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On bulk growth mechanics of solid-fluid mixtures: kinematics and invariance requirements

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Abstract

This work aims at investigating the possibility to account for the volumetric growth of a binary solid-fluid mixture, within the context of biomechanical perspectives in rational mixture theories.

Growth phenomena are coarsely taken into account by describing the time evolution of the solid stress-free configuration, whose introduction contributes a part of the constitutive information to the resulting dynamics, while enriching the kinematical description of the mixture. The issue of invariance requirements under changes in observer is also addressed, and some relevant constitutive implications are briefly outlined.

1 Introduction

Volumetric growth of living tissues, regarded as solid-fluid mixtures, generally occurs through cell division (or death), cell enlargement (or shrinkage), and secretion (or resorption) of extracellular matrix [20]. Whenever mass production and mass resorption are not simply due to local interconvertion, i.e. the growth of one constituent does not

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necessarily occur at the expense of the other, the algebraic sum of mass supply densities per unit volume of the mixture need not vanish. Some admissible constitutive prescriptions for these additional mass supply densities have been recently proposed, for instance, with the aim of describing the volumetric growth of both incompressible and compressible elastic biological materials (see e.g. Taber [20], Klisch and Van Dyke [14]), and the remodeling of living tissues made up of muscles, elastin and collagen (see e.g. Humphrey and Rajagopal [13]).

As the investigation of the historical development of the theory of porous media seems to point out (de Boer [7]), a mathematical theory of mixtures, enriched by the concept of volume fractions, also provides a suitable framework for the development of a consistent macroscopic theory of porous solids saturated with fluids (Bowen [2], Wilmanski [22]), which may be fruitfully employed to describe relevant aspects of bone mechanics (cf. Cowin [6]).

A solid-fluid mixture may be regarded as a couple of body manifolds, embedded into the three-dimensional Euclidean space, so as to share a smooth region of the physical environment while undertaking independent motions (see e.g. Atkin and Craine [1], Bowen [2], Rajagopal and Tao [18], Truesdell [21]). If a smooth region of the Euclidean space is chosen as a reference shape (which need not ever be occupied by any constituent), then a motion of the solid body may be described as a time sequence of mappings which carry it from the reference configuration to the current one. Similarly, the motion of the fluid constituent may be conceived as a time sequence of embeddings into the three-dimensional physical environment.

By virtue of this customary kinematical assumption, any place in the current shape of the mixture results to be simultaneously occupied by a material particle belonging to each constituent. Henceforth, the motion of the fluid-body manifold may be described by taking into account that any fluid particle may be naturally associated with a 1-parameter family of reference places, occupied by the solid particles currently overlapped with it. Accordingly, both the Eulerian fluid and solid velocity fields can be pulled back to the linear vector space associated with the reference shape of the solid constituent, and a referential description of relevant fluid properties can be furthermore considered [17].

Extending the pioneering proposal put forward by Rodriguez, Hoger and McCulloch [19] to binary solid-fluid mixtures, we regard the *bulk growth* of a soft tissue as the *time evolution of its stress-free configuration* [8, 9], described by a smooth (but geometrically noncompatible) tensor field on the reference configuration (section 2).

The issue of invariance requirements under changes in observer and its relevant constitutive implications are also addressed, regarding all stress-free configurations that differ by a rigid displacement as indistinguishable (section 3).

2 A growing solid infused with a fluid

Let us focus our attention on the kinematics of a binary mixture consisting of two smooth three-dimensional[1] material manifolds, $\mathcal{B}_S$ and $\mathcal{B}_F$. In order to avoid any possible confusion between particles belonging to each constituent (see fig.1), we refer to material points $\mathcal{X}_\alpha \in \mathcal{B}_\alpha$ as *α-points*, with $\alpha \in \{\mathrm{S,F}\}$, while calling the motion that they undertake *α-motion.*

2.1 S-motion

By assumption [16], there exists a smooth embedding of the body manifold $\mathcal{B}_S$ into the three-dimensional Euclidean space $\mathcal{E}$,

$$\begin{array}{rrcl} \mathcal{K}_S : & \mathcal{B}_S & \rightarrow & \mathcal{E}\,, \\ & \mathcal{X}_S & \mapsto & X \in \mathcal{K}_S\left(\mathcal{B}_S\right), \end{array}$$

which associates any material S-point with a reference place.

