner E (1960) Allgemeine Kontinuumstheorie der Versetzungen und Eigenspannungen *Arch. Rational Mech. Anal.*, **4**, pp. 273–334.

[7] Stojanović R (1962) On the reference-state problem in the non-linear elasticity theory of continua with dislocations *Physica Status Solidi,* **2**, pp. 566–575.

[8] Mićunović M, Albertini C, Montagnani M, High strain rate viscoplasticity of AISI 316 stainless steel from tension and shear experiments (1997) In: Miljanic, P. (ed) *Solid Mechanics*, *Serbian Acad. Sci. Meetings - LXXXVII*, *Dept. Techn. Sci.*, **3**, pp. 97–106.

[9] Rice J R (1971) Inelastic constitutive relations for solids: an internal variable theory and its application to metal plasticity, *J. Mech. Phys. Solids*, **19**, pp. 433–455.

[10] Vakulenko A A (1970) Superposition in continuum rheology (in Russian), *Izv. AN SSSR Mekhanika Tverdogo Tela*, **1**, 69–74

[11] Fomin V L (1975) *Continuum Mechanics for Engineers* (in Russian), St. Peterburg University Publ., St. Peterburg

[12] Aleksandrović S (1993): *F*ormability of thin sheets for non-monotonous plastic deformation processes, MSc-thesis, Fac.Mech.Engng., Kragujevac.

[13] Marciniak Z, Kuczynsky K (1967) Limit strains in the processes of stretch-forming sheet metal, *International Journal of Mechanical Sciences*, **9**, pp. 609–620

[14] Perzyna P (1971) Fundamental Problems in Viscoplasticity, *Advances in Applied Mechanics* **11**, pp. 313-387.

[15] Chaboche J L and Rousselier G (1983) On the plastic and viscoplastic constitutive equations, *Trans. ASME, PVT*, **105** pp. 153-164.

[16] Rabotnov Yu N (1980) *Elements of Hereditary Solids Mechanics*, Mir Publishers, Moscow

[17] Mićunović M (1992) Normality rule from plastic work extremals?, In: Anthony K.-H. & Wagner H.-J. (eds) *Continuum Models and Discrete Systems CMDS-7*, *Material Science Forum*, **123-125**, pp. 609–616, Trans Tech Publ, Aedermannsdorf

[18] Gambin W, Kowalczyk-Gajewska K, Kudrjavceva L, Mićunović M (2008) Two-scale approach to dynamic localization failure of AISI 316H stainless steel sheets *Theoret.Appl.Mechanics TEOPM7*, **35/1-3**, pp. 93-103

Submitted in October 2014.

Viskoplastičnost niskog reda poprečno izotropnih materiala naizgled nezavisnih od brzine

Eksperimentima je utvrdjeno da materiali naizgled nezavisni od brzine (QRI) veoma dobro opisuju ponašanje čelika u veoma širokom opsegu deformacija i brzina deformacije ([3],[4]). Ova osobina je spregnuta sa modeliranjem tenzorskom reprezentacijom korišćenjem opšteg nepridruženog zakona tečenja zasnovanog ne na funkciji tečenja već na opštijoj funkciji opterećenja. Naizgled nezavisni od brzine QRI–materijali koje sadrže priraštajne jednačine zavisnost od brzine deformacije obezbedjuju pomoću promenljivosti napona tečenja sa naponskom brzinom. Sa druge strane, poprečna izotropnost se javlja pri obradi valjanjem karoserijskih metalnih limova u autoindustriji [18]. Ovde je učinjeno nužno proširenje skupa tenzorskih generatora i invarianata u cilju uključenja zavisnosti od privilegovanog pravca anizotropije. Verujemo da će rezultati ovog rada biti primenjivi i na orthogneiss-stene razmatrane u [5].

doi:10.2298/TAM1403233M Math.Subj.Class.: 74-05,74C20,74C99,74D10,74E10,74L10.