

Dynamic analysis of 2-D and 3-D quasi-brittle solids and structures by D/BEM

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Abstract

A general boundary element methodology for the dynamic analysis of 2-D and 3-D solids and structures exhibiting quasi-brittle material behavior is presented. Inelasticity is modeled with the aid of the elastic damage theory. Strain rate and cyclic loading effects are also considered. The integral formulation of the problem employs the elastostatic fundamental solution and thus both surface and volume integrals due to inertia and inelasticity are created. Consequently the discretization involves both the surface and the interior of the body. The singular integrals are evaluated by advanced numerical integration techniques, while Houbolt's step-by-step time integration scheme is used to obtain the dynamic response. Numerical examples are presented to illustrate the proposed method and demonstrate its accuracy and potential.

1 Introduction

Quasi-brittle materials, such as concrete, rocks, soils, ceramics and masonry, exhibit a behavior under dynamic loading characterized by different strengths in tension and compression, a stress-strain curve with an almost linear ascending branch and a highly nonlinear descending

branch (softening), increasing strength in the compression-compression region, localization of deformation and creation of shear bands, increase of strengths under dynamic load (strain-rate effect) and cyclic material behavior under cyclic loading. All these material characteristics can be successfully simulated by elastic damage theories.

In this work, an elastic damage model called FOM is employed. This model consists of the elastic damage part of the plasticity-damage model of Faria and Oliver [1] expressed in terms of the tension and compression damage indices, which are combined to produce a single damage index on the basis of Mazars [2] theory of damage. Strain rate and cyclic effects are embedded in this model with the aid of experimentally derived empirical relations.

A boundary element method (BEM) employing the elastostatic fundamental solution is developed for the dynamic analysis of 2-D and 3-D solids and structures exhibiting a quasi-static material behavior simulated by the FOM model. Thus both surface and volume integrals are created, the latter due to inertia and inelasticity. This requires both a surface and an interior (domain) discretization of the body. For this reason the method is called Domain/Boundary Element method or D/BEM. The space-discretized equations of motion are finally solved numerically by Houbolt's step-by-step time integration scheme with iterations at every time step.

2 FOM damage model

The starting point is the computation of the principal $\sigma_i (i = 1, 2, 3)$ and the effective $\bar{\sigma}^+$ and $\bar{\sigma}^-$ stresses for tension and compression, respectively. The equivalent stresses $\bar{\tau}^+$ and $\bar{\tau}^-$ in tension and compression, respectively, are then defined in terms of $\bar{\sigma}^+$, $\bar{\sigma}^-$ and σ_i . The damage indices d^+ and d^- in tension and compression, respectively, are subsequently computed in terms of $\bar{\tau}^+$ and $\bar{\tau}^-$ and some material parameters, which are functions of an internal length scale. Finally, the combined damage index d is computed as

$$d = \alpha^+ d^+ + \alpha^- d^- \quad (1)$$

where α^+ and α^- are functions of $\bar{\sigma}^+$, $\bar{\sigma}^-$. For more details one can look at Hatzigeorgiou et al [3].

Strain rate effects can be easily introduced into the FOM model in an empirical way by simply adopting the Suaris and Shah[4] curves, which provide the expressions

$$\begin{aligned} f_{t,d}/f_{t,s} &= \phi(\dot{\varepsilon}^+) \\ f_{c,d}/f_{c,s} &= \phi(\dot{\varepsilon}^+) \end{aligned} \quad (2)$$

In the above, the subscripts t, c, d and s stand for tension, compression, dynamic and static, respectively, f denotes strength and $\dot{\varepsilon}$ denotes strain rate, which is computed during the stepwise integration process of the equations of motion by finite differences in time.

To account for cyclic effects, the damage index d is replaced by the unloading - reloading damage index d^* in the form

$$d^* = d^R \quad (3)$$

where the exponent $R = 2$ for most practical cases. For more details about strain rate and cyclic effects one can consult Hatzigeorgiou et al [3].

Once the damage index d is known, the total stress vector $\{\sigma\}$ is computed from

$$\{\sigma\} = (1 - d) [D] \{\varepsilon\} \quad (4)$$

where $[D]$ is the elasticity matrix and $\{\varepsilon\}$ is the strain vector.

3 Domain/boundary element method

The integral representation of the displacement $u_j = u_j(\boldsymbol{\xi}, t)$ at point $\boldsymbol{\xi}$ and time t inside a body of volume V and surface S has the form (Hatzigeorgiou and Beskos [5])

$$c_{ij}u_j = \int_S (u_{ij}^*p_j - p_{ij}^*u_j) dS - \rho \int_V u_{i,j}^*\ddot{u}_j dV + \int_V \varepsilon_{jki}^* \sigma_{jk}^p dV \quad (5)$$

where $u_{i,j}^*$, p_{ij}^* and ε_{jki}^* are the displacement, traction and strain of the elastostatic fundamental solution, p_j denotes the surface traction, ρ the mass density and σ_{jk}^p the inelastic stress.