As the embedding $\mathcal{K}_S$ does not depend on time, a smooth motion of $\mathcal{B}_S$ may be regarded as a time sequence of mappings,

$$\begin{array}{rrcl} \boldsymbol{\chi}_S\left(\cdot,t\right) : & \mathcal{B} & \rightarrow & \mathcal{E}\,, \\ & X & \mapsto & x \in \boldsymbol{\chi}_S\left(\mathcal{B},t\right)\,, \end{array}$$

[1]We do not deal with Cantor dust (fluid drops or solid slivers) and fractals such as Menger sponges and Sierpinski gaskets.

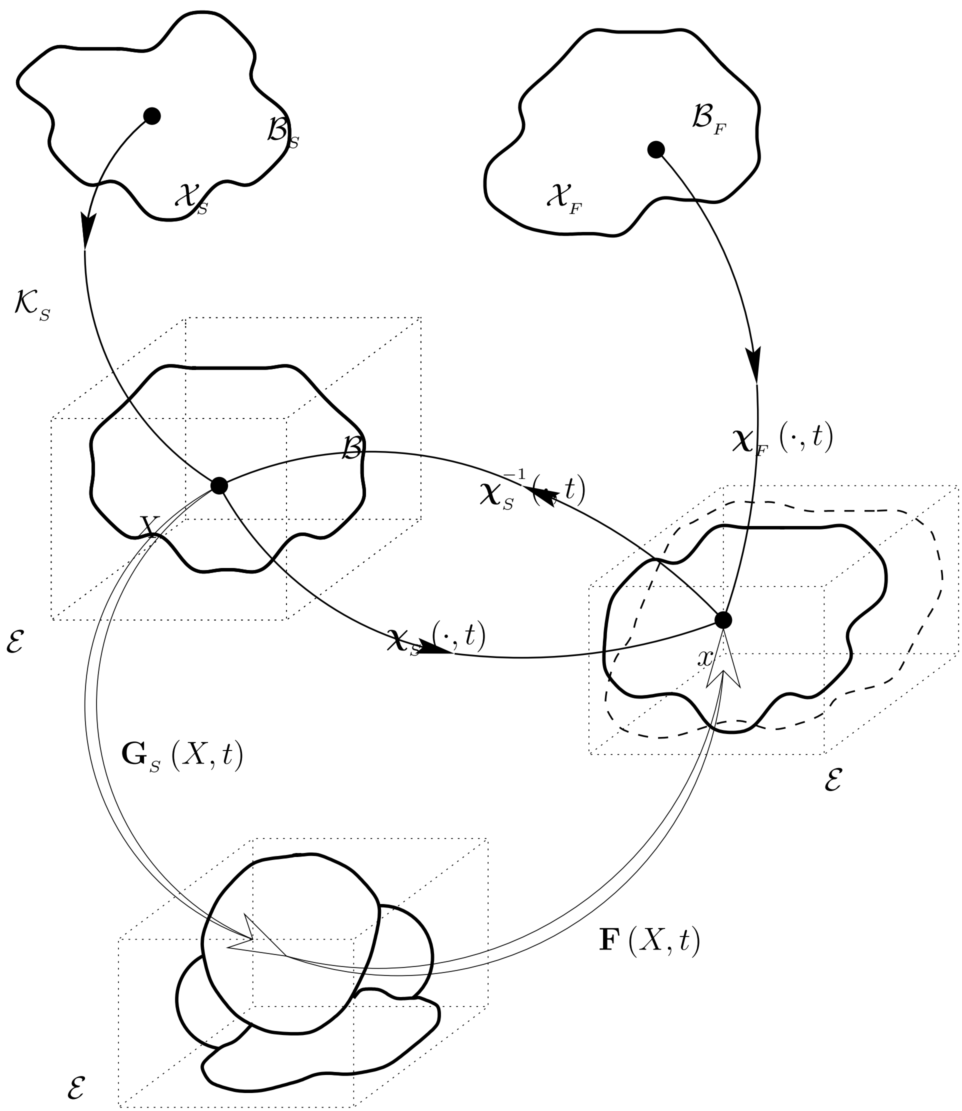

Figure 1: Kinematics of a growing solid infused with a fluid.

