A GENERALIZED RITZ METHOD FOR PARTIAL DIFFERENTIAL EQUATIONS IN DOMAINS OF ARBITRARY GEOMETRY USING GLOBAL SHAPE FUNCTIONS

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ABSTRACT: A new BEM based meshless variational method is presented for the solution of elliptic PDEs describing the mechanical response of general inhomogeneous anisotropic bodies of arbitrary geometry. The equations, which in general have variable coefficients, may be linear or nonlinear. Using the concept of the *analog equation* of Katsikadelis [1,2], the original equation is converted into a linear membrane (Poisson) or a linear plate (biharmonic) equation, depending on the order of the PDE under a fictitious load, which is approximated with radial basis function series of multiquadric (MQ) type. The integral representation of the solution of the substitute equation yields shape functions, which are global and satisfy both essential and natural boundary conditions, hence the name *generalized Ritz method*. The minimization of functional that produces the PDE as the Euler Lagrange equation yields not only the Ritz coefficients but also permits the evaluation of optimal values for the shape parameters of the MQs as well as optimal position of the interior collocation points, minimizing thus the error. In case that a functional does not exists or can not be constructed as it is the usual case of nonlinear PDEs, the Galerkin principle can be applied. Since the arising domain integrals are converted into boundary line integrals, the method is boundary-only and, therefore, it maintains all the advantages of the pure BEM. Example engineering problems are studied, which illustrate the method and demonstrate its efficiency and great accuracy.

Key words : Boundary element method, meshless, partial differential equations, analog equation, Ritz method, variational, optimal multiquadrics, global shape functions, Ritz

References

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