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Effect of variable viscosity on laminar convection flow of an electrically conducting fluid in uniform magnetic field

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Abstract

The flow of a viscous incompressible electrically conducting fluid on a continuous moving flate plate in presence of uniform transverse magnetic field, is studied. The flat plate which is continuously moving in its own plane with a constant speed is considered to be isothermally heated. Assuming the fluid viscosity as an inverse linear function of temperature, the nature of fluid velocity and temperature in presence of uniform magnetic field are shown for changing viscosity parameter at different layers of the medium. Numerical solutions are obtained by using Runge-Kutta and Shooting method. The coefficient of skin friction and the rate of heat transfer are calculated at diffferent viscosity parameter and Prandtl number.

1 Introduction

Ostrach [1] first discussed the combined natural and forced flow of a viscous incompressible fluid through a rigid surface. Later on Grief et al. [2], Gupta et al. [3] and Soundalgekar et al. [4] studied the incompressible flow over a fixed that plate. But this type of flow becomes different when the flow is caused by the motion of the flat plate or rigid surface. Sakiadis [5] discussed the viscous flow of an incompressible fluid due to the motion of rigid surface. Many authors like Gorla [6], Revenkar [7], Igham [8] and Pop [6] discussed the problem of incompressible fluid on a continuous moving flat plate. Both types of flow behave differently-particularly when the fluid viscosity varies with temperature. The fluid properties especially the viscosity depends linearly and inversely to the temperature (see Herwig and Gersten [9]); therefore to characterize the nature of flow and heat transfer, one must consider the variation of fluid viscosity with temperature.

Pop et al. [10] studied the problem of viscous variation for a moving flat plate in an incompressible fluid. In this paper, an attempt is made to study the effect of variable viscosity on the flow of an incompressible electrically conducting fluid on a continuous moving flat plate in presence of a uniform magnetic field. The solutions and results are obtained by similarity transformation.

2 Formulation of the problem

We consider laminar flow of a viscous incompressible electrically conducting fluid on a continuous moving flat plate along $x-$axis. The plate is moving in its own a constant speed U_0 in quiescent fluid. A uniform magnetic field B_0 is applied transversely i.e. along y-axis. The fluid properties except fluid viscosity (μ) are assumed to be isotropic and constant, and the viscosity is inverse linear function of temperature (see Lai and Kulachi [11]) as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma\left(T - T_w\right)\right] \tag{1}$$

$$= \frac{1}{a}\left(T - T_r\right), \tag{2}$$

where

$$a = \frac{\mu_\infty}{\gamma} \quad \text{and} \quad T_r = T_\infty - \frac{1}{\gamma}, \tag{3}$$

both a and T_r being constant. Their values depend in the reference state and the thermal property of the fluid (γ). In general, $a > 0$ for liquid and $a < 0$ for gasses.

3 Assumptions

In order to derive the governing equations of the problem the following assumptions are made.

(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected

(ii) Hall effect and polarization effect are negligible

(iii) The flat plate which is maintained at a constant temperature (T_w) is moving with uniform velocity and the fluid viscosity varies with temperature only, therefore all the physical variables are assumed to be time independent

(iv) The perturbation technique which is used for small values of the magnetic parameter (m) depending on the magnetic field. The second order term is due to the effect of the magnetic field

4 Governing equations

The governing equations of the problem for the fluid medium having small conductivity are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho_\infty}\left[\sigma B_0^2 u - \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial x}\right)\right] = 0, \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} = 0, \tag{6}$$

where u, v are the fluid velocities along $x, y-$axes respectively. The boundary conditions of equations (4-6) of the problem, are as

$$u = U_0, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0, \tag{7}$$

$$u \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \tag{8}$$

Using stream function ψ where

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \tag{9}$$

we get from the equations (4-6)

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0, \tag{10}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} + m u_0 \frac{\partial \psi}{\partial y} - \frac{1}{\rho_\infty}\frac{\partial \mu}{\partial y}\frac{\partial^2 \psi}{\partial y^2} - \mu \frac{\partial^3 \psi}{\partial y^3} = 0, \tag{11}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} - \alpha \frac{\partial^2 T}{\partial y^2} = 0, \tag{12}$$

where $(\sigma B_0^2 u) / \rho_\infty = m u_0$, m being the magnetic parameter.