which carry the solid body from its reference shape, $\mathcal{B} \subset \mathcal{E}$, to its current shape, $\boldsymbol{\chi}_S(\mathcal{B}, t) \subset \mathcal{E}$. Consistently, at any given time t, the smooth tensor field on $\mathcal{B}$

$$\mathbf{F}_S := \operatorname{Grad} \boldsymbol{\chi}_S$$

linearly maps the set of all vectors tangent to $\mathcal{B}$ at X (denoted by $\mathrm{T}_X\mathcal{B}$ for the sake of conciseness), onto the set of free vectors of the Euclidean space $\mathcal{E}$ (denoted by $\mathrm{V}\mathcal{E}$):

$$\mathbf{F}_S(X, t) : \ \mathrm{T}_X\mathcal{B} \ \rightarrow \ \mathrm{V}\mathcal{E} .$$

Moreover, as we exclude a priori the possibility that a three-dimensional region of the reference shape can collapse under the motion $\boldsymbol{\chi}_S$ (namely, $\det \mathbf{F}_S > 0$), there exists a smooth inverse mapping,

$$\boldsymbol{\chi}_S^{-1}(\cdot, t) : \ \boldsymbol{\chi}_S(\mathcal{B}, t) \ \rightarrow \ \mathcal{E} ,$$

which satisfies the identity

$$X = \boldsymbol{\chi}_S^{-1}\big(\boldsymbol{\chi}_S(X, t), t\big) , \quad \text{for any} \quad X \in \mathcal{B} .$$

Since the reference shape of the solid constituent does not depend on time, the Eulerian velocity of any given S-point $\mathcal{X}_S \in \mathcal{B}_S$ can be obtained by taking the partial derivative of the motion $\boldsymbol{\chi}_S$ with respect to time,

$$\mathbf{v}_S(x, t) \ := \ \frac{\mathcal{D}^S}{\mathcal{D}t}\left[\boldsymbol{\chi}_S(\cdot, t) \circ \mathcal{K}_S\right](\mathcal{X}_S) \ = \ \left.\frac{\partial \boldsymbol{\chi}_S}{\partial t}\right|_X (X, t) ,$$

with $X = \mathcal{K}_S(\mathcal{X}_S)$ and $x = \boldsymbol{\chi}_S(X, t)$.

For the sake of brevity, we denote the material derivative following the S-motion by a superposed dot, i.e.

$$\dot{\boldsymbol{\chi}}_S(X, t) \ := \ \left.\frac{\partial \boldsymbol{\chi}_S}{\partial t}\right|_X (X, t) .$$

2.2 Bulk growth

Following the pioneering proposal put forward by Rodriguez, Hoger and McCulloch [19], we define the *bulk growth* of the solid constituent as the *time evolution of its stress-free configuration*, described by a smooth (but geometrically noncompatible) tensor field on $\mathcal{B}$, here denoted by $\mathbf{G}_S$ (fig.1).

The geometrical incompatibility of this additional descriptor (Lee [15], cf. Taber [20], Epstein and Maugin [9], Di Carlo and Quiligotti [8]) is mostly due to the fact that if, for any reference place $X \in \mathcal{B}$, we allowed the corresponding *body element* $\mathrm{T}_X\mathcal{B}$ to grow independently of neighbouring ones, in the absence of external applied load we would generally find out that body elements may no longer be geometrically compatible, after growing. Hence, in order to make them fit together again, it may be necessary to deform them [20], giving rise to a residual stress field whose existence in biological tissues has been experimentally observed and investigated (see, for instance, Fujie, Yamamoto et al. [10], Yasuda and Hayashi [23]).

In the light of former remarks, the *kinematical* descriptor $\mathbf{G}_S$ seems to have a peculiar twofold nature [8]: being closely related to the standard notion of stress, it contributes a relevant piece of constitutive information to the resulting dynamics, while enriching the kinematical description of the mixture.

We notice in passing that for any given reference place $X \in \mathcal{B}$ and tangent vector $\mathbf{u} \in \mathrm{T}_X\mathcal{B}$, the material derivatives of the corresponding vectors:

$$\begin{aligned} \mathbf{u}'(t) &:= \mathbf{G}_S(X,t)\;\mathbf{u} \;\in \mathrm{V}\mathcal{E} \\ \mathbf{u}''(t) &:= \mathbf{F}_S(X,t)\;\mathbf{u} = \mathbf{F}\,(X,t)\;\mathbf{u}'\,(t) \quad \in \mathrm{V}\mathcal{E}\,, \end{aligned}$$

following the motion of the material S-point associated with the reference place X, respectively result in the expressions (dropping the arguments for the sake of conciseness):

$$\begin{aligned} \dot{\mathbf{u}}' &= \dot{\mathbf{G}}_S\,\mathbf{u} = \left(\dot{\mathbf{G}}_S\mathbf{G}_S^{-1}\right)\mathbf{u}' \quad \in \mathrm{V}\mathcal{E} \\ \dot{\mathbf{u}}'' &= \dot{\mathbf{F}}_S\,\mathbf{u} = \left(\dot{\mathbf{F}}\,\mathbf{F}^{-1}\right)\mathbf{u}'' + \mathbf{F}\,\dot{\mathbf{u}}' \quad \in \mathrm{V}\mathcal{E}\,. \end{aligned}$$

