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Unsteady incompressible MHD boundary layer on porous aerofoil in high accelerating fluid flow

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Abstract

The fluid, flowing past the surface, is incompressible and its electro-conductivity is constant. The present magnetic field is homogenous and perpendicular to the surface and through the porous contour the fluid has been injected or ejected. In order to study this problem, a polyparametric method known as generalized similarity method has been established. The corresponding equations of unsteady boundary layer, by introducing the appropriate variable transformations, momentum and energy equations and three similarity parameters sets, being transformed into generalized form. The numerical integration of the generalized equation with boundary conditions has been performed by means of the difference schemes and by using Tridiagonal Algorithm Method with iterations in the four parametric and twice localized approximation. So obtained generalized solutions are used to calculate the shear stress distribution in laminar-turbulent transition of unsteady boundary layer on porous high accelerating aerofoil.

It's shown that for both in confuser and in diffuser regions the ejection of fluid postpones the boundary layer separation, and vice versa the fluid injection favours the separation. For both injection and ejection of fluid, the magnetic fielf increases the friction and postpones the laminar-turbulent transition.

1 Introduction

The results obtained by means of boundary layer theory dispose, in comparison with numerical solutions of complete Navier-Stokes equations, significant preference, because they are exact corresponding to the structure of solutions for a great Re numbers, i.e. represent solution which possess the boundary layer character. That is why the numerical method for calculation of Navier-Stokes equations, for a great Re numbers, is only appropriate if algorithm formed for their solving, concerning asymptotic behaviour, gives results which cover up solutions of boundary layer equations. The generalized similarity equation represents one of the ways for improvement of modern analytical methods for the calculation of boundary layers. As a result of this procedure which consists of introducing a conveniently chosen set of parameters, quantities characterizing any special problem are eliminated from the governing set of equations and the corresponding boundary conditions. A numerical solution of this equation can be found once for all and then it can be used in any special problem of the boundary layer theory.

The MHD boundary layer theory has a significant place in the development of the magnetic hydrodynamics. The results of this theory have a wide application in technical practice, especially in nuclear reactors, MHD-generators, as well as in different devices in chemical technology etc.[1,2]. The plane laminar unsteady MHD boundary layer on a porous surface, has been studied. It is assumed that the outer magnetic field is homogeneous, perpendicular and stationary with respect to the porous contour. The velocity in the basic fluid flow U is an arbitrary analytic function of the longitudinal coordinate x and time t. The fluid is incompressible and its electro-conductivity is constant. Through the surface in perpendicular direction, the fluid of the same properties as fluid in basic flow, has been injected or ejected with velocity v_w. The injection or ejection velocity is a function of the coordinate x and time t. The described MHD boundary layer has been considered in inductionless approximation.

2 Mathematical model and generalized similarity equation

The mathematical model of the noticed problem [2, 3, 7, 9, 11] is described by the following equation:

$$\Psi_{ty} + \Psi_y \Psi_{xy} + (v_w - \Psi_x)\Psi_{yy} = U_t + UU_x + \upsilon\Psi_{yyy} - N(\Psi_y - U) \quad (1)$$

with boundary and initial conditions:

$$\begin{aligned} &y = 0 : \Psi = \Psi_y = 0; \quad y \to \infty : \Psi_y \to U(x,t); \\ &t = t_0 : \Psi_y = u_1(x,y); \quad x = x_0 : \Psi_y = u_0(t,y), \end{aligned} \quad (2)$$

where we use: $\Psi(x,y,t)$ - stream function, $U(x,t)$ - free-stream velocity, υ -kinematic viscosity, $u_1(x,y)$ - the streamwise velocity distribution in boundary layer in some determined point of time $t = t_0$, $u_0(t,y)$ - the streamwise velocity distribution in boundary layer in cross-section $x = x_0$, x - streamwise coordinate, y - crosswise coordinate, t - time, $N = \sigma B^2/\rho$, B -magnetic induction, ρ -fluid density.

