ON THE APPLICATION OF FRACTIONAL CALCULUS IN MECHANICS

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Abstract:

In this reviewing paper some of the basic applications of fractional calculus in mechanics are elucidated. In addition, the underlying mathematical background and some historical remarks are given. The point in the paper is on the application of fractional calculus in viscoelasticity, i.e. on the use of four- parameter generalized Zener model in exploring viscoelastic properties of various materials. Also, it is shown in the paper that fractional derivatives can be successfully used in modeling some of the boundary conditions in fluid mechanics, in particular in rarefied gas dynamics, in which, as shown, the entire range of variations of the Knudsen number can be covered by a single fractional slip boundary condition. As a contribution to the mathematical theory of fractional nonlinear differential equations we show that some of them, like heat conduction equation, and a combination of Burgers and Korteweg-de Vries equation, possess similarity solutions. We present some exact and numerical solutions of the resulting nonlinear fractional ordinary differential equations.

Key Words: fractional calculus, viscoelasticity, generalized Zener model, rarefied gas dynamics, slip boundary conditions. Similarity solutions.

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