2.3 F-motion

A smooth motion of the fluid constituent may similarly be described by a time sequence of embeddings,

$$\begin{aligned} \boldsymbol{\chi}_{F}(\cdot,t): \ \mathcal{B}_{F} &\to \mathcal{E} \\ \mathcal{X}_{F} &\mapsto x \in \boldsymbol{\chi}_{F}(\mathcal{B}_{F},t) \ , \end{aligned}$$

which map the body manifold $\mathcal{B}_{F}$ onto its current shape $\boldsymbol{\chi}_{F}(\mathcal{B}_{F},t) \subset \mathcal{E}$.

Consistently, the Eulerian velocity of any given fluid particle $\mathcal{X}_{F} \in \mathcal{B}_{F}$ is defined by the relation

$$\mathbf{v}_{F}(x,t) := \left.\frac{\partial \boldsymbol{\chi}_{F}}{\partial t}\right|_{\mathcal{X}_{F}} (\mathcal{X}_{F},t) \ , \quad \text{with} \quad x = \boldsymbol{\chi}_{F}(\mathcal{X}_{F},t) \ .$$

According to the classical theory of mixtures (see, for instance, Atkin and Craine [1], Bowen [2], Rajagopal and Tao [18], Truesdell [21]), any place in the current shape

$$\mathcal{B}_{t} \ := \ \left\{\boldsymbol{\chi}_{S}(\mathcal{K}_{S}(\mathcal{B}_{S}),t)\right\} \ \bigcap \ \left\{\boldsymbol{\chi}_{F}(\mathcal{B}_{F},t)\right\}$$

is simultaneously occupied by a material particle belonging to each constituent, $\mathcal{X}_{S} \in \mathcal{B}_{S}$ and $\mathcal{X}_{F} \in \mathcal{B}_{F}$, such that

$$x = \boldsymbol{\chi}_{S}(\mathcal{K}_{S}(\mathcal{X}_{S}),t) = \boldsymbol{\chi}_{F}(\mathcal{X}_{F},t) .$$

In order to deal with a description of the motion of F-points through the reference shape of the solid constituent, we notice that any F-point which belongs to the mixture at the given time t, namely

$$\mathcal{X}_{F} \in \boldsymbol{\chi}_{F}^{-1}(\mathcal{B}_{t},t) \ \subset \ \mathcal{B}_{F} \ ,$$

interacts with a 1-parameter family of S-points, moving along the curve

$$\boldsymbol{\chi}_{S}^{-1}\left(\boldsymbol{\chi}_{F}(\mathcal{X}_{F},\cdot),\cdot\right): \ t \mapsto X \ ,$$

at the velocity $\mathbf{w}_{F}(X,t)$, defined by

$$\mathbf{v}_{F}(x,t) \ = \ \mathbf{F}_{S}(X,t) \ \mathbf{w}_{F}(X,t) \ + \ \mathbf{v}_{S}(x,t) \ ,$$

with $x = \boldsymbol{\chi}_{F}(\mathcal{X}_{F},t) = \boldsymbol{\chi}_{S}(X,t) \in \mathcal{B}_{t}$.

3 Invariance requirements

With the aim of addressing the issue of invariance requirements, let us consider the action of the group of change in observers on the manifold of admissible motions as given by the relations:

$$\tilde{\boldsymbol{\chi}}_S(X,t) = \tilde{\boldsymbol{o}}(t) + \tilde{\mathbf{Q}}(t)\left(\boldsymbol{\chi}_S(X,t) - \boldsymbol{o}(t)\right), \quad \forall X \in \mathcal{B} \tag{1}$$

$$\tilde{\boldsymbol{\chi}}_F(\mathcal{X}_F,t) = \tilde{\boldsymbol{o}}(t) + \tilde{\mathbf{Q}}(t)\left(\boldsymbol{\chi}_F(\mathcal{X}_F,t) - \boldsymbol{o}(t)\right), \quad \forall \mathcal{X}_F \in \mathcal{B}_F \tag{2}$$