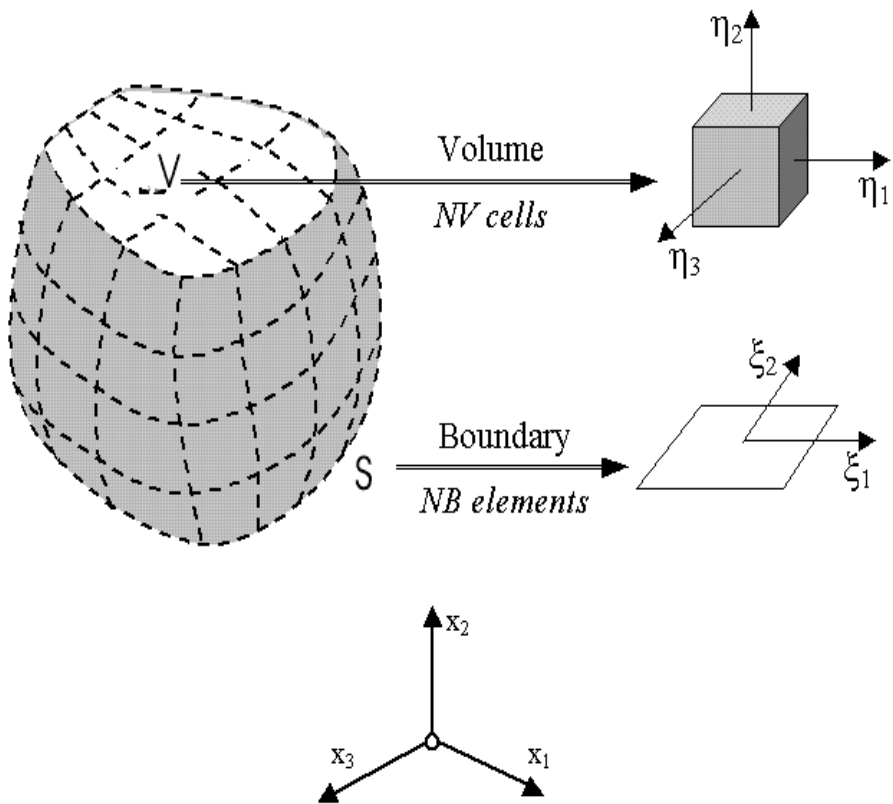


Figure 1: Discretization of a general three-dimensional body

Discretization of the surface S into a number of linear quadrilateral boundary elements and the volume V into a number of linear hexahedral interior cells (see Fig. 1) enables one to write Eq. (6) in the discretized matrix form

$$[G]\{p\} - [H]\{u\} - [M]\{\ddot{u}\} + [Q]\{\sigma^p\} = \{0\} \quad (6)$$

The regular space integrations are done by standard Gauss quadrature, while the singular ones by advanced direct algorithms based on Guiggiani and Gigante[6]. Employment of Houbolt's stepwise in time integration scheme enables one to replace \ddot{u} at time step $n + 1$ by u at previous time steps. In addition, use of the initial and boundary conditions of the problem and rearrangement, finally transforms Eq. (6) into

$$[GH] \begin{Bmatrix} p \\ u \end{Bmatrix}_{n+1} = [M]\{-5u_n + 4u_{n-1} - u_{n-2}\} - [Q^*]\{\sigma_{n+1}^p\} + \{B\} \quad (7)$$

where $\{B\}$ arises from the known boundary conditions.

Since σ_{n+1}^p is not known, u_{n+1} and p_{n+1} have to be computed iteratively with the aid of Eq. (7) and the constitutive equation of the material at every time step. This procedure appears in Fig. 2.

4 Numerical examples

4.1 Dynamic analysis of a mortar beam

A simply supported mortar beam subjected to a central impact loading is analyzed by the present method. The material parameters are $E = 22000N/mm^2$, $\nu = 0.15$, $f_t = 3.91N/mm^2$, $f_c = 31.0N/mm^2$, $f_{c-2D} = 36.0N/mm^2$, $\rho = 2410Kg/m^3$ and $G_f = 103.7N/m$, where E, ν, f_{c-2D} and G_f stand for modulus of elasticity, Poisson's ratio, compressive strength under biaxial conditions and fracture energy, respectively. Figure 3 shows the geometry and the 3-D BEM discretization of the structure.

Figure 4 provides the time history of the vertical displacement at the load point, as obtained by the present D/BEM and the FEM and experiments of Du et al [7]. The agreement is very good.