The boundary conditions of the equations (10-12) can be written as

$$\frac{\partial \psi}{\partial y} = U_0, \quad \psi = 0, \quad T = T_w \quad \text{at} \quad y = 0, \tag{13}$$

$$\frac{\partial \psi}{\partial y} \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \tag{14}$$

Using the relations (1-3), the equations (10-12) can be written as

$$\frac{(\theta - \theta_r)^2}{\theta_r}\left[x \frac{\partial F}{\partial \eta}\frac{\partial^2 F}{\partial x \partial \eta} - \frac{1}{2} F \frac{\partial^2 F}{\partial \eta^2} - x \frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial \eta^2} + \right.$$

$$mx\frac{\partial F}{\partial \eta}\bigg] - \frac{\partial \theta}{\partial \eta}\frac{\partial^2 F}{\partial \eta^2} + (\theta - \theta_r)\frac{\partial^3 F}{\partial \eta^3} = 0, \tag{15}$$

$$\frac{\partial F}{\partial \eta}\left(\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial \theta}{\partial x}\right) - \frac{1}{2x}\frac{\partial \theta}{\partial \eta}\left(F - \eta\frac{\partial F}{\partial \eta}\right) + \frac{\partial \theta}{\partial \eta}\frac{\partial F}{\partial x} - \frac{\alpha}{\gamma_\infty x}\frac{\partial^2 \theta}{\partial \eta^2} = 0, \tag{16}$$

where (Re is Reynolds number, θ_r—viscosity parameter and α— thermal diffusivity):

$$\psi = v_\infty \overset{1/2}{\mathrm{Re}} F(\eta), \quad \eta = \frac{y}{x}\overset{1/2}{\mathrm{Re}}, \ \ \mathrm{Re} = \frac{U_0 x}{v_\infty} \tag{17}$$

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \ \theta_r = (T_r - T_\infty)/(T_w - T_\infty), \tag{18}$$

$$\gamma = 1/(T_\infty - T_r). \tag{19}$$

The boundary condition (13, 14) are now as follows

$$\frac{\partial F}{\partial \eta} = 1, \quad F(\eta) = 0, \quad \theta(\eta) = 1 \quad \text{at} \quad \eta = 0, \tag{20}$$

$$\frac{\partial F}{\partial \eta} \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \tag{21}$$

The following expansions for $F(\eta)$ and $\theta(\eta)$ are assumed:

$$F(\eta, x) = f_0(\eta) + mx\ f_2(\eta) + (mx)^2\ f_4(\eta) + ... \tag{22}$$

$$\theta(\eta, x) = \theta_0(\eta) + mx\ \theta_2(\eta) + (mx)^2\ \theta_4(\eta) + ... \tag{23}$$

These expansions are valid for small values of magnetic parameter (m), which show the degree of magnetic field interactions to the flow and the temperature of the fluid. Using the expansions (22), (23) in the equations (15) and (16), we have different sets of non-linear equations according to the degree of magnetic parameter (m) as given below.

System(I)

The first pair of equations which is independent of m, gives the velocity and temperature distribution in absence of magnetic field. These equations are

$$f_0'''(\eta) - \frac{\theta_0(\eta)-\theta_r}{2\theta_r} f_0(\eta)\ f_0''(\eta) - \frac{1}{\theta_0(\eta)-\theta_r}\ f_0''(\eta) = 0, \tag{24}$$

$$\theta_0'''(\eta) - \frac{\mathrm{Pr}}{2}\ f_0(\eta)\ \theta_0'(\eta) = 0. \tag{25}$$

System(II)

The second pair of equations for the first degree of magnetic interaction are

$$f_2'''(\eta) + \frac{\theta_2(\eta)}{\theta_0(\eta)-\theta_r}\ f_0'''(\eta) + \frac{\theta_0(\eta)-\theta_r}{\theta_0(\eta)}\ [f_0'(\eta)f_2'(\eta) -$$

$$\frac{1}{2}f_0(\eta)f_2''(\eta) - \frac{3}{2}f_2(\eta)f_0''(\eta) + f_0'(\eta)\Big] - \tag{26}$$

$$\frac{\theta_2(\eta)}{\theta_r} f_0(\eta)f_0''(\eta) - \frac{1}{\theta_2(\eta)-\theta_r}\ [\theta_0'(\eta)f_2''(\eta) + \theta_2'(\eta)f_0''(\eta)] = 0,$$

$$\frac{1}{\mathrm{Pr}}\theta_2''(\eta) + \frac{3}{2}\theta_0'(\eta)f_2(\eta) + \frac{1}{2}\theta_2'(\eta)f_0(\eta) - f_0'(\eta)\theta_2(\eta) = 0, \tag{27}$$

where the prime denotes differentiation with respect to η.