Introducing new variables in the form [2, 3, 7, 8, 9, 10] :

$$\begin{aligned} &x = x,\ t = t,\ \eta = yU^{b_0/2}\left(a_0\upsilon\int_0^x U^{b_0-1}dx\right)^{-1/2} \\ &\Phi = \Psi U^{b_0/2-1}\left(a_0\upsilon\int_0^x U^{b_0-1}dx\right)^{-1/2} \end{aligned} \quad (3)$$

where [8] $a_0 = 0.4408$, $b_0 = 5.714$, we transform the equation (1) to the new form. For this sake, we introduce a group of parameters:

$$f_{k,n} = U^{k-1} U^{(k+n)}_{x^{(k)}t^{(n)}} z^{**k+n} \qquad (k,n = 0,1,2,..; k \vee n \neq 0)$$

$$\lambda_{k,n} = -\upsilon^{-1/2} U^k \upsilon_{w}{}^{(k+n)}_{x^{(k)}t^{(n)}} z^{**k+n+1/2} \qquad (k,n = 0,1,2...) \tag{4}$$

$$g_{k,n} = U^{k-1} N^{(k-1+n)}_{x^{(k-1)}t^{(n)}} z^{**k+n} \quad (k,n = 0,1,2,..; k \neq 0)$$

as new independent variables, where:

$$z^{**} = \delta^{**2}/\upsilon, \quad \delta^{**} = \left(a_0 \upsilon U^{-b_0} \int_0^x U^{b_0-1} dx \right)^{1/2} B,$$

$$B = \int_0^\infty \Phi_\eta \left(1 - \Phi_\eta\right) d\eta. \tag{5}$$

Now, the already transformed equation (1) being transformed to the new form:

$$B^2 \Phi_{\eta\eta\eta} + 0.5 \left[a_0 B^2 + (2 - b_0) f_{1,0}\right] \Phi \Phi_{\eta\eta} + f_{1,0} \left(1 - \Phi_\eta^2\right) +$$

$$(f_{0,1} + g_{1,0}) (1 - \Phi_\eta) + (0.5 \eta T^{**} + B\lambda_{0,0}) \Phi_{\eta\eta} =$$

$$\eta B^{-1} \Big(\sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} C_{k,n} B_{f_{k,n}} + \sum_{\substack{k=1 \\ n=0}}^{\infty} D_{k,n} B_{g_{k,n}} + \sum_{k,n=0}^{\infty} R_{k,n} B_{\lambda_{k,n}} \Big) \Phi_{\eta\eta} +$$

$$\Big\{ \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left[C_{k,n} \Phi_{\eta f_{k,n}} + A_{k,n} (\Phi_\eta \Phi_{\eta f_{k,n}} - \Phi_{f_{k,n}} \Phi_{\eta\eta})\right] + \tag{6}$$

$$\sum_{\substack{k=1 \\ n=0}}^{\infty} \left[D_{k,n} \Phi_{\eta g_{k,n}} + B_{k,n} (\Phi_\eta \Phi_{\eta g_{k,n}} - \Phi_{g_{k,n}} \Phi_{\eta\eta})\right] +$$

$$\sum_{k,n=0}^{\infty} \left[R_{k,n} \Phi_{\eta \lambda_{k,n}} + E_{k,n} (\Phi_\eta \Phi_{\eta \lambda_{k,n}} - \Phi_{\lambda_{k,n}} \Phi_{\eta\eta})\right] \Big\},$$

with corresponding boundary conditions:

$$\eta = 0 : \Phi = \Phi_\eta = 0; \eta \to \infty : \Phi_\eta \to 1;$$

$$f_{k,n} = \lambda_{k,n} = g_{k,n} = 0 : \Phi = \Phi_0(\eta), \tag{7}$$

where $\Phi_0(\eta)$ is Blasius's solution for the problem of flat plate. In the equation (6) the following notations have been used:

$$A_{k,n} = (k-1)f_{1,0}f_{k,n} + f_{k+1,n} + (k+n)f_{k,n}\overset{**}{F};$$

$$B_{k,n} = (k-1)f_{1,0}g_{k,n} + g_{k+1,n} + (k+n)g_{k,n}F^{**};$$

$$E_{k,n} = kf_{1,0}\lambda_{k,n} + \lambda_{k+1,n} + (k+n+0.5)\lambda_{k,n}\overset{**}{F};$$

$$C_{k,n} = (k-1)f_{0,1}f_{k,n} + f_{k,n+1} + (k+n)f_{k,n}\overset{**}{T}; \tag{8}$$

$$D_{k,n} = (k-1)f_{0,1}g_{k,n} + g_{k,n+1} + (k+n)g_{k,n}T^{**};$$

$$R_{k,n} = kf_{0,1}\lambda_{k,n} + \lambda_{k,n+1} + (k+n+0.5)\lambda_{k,n}\overset{**}{T};$$

$$\overset{**}{T} = z_t^{**};\ \overset{**}{F} = Uz_x^{**}.$$

In order to take the equation (6) universal, the multipliers $\overset{**}{F}$ and $\overset{**}{T}$ have to be expressed by means of quantities which are explicit functions only of parameters (4). In the determination of this functions, one can use the momentum and energy equations of the considered problem:

$$(U\delta^*)_t + (U^2\delta^{**})_x + U(U_x+N)\delta^* - Uv_w - \tau_w/\rho = 0;$$
$$(U^2\delta^{**})_t + U^3\delta^{**}_{1_x} + U^2(\delta^*_t + 3\delta^{**}_1 U_x + 2N\delta^{**} - 2ve) = 0 \tag{9}$$

where is:

$$\delta^* = L^{1/2}\int_0^\infty (1-\Phi_\eta)\,d\eta;\quad \tau_w = \rho v U^{b_0/2+1}L^{-1/2}(\Phi_{\eta\eta})_{\eta=0};$$

$$\delta_1^{**} = L^{1/2}\int_0^\infty \Phi_\eta\left(1-\Phi_\eta^2\right)d\eta;\quad e = L^{-1/2}\int_0^\infty \Phi_{\eta\eta}^2 d\eta; \tag{10}$$

$$L = a_0 v U^{-b_0}\int_0^x U^{b_0-1}dx$$

After certain transformations, the expressions for F^{**} and T^{**} have been obtained as universal, i.e. they do not depend on outer flow characteristics. In equation (6), the velocity at outer edge of the boundary layer and its derivatives, as well as ejection or injection fluid velocity and magnetic induction are not involved in explicit form, thus this equation can be called the generalized i.e. universal equation. The universal boundary conditions have the form as (7).

3 Approximative generalized similarity equation

The numerical integration of the equation (6), with the corresponding universal boundary conditions (7), can be performed "once and forever" only for its approximative form. It means, that the solution of universal equation in practice needs limitation of the number of the independent variables. It leads to the necessity of application of the "segment" method, in which all variables have to be set to be equal to zero. In such a way, the approximative universal equation is obtained. Having the above procedure in mind, the parameters $f_{1,0}$, $f_{0,1}$, $\lambda_{0,0}$ and $g_{1,0}$ will remain, while all others will be let to be equal to zero. Also, the derivatives with respect to the first parameters: porous $\lambda_{0,0}$ and magnetic $g_{1,0}$ will be considered as equal to zero. The equation (6) in these four parametric and twice localized approximation, has the form:

$$
\begin{gathered}
B^2\Phi_{\eta\eta\eta} + 0.5\left[a_0B^2 + (2-b_0)\,f_{1,0}\right]\Phi\Phi_{\eta\eta} + f_{1,0}\left(1-\Phi_\eta^2\right) + \\
\left(f_{0,1}+g_{1,0}\right)\left(1-\Phi_\eta\right) + \left(0.5\eta T^{**} + B\lambda_{0,0}\right)\Phi_{\eta\eta} = \\
\eta B^{-1}[T^{**}\left(f_{1,0}B_{f_{1,0}} + f_{0,1}B_{f_{0,1}}\right) - f_{0,1}^2 B_{f_{0,1}}]\Phi_{\eta\eta} + \\
[T^{**}(f_{1,0}\Phi_{\eta f_{1,0}} + f_{0,1}\Phi_{\eta f_{0,1}}) - f_{0,1}^2\Phi_{\eta f_{0,1}} + f_{1,0}F^{**}(\Phi_\eta\Phi_{\eta f_{1,0}} - \\
\Phi_{f_{1,0}}\Phi_{\eta\eta}) + f_{0,1}\left(F^{**}-f_{1,0}\right)\left(\Phi_\eta\Phi_{\eta f_{0,1}} - \Phi_{f_{0,1}}\Phi_{\eta\eta}\right)\right],
\end{gathered}
\tag{11}
$$

and the corresponding boundary conditions (7) are reduced to the following:

$$
\begin{gathered}
\eta = 0 : \Phi = \Phi_\eta = 0; \eta \to \infty : \Phi_\eta \to 1; \\
f_{1,0} = f_{0,1} = \lambda_{0,0} = g_{1,0} = 0 : \Phi = \Phi_0(\eta),
\end{gathered}
\tag{12}
$$

where the functions T^{**} and F^{**}, after same approximation have the following forms:

$$T^{**} = \{2[2(f_{1,0}H^{**}_{1_{f_{1,0}}} + f_{0,1}H^{**}_{1_{f_{0,1}}}) + H^{**}_1][(\zeta - 2f_{1,0} -$$
$$H^{**}(f_{1,0} + f_{0,1} + g_{1,0}) - \lambda_{0,0}) + f^2_{0,1}H^{**}_{f_{0,1}}] + 2[f_{0,1}(2-$$
$$f_{1,0}H^{**}_{1_{f_{0,1}}} - f_{0,1}H^{**}_{1_{f_{0,1}}}) + 6H^{**}_1 f_{1,0} + 4(g_{1,0} - \alpha)\}/\{[H^{**}+ \tag{13}$$
$$2(f_{1,0}H^{**}_{f_{1,0}} + f_{0,1}H^{**}_{f_{0,1}})][2(f_{1,0}H^{**}_{1_{f_{0,1}}} + f_{0,1}H^{**}_{1_{f_{0,1}}}) + H^{**}_1 - 1\}^{-1};$$
$$F^{**} = 2\{\zeta - 2f_{1,0} - H^{**}(f_{1,0} + f_{0,1} + g_{1,0} + 0.5T^{**}) -$$
$$\lambda_{0,0} + f_{0,1}H^{**}_{f_{0,1}}(f_{0,1} - T^{**}) - T^{**}f_{1,0}H^{**}_{f_{1,0}}\}$$

where is:

$$H^{**} = B^{-1}\int_0^\infty (1 - \Phi_\eta)\, d\eta = A/B;$$
$$H^{**}_1 = B^{-1}\int_0^\infty \Phi_\eta \left(1 - \Phi^2_\eta\right) d\eta; \tag{14}$$
$$\zeta = B\left(\Phi_{\eta\eta}\right)_{\eta=0}; \alpha = B\int_0^\infty \Phi^2_{\eta\eta} d\eta$$

The numerical integration of the equation (11) with boundary conditions (12) has been performed by means of the difference schemes and by using Tridiagonal algorithm method with iterations. The obtained results can be used in drawing about general conclusions of boundary layer development and in calculation of particular problems.

4 Unsteady boundary layer on porous aerofoil

Universal solutions of the equation (11) $\Phi''(0)$, A, B are used to calculate the characteristic properties of unsteady boundary layer on wing aerofoil whose center velocity changes with time as a degree function. Substituting nondimensional coordinates: $\tilde{x} = x/l$ and $\tilde{t} = \tilde{U}_\infty t/l$,

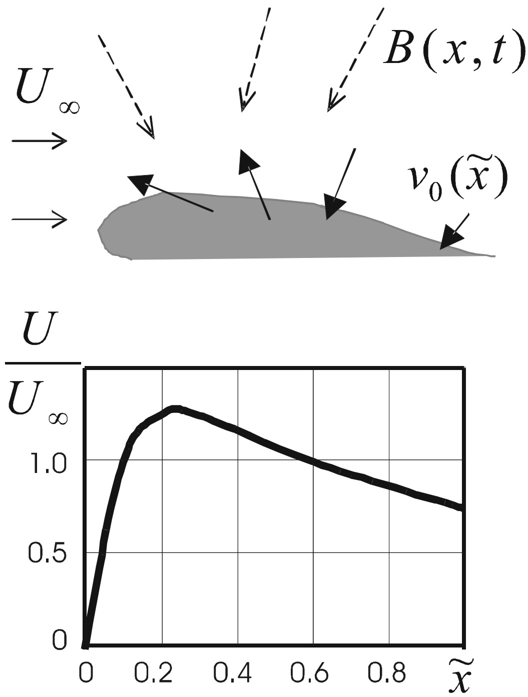

Figure 1: Potential velocity on aerofoil

where is l-chord and U_∞-endlessly velocity, nondimensional potential external velocity seems

$$\tilde{U}\left(\tilde{x},\tilde{t}\right)=\tilde{U}_1\left(\tilde{t}\right)\tilde{U}_2\left(\tilde{x}\right)=\left(\tilde{B}+\tilde{A}\,\tilde{t}^n\right)\tilde{U}_2\left(\tilde{x}\right) \tag{15}$$

with constant values for $\tilde{A}, \tilde{B}, n$.

The Figure 1. shows potential external velocity $\tilde{U}_2\left(\tilde{x}\right) = U/U_\infty$ on wing aerofoil measured by J.Stueper in free flight [11], where is lift coefficient $c_1 = 0.4$, Reynolds number $\mathrm{Re} = 4 \times 10^6$ and chord $l = 1800mm$. Substituting (14) in (4), (5) yields the following relations for the universal functions:

$$f_{1,0}/B^2 = a_0\tilde{U}^{-b_0}\tilde{U}_x Q; \qquad f_{0,1}/B^2 = a_0\tilde{U}^{-(b_0-1)}\tilde{U}_t Q;$$