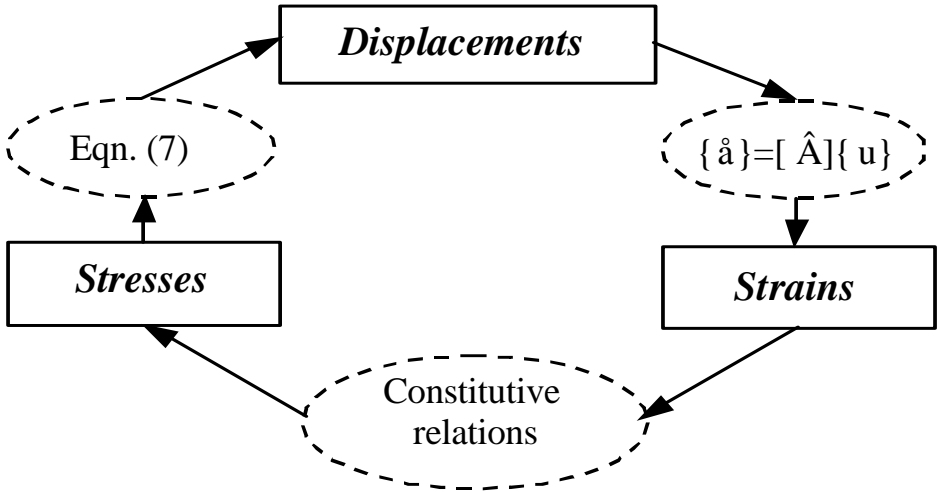


Figure 2: Iterative procedure for the computation of stresses

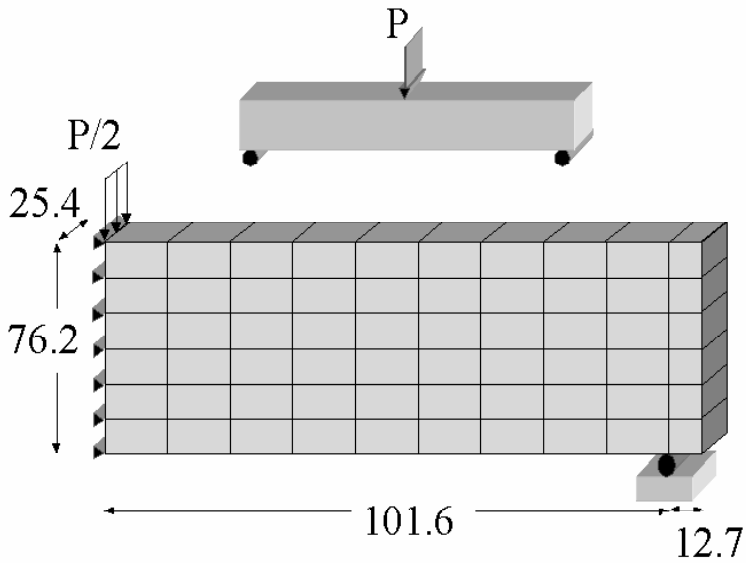


Figure 3: Geometry and discretization of Example 1 (dimensions in mm)

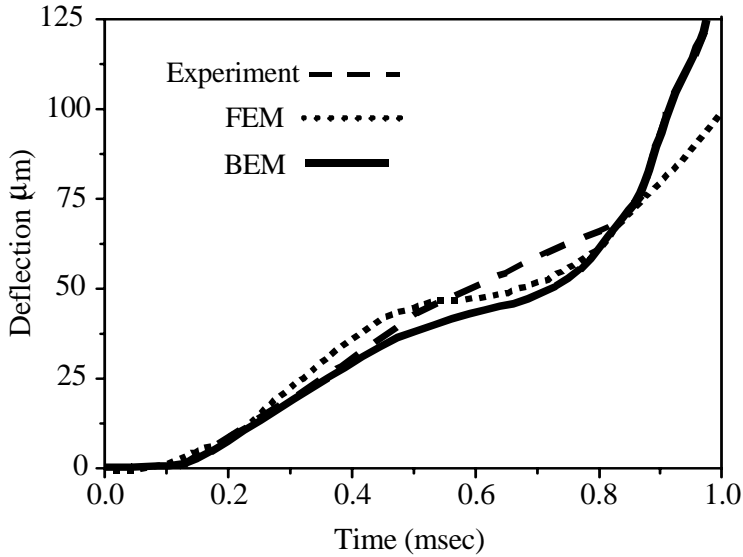


Figure 4: Time history of deflection of load point

4.2 Seismic analysis of the Arta bridge

The historic Arta bridge made of masonry has been analyzed by the proposed D/BEM under plane stress conditions for the loading case of self weight plus the first 5secs of the NS El-Centro (1940) earthquake with maximum horizontal and vertical acceleration of $0.16g$ and $0.10g$, respectively to match local conditions. The bridge consists of four main arches with spans of 23.95 m, 15.83 m, 15.43 m and 16.16 m, while its width is 3.70 m. Its material parameters are:

- $E = 3.0GPa$,
- $\nu = 0.22$,
- $f_c = 30.00MPa$,
- $f_{c-2D} = 34.8MPa$,
- $\rho = 2700Kg/m^3$,

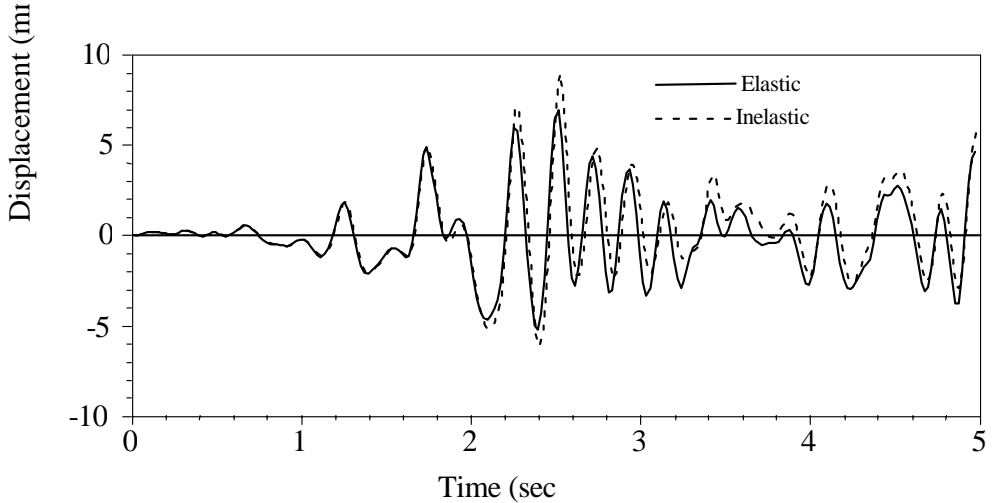


Figure 5: Horizontal displacement [mm] versus time [sec]

- $f_t = 0.3MPa$, and
- $G_f = 20N/m$

Figure 5 shows the time history of the horizontal displacement at the top of the largest arch as obtained under elastic and inelastic (FOM model) material behavior.

The discretization of the Arta bridge is shown in Fig. 6. The same figure (Fig. 6) shows the distribution of damage (dark colour) in the bridge, on the assumption that an element is considered failed (or fully damaged) when $d = 1$.

5 Conclusions

A D/BEM has been developed for the dynamic analysis of 2-D and 3-D solids and structures exhibiting quasi-brittle material behavior, which is simulated by the elastic-damage model FOM. The method has been

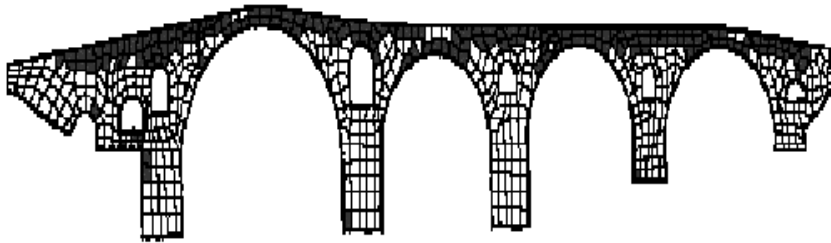


Figure 6: Damaged region of the structure: 'self weight + earthquake' load combination

illustrated by means of two examples, which have also demonstrated its accuracy and potential.

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**Dinamička analiza 2-D and 3-Dd skoro krutih čvrstih tela i
struktura pomoću D/BEM**

UDK 517.95; 517.968

Prikazana je opšta metodologija graničnih elemenata za dinamičku analizu 2-D i 3-D čvrstih tela i struktura koji ispoljavaju skoro krto materijalno ponašanje. Neelastičnost je modelirana pomoću teorije elastičnog zamora. Takođe se razmatraju efekti brzine deformacije i cikličnog opterećenja. Integralna formulacija problema koristi fundamentalno rešenje elastostatike. Dakle, i površinski i zapreminski integrali koji potiču od inercije i neelastičnosti su sačinjeni. Prema tome, diskretizacija obuhvata i površ i unutrašnjost tela. Singularni integrali su izračunati naprednim tehnikama numeričke integracije, dok se Houbolt-ova korak-po-korak vremensko integraciona šema koristi za dobijanje dinamičkog odgovora. Numerički primeri su prikazani kako radi ilustracije predloženog metoda tako i zbog demonstracije njegove tačnosti i sposobnosti.