The corresponding boundary conditions are

$$\begin{gathered} f_0(\eta) = f_2(\eta) = f_4(\eta) = \ldots = 0, \\ f_0' = 1, \quad f_2' = f_4' = \ldots = 0 \quad \text{at} \ \ \eta = 0, \end{gathered} \tag{28}$$

$$\theta_0(\eta) = 1, \quad \theta_2(\eta) = \theta_4(\eta) = \ldots = 0 \quad \text{at} \ \ \eta = 0, \tag{29}$$

$$f_0'(\eta) \to 0, \quad f_2'(\eta) = f_4'(\eta) = \ldots = 0 \quad \text{as} \ \ \eta \to \infty, \tag{30}$$

$$\theta_0(\eta) \to 0, \quad \theta_2(\eta) = \theta_4(\eta) = \ldots = 0 \quad \text{as} \ \ \eta \to \infty. \tag{31}$$

5 Skin Friction and Rate of Heat transfer

The physical quantities of this problem are the Skin friction coefficient (C_f) and the Nisselt number (N_u) which are defined by

$$C_f = \frac{2\tau_w}{\rho U_0^2} \quad \text{and} \quad N_u = \frac{x q_w}{K\left(T_w - T_\infty\right)}, \tag{32}$$

where

$$\tau_w = \mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{33}$$

and K is the thermal conductivity of the fluid.

Using relations (2), (17) and (18), (C_f) and (N_u) are written as

$$C_f = \overset{1/2}{\text{Re}}\left[\frac{2\theta_r}{\theta_r - 1}\left(f_0'' + m x f_2'' + \ldots\right)\right] \tag{34}$$

$$= C_{f,1} + C_{f,2} + \ldots \tag{35}$$

and

$$N_u = -\overset{1/2}{\text{Re}}\left(\theta_0' + m x \theta_2' + \ldots\right) \tag{36}$$

$$= N_{u,1} + N_{u,2} + \ldots \tag{37}$$

where $(C_{f,1}), (N_{u,1})$ are meant for skin friction and the rate of heat transfer in absence of magnetic field respectively and $(C_{f,2}), (N_{u,2})$ are meant for the same in the presence of the field.

	Table (I)		Table (II)	
	$f_2''(\eta = 0)$		$\theta_2'(\eta = 0)$	
θ_r	Pr = 0.71	Pr = 10.0	Pr = 0.71	Pr = 10.0
-10.0	0.7679	0.8356	0.2120	0.2129
-8.0	0.7700	0.8472	0.2100	0.2151
-6.0	0.7737	0.8660	0.2069	0.2188
-4.0	0.7821	0.9021	0.2008	0.2259
-2.0	0.8132	0.9991	0.1852	0.2454
-1.0	0.8934	1.1819	0.1626	0.3020
-0.1	3.9084	4.5587	0.0385	0.8446

6 Results and Discussion

The physical quantities of our interest are $f_2, (C_{f,2})$ and $\theta_2, (N_{u,2})$ which are the first degree magnetic interaction to the flow and the heat transfer respectively. Numerical solutions are obtained using Runge-Kutta and Shooting method of the non-linear equations of the system (II) from two different values of Prandtl number ($\mathrm{Pr} \in \{0.71, 10.0\}$). The variation of viscosity parameter θ_r means the variation of fluid viscosity with respect to the fluid temperature, and our aim is to show the nature of fluid velocity and temperature in the presence of uniform magnetic field under the action of variable viscosity. Figures (1-4) are plotted for f_2 and θ_2 against the viscosity parameter θ_r. Further negative values of viscosity parameter make $(T_w - T_\infty)$ negative, and $(T_w - T_\infty)$ is always negative for an incompressible fluid therefore we have calculated f_2'' and θ_2' for negative values of θ_r varying from -10.0 to -0.10. The values of $f_2''(\eta)$ and $\theta_2'(\eta)$ which are the factors for skin friction and rate heat transfer respectively of our problem are given in the tables (I, II).

Following are the results obtained from the figures and the tables:

1. (i) Figures (1,2) show the variation of f_2 against θ_r at different values of η. It is observed that the change of f_2 with the

increase of θ_r from -10 to -0.1 is negligibly small within the boundary layers ($\eta \in (0,1)$ approximately), after which it increases steadily for $\theta_r \in (-1,0)$. At constant θ_r, f_2 increases with the increase of η.