$$\lambda_{0,0}/B = -\upsilon_w\left(a_0 Q/\upsilon\tilde{U}^{b_0}\right)^{1/2}; \quad g_{1,0}/B^2 = a_0 N\tilde{U}^{-b_0}Q; \qquad (16)$$

$$Q = \int_0^{\tilde{x}} \tilde{U}^{b_0-1}d\tilde{x}.$$

Using (10) and (13) the expression for the dimensionless skin friction $\tilde{\tau}_w$ has the form

$$\tilde{\tau}_w = 2\tau_w \overset{1/2}{\mathrm{Re}} / \left(\rho U_\infty^2\right) = 2\tilde{U}^{(b/2)+1}\left(a_0\int_0^{\tilde{x}}\tilde{U}^{b_0-1}d\tilde{x}\right)^{-1/2}\Phi''(0). \quad (17)$$

Now, we select a given set of the constants $\tilde{A}, \tilde{B}, n$ and for particular point on contour $\tilde{x}_0$ and time $\tilde{t}_0$ searching by (15) the obtained universal functions $(f_{1,0}/B^2)_0$, $(f_{0,1}/B^2)_0$, $(\lambda_{0,0}/B)_0$, $(g_{1,0}/B^2)_0$ concerning $[\Phi''(0)]_0$ for different values of porous parameter $\lambda_{0,0}$ and magnetic parameter $g_{1,0}$. Afterwards, using (16) one can determine $\tilde{\tau}_w$ distribution on contour. Preliminary calculations of expressions (15) and (16) have been made for a great accelerating fluid flow: $\tilde{A} = \tilde{B} = 1$; $n = 0.5$, 1.0; $\tilde{t} = 0.0$, 0.1, 0.2; $\lambda_{0,0} = 0.0$, ± 0.1, ± 0.2; $g_{1,0} = 0.0$; 0.1; 0.2. For all n values sufficient universal quantities could be found to cover the contour of wing aerofoil, and it means that $\tilde{\tau}_w$ can be calculated for all variations of great contour accelerating through the fluid. It was not the case in reference [10] in which there were no porous and magnetic parameter, and where the universal solutions are obtained using the approximative momentum and energy equation, so it was reason why we considered only very slow cylinder accelerating through the fluid.

5 Conclusions

It's found that for both in cofuser and in diffuser contour regions the accelerating flow ($\tilde{A} = 1$) increases the shear stress and postpones the separation of boundary layer i.e. laminar-turbulent transition section, and vice versa the decelerating flow reduces the shear stress and favors the separation of flow. It can be noted that the unsteady parameter has

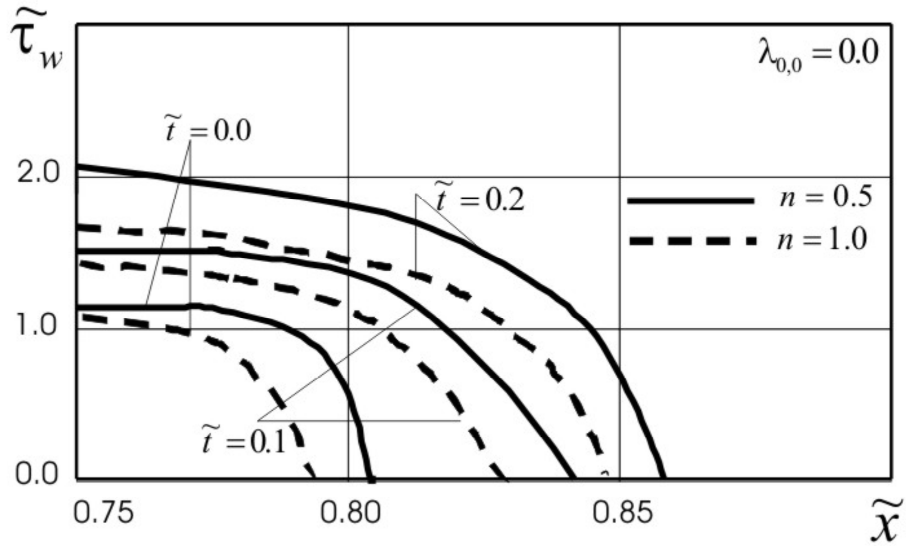

Figure 2: Shear stress distribution on unporous aerofoil

a significant influence on a shear stress distribution and especially on the laminar-turbulent transition location obtained by zero skin friction criteria. When this parameter is increasing ($\tilde{t} = 0.1; 0.2$) the shear stress magnitude is increasing on whole contour and the separation point is removing along the surface. It means, that the acceleration for $n = 1.0$ leads to the postponing of the boundary layer in diffuser region from 78.8% for steady flow i.e. $\tilde{t} = 0.0$ to the 82.9% of contour for $\tilde{t} = 0.1$ and to 84.1% of contour for $\tilde{t} = 0.2$.