with (see, for instance, Casey [3], Casey and Naghdi [4, 5])

$$\tilde{\mathbf{F}}_S(X,t) = \tilde{\mathbf{Q}}(t)\ \mathbf{F}_S(X,t) \tag{3}$$

$$\tilde{\mathbf{F}}(X,t) = \tilde{\mathbf{Q}}(t)\ \mathbf{F}(X,t)\ \bar{\mathbf{Q}}^{\mathsf{T}}(t) \tag{4}$$

$$\tilde{\mathbf{G}}_S(X,t) = \bar{\mathbf{Q}}(t)\ \mathbf{G}_S(X,t)\ , \tag{5}$$

for all orthogonal tensor-valued functions of time $\tilde{\mathbf{Q}}(t), \bar{\mathbf{Q}}(t) \in \mathrm{Orth}$, such that $\tilde{\mathbf{F}}_S = \tilde{\mathbf{F}}\,\tilde{\mathbf{G}}_S$. These general invariance requirements, observed by Green and Naghdi [12] in the context of elastic-plastic deformation at finite strain, are intended to formalize the idea that two stress-free configurations differing by a rigid displacement are indistinguishable, and no physical argument can be invoked to support the choice of one of them rather than another.

By taking the partial time derivative of relations (1)-(2), we can straightforwardly deduce that

$$\tilde{\mathbf{v}}_S(\tilde{x},t) = \boldsymbol{\omega}(t) + \tilde{\boldsymbol{\Omega}}(t)(\tilde{x} - \tilde{\boldsymbol{o}}(t)) + \tilde{\mathbf{Q}}(t)\ \mathbf{v}_S(x,t) \tag{6}$$

$$\tilde{\mathbf{v}}_F(\tilde{x},t) = \boldsymbol{\omega}(t) + \tilde{\boldsymbol{\Omega}}(t)(\tilde{x} - \tilde{\boldsymbol{o}}(t)) + \tilde{\mathbf{Q}}(t)\ \mathbf{v}_F(x,t)\ , \tag{7}$$

with

$$\begin{aligned} \tilde{x} &:= \tilde{\boldsymbol{\chi}}_S(X,t) = \tilde{\boldsymbol{\chi}}_F(\mathcal{X}_F,t) \\ x &:= \boldsymbol{\chi}_S(X,t) = \boldsymbol{\chi}_F(\mathcal{X}_F,t) \\ \boldsymbol{\omega}(t) &:= \dot{\tilde{\boldsymbol{o}}}(t) - \tilde{\mathbf{Q}}(t)\,\dot{\boldsymbol{o}}(t) \\ \tilde{\boldsymbol{\Omega}}(t) &:= \dot{\tilde{\mathbf{Q}}}(t)\,\tilde{\mathbf{Q}}^{\mathsf{T}}(t) \ \in \ \mathrm{Skw}. \end{aligned}$$

Finally, we notice that the time derivatives of expressions (3)-(5) yield:

$$\dot{\tilde{\mathbf{F}}}_{\mathbf{s}}\,\tilde{\mathbf{F}}_{\mathbf{s}}^{-1} \;=\; \tilde{\mathbf{\Omega}} + \tilde{\mathbf{Q}}\left\{\dot{\mathbf{F}}_{\mathbf{s}}\,\mathbf{F}_{\mathbf{s}}^{-1}\right\}\tilde{\mathbf{Q}}^{\mathsf{T}}, \quad \tilde{\mathbf{\Omega}} := \dot{\tilde{\mathbf{Q}}}\tilde{\mathbf{Q}}^{\mathsf{T}} \in \mathrm{Skw} \tag{8}$$

$$\dot{\tilde{\mathbf{G}}}_{\mathbf{s}}\tilde{\mathbf{G}}_{\mathbf{s}}^{-1} \;=\; \bar{\mathbf{\Omega}} + \bar{\mathbf{Q}}\left\{\dot{\mathbf{G}}_{\mathbf{s}}\mathbf{G}_{\mathbf{s}}^{-1}\right\}\bar{\mathbf{Q}}^{\mathsf{T}}, \quad \bar{\mathbf{\Omega}} := \dot{\bar{\mathbf{Q}}}\bar{\mathbf{Q}}^{\mathsf{T}} \in \mathrm{Skw} \tag{9}$$