(ii) Figure (3) shows variation of f_2 with the increases of η at constant θ_r. It is observed that when θ_r remains unchanged, f_2 increases with the increases of η and almost vanishes for $\eta = 0$ (i.e. at the plate). For all values of $\eta \leq 1$ approximately.), the magnitude of f_2 remains same for different values of $\theta_r \in (-10,-1)$ and for all values of $\eta > 1$ approximately.) the magnitude of f_2 decreases with the increase of θ_r.

2. Figure (4) shows the variation of θ_2 with η at constant values of θ_r . It is seen that θ_2 rises from a minimum value ($\cong 0$) with the increase of η , attains maximum and then gradually decreases to minimum value.

3. The tables (I) and (II) show the values of f_2'' and θ_2' which are the factors for skin friction and rate of heat transfer respectively at $\eta = 0$ for Pr $= 0.71$ and 10.0. It has been observed that the magnitude of f_2'' increases with the increase of θ_r; on the other hand, θ_2' decreases for Pr $= 0.71$ and increases for Pr $= 10.0$ as θ_r changes from -10 to -0.1 . The variation in the values of θ_2' is negligibly small as Pr changes from 0.71 to 10.0 when θ_r is small ($\theta_r \sim -10$).

7 Conclusions

From the above discussions we can draw the following conclusions..

(a) The effect of viscosity parameter change in the range $(-10, 1)$ on the fluid velocity is insignificant within the boundary layers $\eta \in (0,1)$, while the outside ($\eta > 1$) the fluid velocity gradually decreases with the increase of viscosity parameter.