Also for $n = 0.5$ the separation point is moving from $80,8\%$ of contour ($\tilde{t} = 0.0$) to 84.1% ($\tilde{t} = 0.1$) and to 85.8% of contour for $\tilde{t} = 0.2$, Fig.2, and everything what is said is when there are no fluid injection or ejection through the porous contour and when magnetic field is absent. It is important fact, because the achievement of laminar flow on $73.8\% - 85.2\%$ of contour in different time significantly reduce the contour drag.

For accelerating flows, the ejection of fluid increases the shear stress, especially in cofuser region about stagnation point, where shear stress is dramatically increased in time. It's not good for drag, so one can

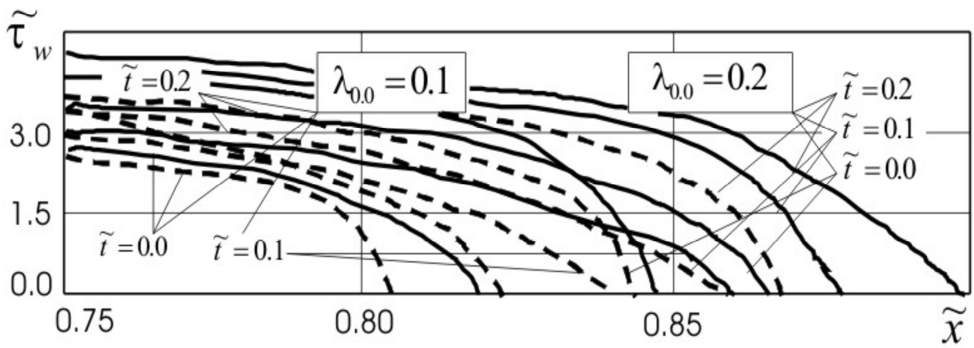

Figure 3: Shear stress distribution on porous aerofoil for fluid ejection ($\lambda_{0,0} = 0.1,\ 0.2;\ g_{1,0} = 0.0$)

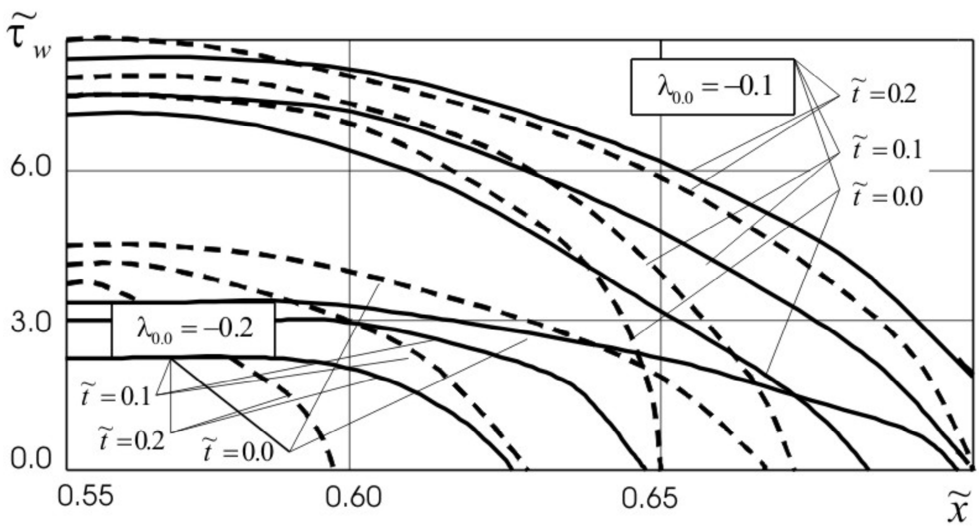

Figure 4: Shear stress distribution on porous aerofoil for fluid injection ($\lambda_{0,0} = -0.1,\ -0.2;\ g_{1,0} = 0.0$)