$$\dot{\tilde{\mathbf{F}}}\,\tilde{\mathbf{F}}^{-1} \;=\; \tilde{\mathbf{\Omega}} + \tilde{\mathbf{Q}}\left\{\dot{\mathbf{F}}\,\mathbf{F}^{-1}\right\}\tilde{\mathbf{Q}}^{\mathsf{T}} - \tilde{\mathbf{F}}\,\bar{\mathbf{\Omega}}\,\tilde{\mathbf{F}}^{-1},$$

whereas

$$\left.\frac{\partial\tilde{\mathbf{v}}_F}{\partial\tilde{x}}\right|_t(\tilde{x},t) \;=\; \tilde{\mathbf{\Omega}}(t) + \tilde{\mathbf{Q}}(t)\left\{\left.\frac{\partial\mathbf{v}_F}{\partial x}\right|_t(x,t)\right\}\tilde{\mathbf{Q}}^{\mathsf{T}}(t)\;. \tag{10}$$

The proposed extension of the action of the group of changes in observer (1)-(9) yields some remarkable selection rules on the set of available constitutive prescriptions for internal (generalized) forces. For instance,[2] we may assume the internal power density per unit current volume of the mixture, expended on any set of test velocity fields ($\hat{\mathbf{v}}_S$, $\hat{\mathbf{v}}_F$, $\hat{\mathbf{V}}_S$), to be given by the expression

$$\boldsymbol{\tau}_S\cdot\hat{\mathbf{v}}_S + \boldsymbol{\tau}_F\cdot\hat{\mathbf{v}}_F + \boldsymbol{\sigma}_S\cdot\mathrm{grad}\,\hat{\mathbf{v}}_S + \boldsymbol{\sigma}_F\cdot\mathrm{grad}\,\hat{\mathbf{v}}_F + \boldsymbol{\pi}_S\cdot\hat{\mathbf{V}}_S\,, \tag{11}$$

which is required to be invariant under superposed rigid-body velocity fields[3] [11]

$$\mathbf{v}^R_S(x,t) \;:=\; \mathbf{v}^R_F(x,t) \;:=\; \boldsymbol{\omega}(t) + \tilde{\mathbf{\Omega}}(t)\,(x - o(t))$$

$$\mathrm{grad}\,\mathbf{v}^R_S(x,t) \;=\; \mathrm{grad}\,\mathbf{v}^R_F(x,t) \;=\; \tilde{\mathbf{\Omega}}(t) \tag{12}$$

$$\mathbf{V}^R_S(x,t) \;:=\; \bar{\mathbf{\Omega}}(t)\;. \tag{13}$$

As a consequence, it is possible to deduce that

$$\boldsymbol{\tau}_S + \boldsymbol{\tau}_F \;=\; \mathbf{0}$$

$$\mathrm{skw}\,(\boldsymbol{\sigma}_S + \boldsymbol{\sigma}_F) \;=\; \mathbf{O} \tag{14}$$

$$\mathrm{skw}\,(\boldsymbol{\pi}_S) \;=\; \mathbf{O}\,, \tag{15}$$

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[2] Compare Quiligotti et al. [17] with Di Carlo et al. [8].

[3] We focus attention on two synchronized moving observers, whose frames coincide at the given time t (namely, $\tilde{\mathbf{Q}}(t) = \bar{\mathbf{Q}}(t) = \mathbf{I}$, and $\tilde{x} = x$ in expressions (6)-(7) and (8)-(10).

where the dynamical descriptors $\boldsymbol{\sigma}_\alpha$ and $\boldsymbol{\tau}_\alpha$ represent, respectively, the peculiar Cauchy stress tensor and the zeroth-order interaction associated with the α-th constituent of the mixture [17], while the descriptor $\boldsymbol{\pi}_S$ expends power on the evolution of the stress-free configuration of the solid constituent [8].

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O mehanici rasta u mešavinama čvrsto telo-fluid: kinematika i zahtevi invarijantnosti

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Cilj ovog rada je istraživanje mogućnosti uzimanja u obzir zapreminskog rasta binarne mešavine čvrsto telo-fluid u kontekstu biomehaničkih perspektiva racionalnih teorija mešavina.

Fenomeni rasta se grubo uzimaju u obzir opisivanjem vremenske evolucije beznaponske konfiguracije čvrstof tela, čije uvodjenje učestvuje u delu konstitutivne informacije na rezultujuću dinamiku, pri čemu obogaćuje kinematski opis mešavine. Problem zahteva invarijantnosti pri promeni posmatrača je takodje razmatran, pa su neke relevantne konstitutivne implikacije kratko izložene.