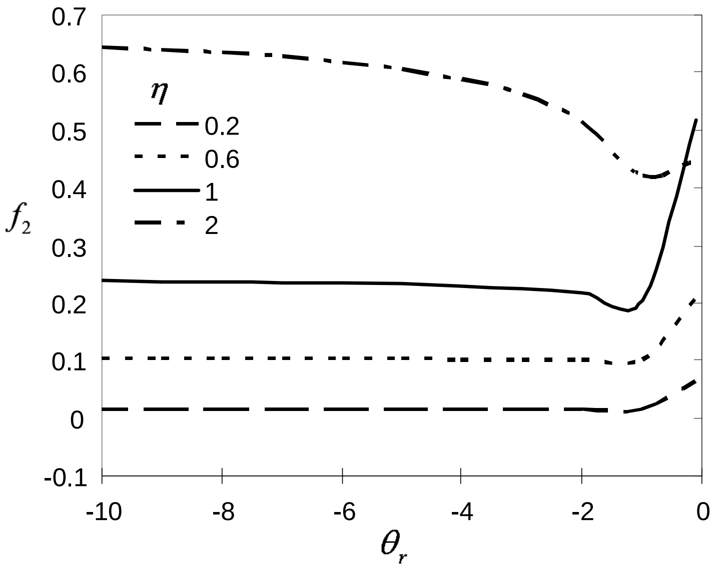

Figure 1: Variation of f_2 with θ_r for $\mathrm{Pr} = 0.71$

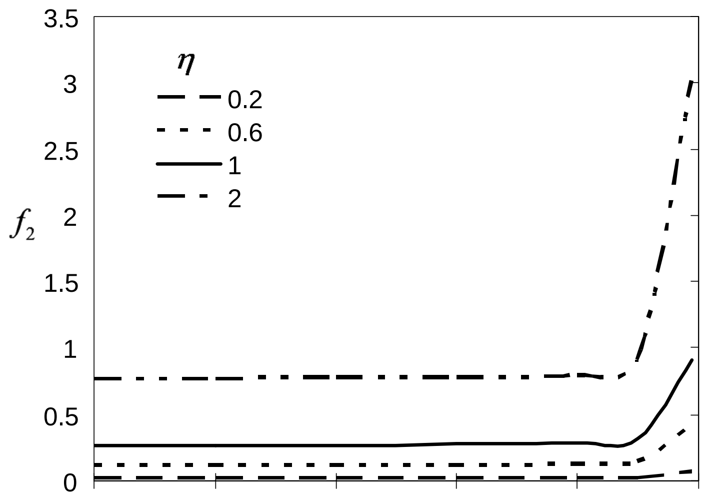

Figure 2: Variation of f_2 with θ_r for $\mathrm{Pr} = 10.0$

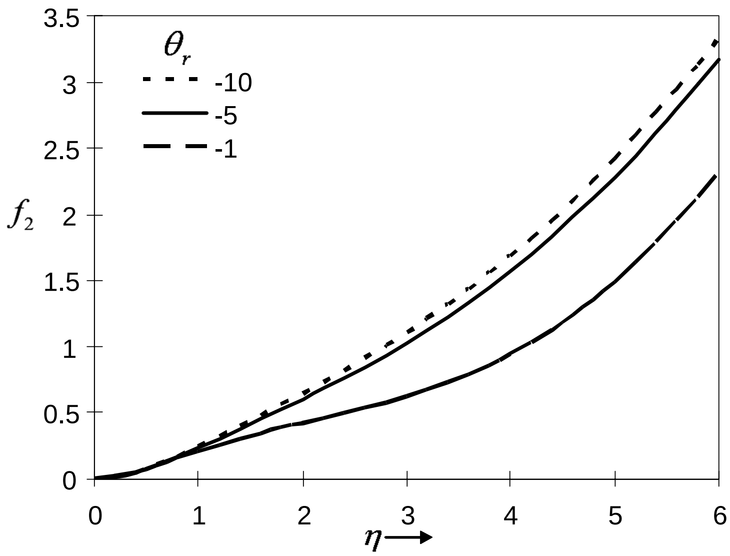

Figure 3: Velocity distribution (f_2) for $\theta_r = (-1.0, -5.0, -10.0)$

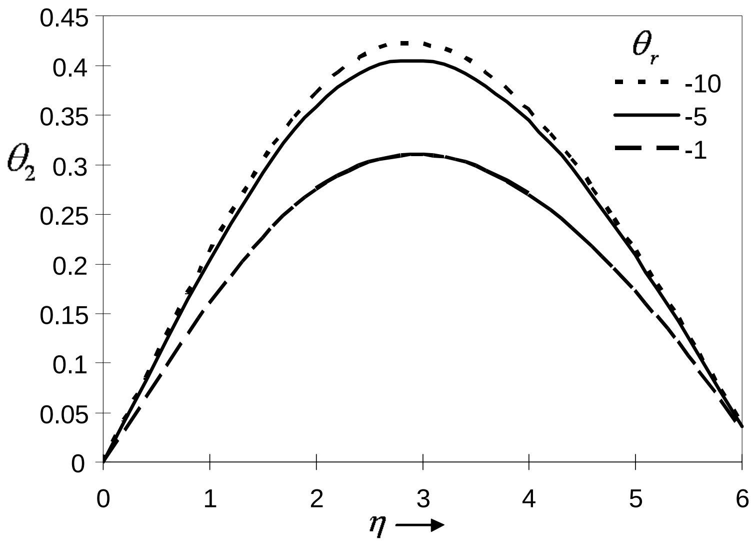

Figure 4: Variation of θ_2 with η for $\theta_r = (-10.0, -5.0, -1.0)$ and $\mathrm{Pr} = 0.71$

(b) The fluid temperature gradually decreases with the increase of viscosity parameter (from -10 to -1).

(c) The skin friction increases with the increase of viscosity parameter (from -10 to -1).

(d) The heat transfer decreases with the increase of viscosity parameter at small value of Prandtl number (i.e. $\mathrm{Pr} = 0.71$) and increases at high value of Prandtl number ($\mathrm{Pr} = 10.0$). At small values of the viscosity parameter, the heat transfer is less dependent on Prandtl number.

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Uticaj promenljive viskoznosti na laminarnu konvekciju tečenja elektroprovodnog fluida u uniformnom magnetnom polju

UDK 532.13; 537.84

Proučava se tečenje viskoznog nestišljivog elektroprovodnog fluida na neprekidnoj pokretnoj ravanskoj ploči u prisustvu uniformnog poprečnog magnetnog polja. Smatramo da se ravanska ploča, koja se neprekidno konstantnom brzinom kreće u svojoj sopstvenoj ravni, izotermski zagreva. Pod pretpostavkom da je viskoznost fluida inverzna funkcija temperature, na različitim slojevima sredine su prikazane viskoznost i temperatura fluida u prisustvu uniformnog magnetnog polja. Numerička rešenja su dobijena korišćenjem metoda Runge-Kutta i Shooting-a. Koeficijent trenja površinskog sloja i brzina prenosa toplote su izračunati pri različitim vrednostima parametra viskoznosti i Prandtl-ovog broja.