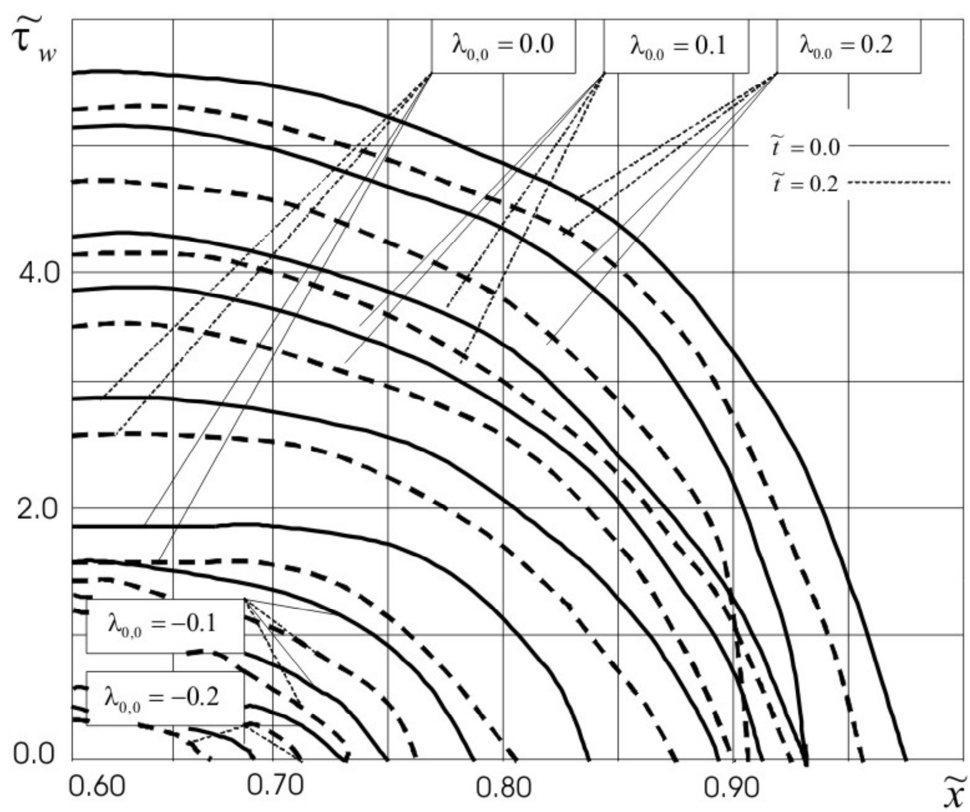

Figure 5: Magnetic field influence ($g_{1,0} = 0.1$) on shear stress distribution for fluid injection-ejection ($\lambda_{0,0} = 0.0; 0.1; 0.2; -0.1; -0.2$) through the porous contour

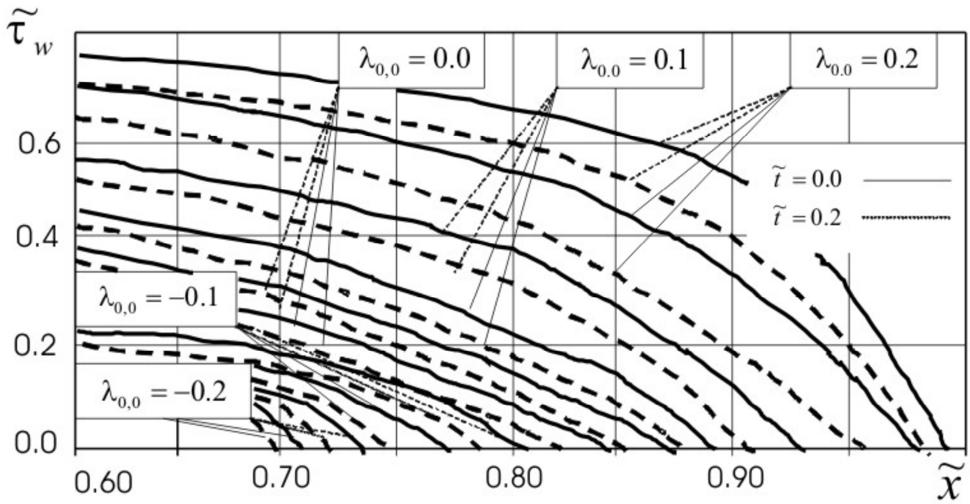

Figure 6: Shear stress distribution forfluid injection-ejection ($\lambda_{0,0} = 0.0; 0.1; 0.2; -0.1; -0.2$) and high magnetic field influence ($g_{1,0} = 0.2$)

control this great shear stress with fluid injection, when his value is noticeably reduction. Also, the ejection of fluid postpones the boundary layer separation, and vice versa the injection of fluid reduces the shear stress and favors the flow separation.

The fluid ejection, i.e. when the porous parameter is $\lambda_{0,0} = 0.1$, leads to the postponing of separation to 84.4% of contour for $\tilde{t} = 0.1$and for $n = 1.0$. For $n = 0.5$ in that time and for that porous parameter the separation point is on 86.1% of contour. For $\tilde{t} = 0.2$ and for this ejection, separation is on 86.2% of contour for $n = 1.0$ and on 87.3% for greater acceleration $n = 0.5$, Fig.2.

For the greater fluid ejection ($\lambda_{0,0} = 0.2$) the separation point is moving from 82.7% of contour for $\tilde{t} = 0.0$ and for $n = 1.0$, to 84.3% in $\tilde{t} = 0.1$ and to 85.9% for $\tilde{t} = 0.2$. For the greater acceleration ($n = 0.5$) an for same ejection ($\lambda_{0,0} = 0.2$) the separation point moves from 84.6% of contour ($\tilde{t} = 0.0$) to the greater contours values, 87.1% for $\tilde{t} = 0.1$ and 89.5% for $\tilde{t} = 0.2$, Fig.3.

Opposite, for a fluid injection ($\lambda_{0,0} = -0.1$) the separation is occurring at lower contour values, i.e. on 69.4% of contour for $n = 1.0$ and for $\tilde{t} = 0.0$ and moves toward stagnation point to 66.8% of contour for $\tilde{t} = 0.1$ and to 65.1% in $\tilde{t} = 0.2$. For the same fluid injection and for the greater acceleration ($n = 0.5$) the moving of separation point toward the stagnation point is slow, so from 73.2% in $\tilde{t} = 0.0$ the separation is on 70.2% for $\tilde{t} = 0.1$ and on 68.8% for $\tilde{t} = 0.2$, Fig.4.

For a greater fluid injection ($\lambda_{0,0} = -0.2$) the moving of separation point toward the stagnation point is faster in time than it was in previous case ($\lambda_{0,0} = -0.1$), so the separation for $n = 1.0$ is on 66.2% of contour for $\tilde{t} = 0.0$, 62.7% in $\tilde{t} = 0.1$ and 59.3% for $\tilde{t} = 0.2$, and for a greater acceleration ($n = 0.5$) there are: 69.8% ($\tilde{t} = 0.0$), 65.4% ($\tilde{t} = 0.1$), 63.1% ($\tilde{t} = 0.2$), Fig.4. On Figures 2., 3. and 4. are shown the cases when there is no the magnetic field, i.e. the magnetic parameter is $g_{1,0}$=0.0.

But when the magnetic field is perpendicular to the contour, and when the magnetic parameter $g_{1,0}$ is 0.1 (Fig.5) and 0.2 (Fig.6), one can see that for both fluid ejection and injection the shear stress increases and separation point moves to the downstream of contour.

An important advantage of the generalized similarity method demonstrated in this paper is that the skin friction and laminar-turbulent transition with separation point are found directly, no further numerical integration of momentum equation being involved as it was done in references [2, 3, 5, 11].

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Nestacionarni nestišljivi MHD granični sloj na poroznom aeroprofilu u visoko ubrzavanom strujnom toku

UDK 537.84

Fluid koji opstrujava čvrstu konturu je nestišljiv konstantne elektrokonduktivnosti. Prisutno magnetno polje je homogeno i normalno na površinu kroz koju se, normalno na nju, fluid ubrizgava u granični sloj, odnosno isisava iz sloja. U cilju proučavanja ovog problema, razvijena je poluparametarska metoda poznata kao metoda uopštene sličnosti. Odgovarajuće jednačine nestacionarnog magnetohidrodinamičkog graničnog sloja, uvođenjem svrsishodnih transformacija promenljivih, impulsne i energijske jednačine, kao i tri skupa parametara sličnosti, prelaze u univerzalni oblik. Numerička integracija ovako dobijene jednačine uopštene sličnosti sa početnim i graničnim uslovima u četvoro parametarskoj - dvaput lokalizovanoj aproksimaciji, urađena je koristeći metodu konačnih razlika tj. Tridiagonal Algorithm Method. Dobijena rešenja uopštene sličnosti su upotrijebljena za sračunavanje raspodjele trenja u laminarno-turbulentnoj tranziciji nestacionarnog graničnog sloja na poroznom visoko ubrzavajućem aeroprofilu. Pokazano je da i u konfuzorskoj, kao i u difuzorskoj oblasti konture, isisavanje fluida iz graničnog sloja odlaže njegovu separaciju, dok obrnuto, ubrizgavanje fluida kroz poroznu konturu u sloj favorizuje njegovo odvajanje. Magnetno polje povećava trenje na konturi i istovremeno odlaže pojavu laminarno-turbulentne tranzicije i u slučaju ubrizgavanja odnosno isisavanja fluida iz sloja.